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ON WEAKLY SOFT SOMEWHAT OPEN SETS

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ABSTRACT. It is well known that certain topological concepts such as the sum of topologies and the product of topologies are defined by particular classes of other topologies. In such topologies and others, it is crucial to look into the connections between these classes and the topology generated by them. One of the topologies that creates a family of general topologies is soft topology, which is defined over a family of soft sets satisfying the basic axioms of general topology. It is interesting to investigate how topological notions behave when applied to soft topologies because it enables us to study topological qualities by comparing them to their counterparts in other topologies. In this paper, we benefit from the fruitful variety via soft topology to introduce the concept of weakly soft somewhat open sets (briefly, weakly soft *so*-sets), which is constructed using its corresponding notion via parametric topologies. We demonstrate that the class of soft somewhat open sets is a stronger generalization of soft open sets than this type, and we also offer a condition that guarantees their equality. Then, we present new soft operators and discuss their essential characteristics by utilizing the notion of weakly soft *so*-sets and their complements. One of the unique characteristics that we prove of these operators is that the *so*-interior of a soft subset is the soft subset itself or the null soft set, whereas *so*-closure of a soft subset is the soft subset itself or the absolute soft set. Finally, we provide new categories of soft mappings and explore their main characteristics. We provide a few examples and counterexamples to clarify the findings and relationships obtained herein.

1. Introduction

Molodtsov [31] presented the idea of soft sets as a brand-new mathematical strategy to deal with uncertainty in 1999. He looked into its benefits over fuzzy sets and demonstrated how it could be used in various fields. Then, Maji et al. [28] used this strategy to address decision-making problems. Additionally, they [29] introduced a few operators and operations between soft sets that were modified and expanded in a number of published papers [4, 13, 35].

Soft sets were used to structure the idea of soft topology in 2011 by Shabir and Naz [36] and Çağman et al. [19]. Regarding whether or not a set of parameters should be chosen as a constant, their definitions differ. Following Shabir and Naz's methodology, which imposed the requirement of a constant set of parameters, we proceed as follows. The space of soft topology was then explored by various researchers using topological notions and concepts. For example, Min [30] identified additional characteristics of soft separation axioms, while El-Shafei et al. [23] examined these axioms in regard to the relations of total belonging and non-belonging. An interesting application of some types of soft separation axioms to select the optimal program of tourism was presented by Al-shami [11]. Compact spaces were first introduced by Aygünoğlu and Aygün [18] and later another kind of soft compact

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spaces were described by Hida in [25]. Recently, the concepts of compact and Lindelöf spaces have been established via soft regular closed [5] and soft somewhat open sets [14]. According to Al-shami [7] some of the features and results provided in the frame of soft topologies of compactness and separation axioms need refinement. Al-shami [8] successfully applied soft compactness on ordered settings to expect the missing values on the information systems. Turanlı et al. introduced soft paracompactness via soft ideals [37]. Some authors in [17, 38] explored soft connectedness and its expansions. According to Al-shami and Kočinac [15], who established the correspondence between enriched and extended soft topologies and demonstrated that these types of soft topologies guarantee that many topological properties are interchangeable between soft and parametric topologies. Kharal and Ahmad [26] introduced soft mapping, and Al-shami [9] updated it using soft points. In [39] soft continuity and its main characteristics were examined. Demir [21] studied the concepts of vietoris topology via soft settings. Some results related to fixed soft point theorems were provided in [10, 22]. Kočinac et al. [27] studied Menger spaces in the context of soft topologies. Recently, Rawshdeh et al. [33] have introduced a new sort of spaces called soft expandable spaces.

Generalizations of soft open sets are another intriguing topic via soft topologies. All of the topological concepts have been reproduced using these generalizations. Chen [20] familiarized the concept of soft semi-open sets and explored its basic features. Akdag and Ozkan [1] presented the concept of soft α-open sets using the interior and closure operators in the same method of classical α-open sets. Al-shami [6] put forward the concept of somewhere dense sets, it was then used to introduce new types of soft compactness and soft continuity. Al-Ghour studied the concepts of soft Q-sets [2] and minimal soft sets [3]. Recently, Ameen et al. [16] have provided several soft functions motivated by the family of soft somewhat open sets, which were exploited to initiate some classes of separation axioms by Al-shami [12].

This work is displayed as follows. Following this introduction, we review the definitions and conclusions required to comprehend the information in Section 2. As a new extension of soft open sets, weakly soft *so*-sets are defined and their behavior is illustrated in Section 3. The interior, closure, boundary, and limit soft points described with regard to weakly soft *so*-sets and weakly soft *sc*-sets are shown in Section 4. The last main part is Section 5 which introduces and discusses new sorts of soft continuity, irresolute, openness, closeness, and homeomorphism. Finally, in Section 6, we summarize the main contributions and offer some suggestions for the future.

2. Preliminaries

Definition 2.1. [31] *An ordinary mapping λ from nonempty set of parameters $\mathscr{P}$ to the power set $2^{\mathscr{Y}}$ of the universal set of objects $\mathscr{Y}$ is called a "soft set". It is denoted by the pair $(\lambda,\mathscr{P})$ and one way of writing is $(\lambda,\mathscr{P})=\{(p,\lambda(p)) : p\in\mathscr{P}$ and $\lambda(p)\in 2^{\mathscr{Y}}\}$. We call $\lambda(p)$ a p-component of $(\lambda,\mathscr{P})$. The family of all soft sets over $\mathscr{Y}$ with a set of parameters $\mathscr{P}$ is denoted by $2^{\mathscr{Y}_{\mathscr{P}}}$*

Through this manuscript, $(\lambda,\mathscr{P}),(\delta,\mathscr{P})$ denote soft sets over $\mathscr{Y}$.

Definition 2.2. [29, 32] *A soft set $(\lambda,\mathscr{P})$ is called:*

(i): *absolute, symbolized by $\widetilde{\mathscr{Y}}$, if $\lambda(p)=\mathscr{Y}$ for all $p\in\mathscr{P}$.*
(ii): *null, symbolized by ϕ, if $\lambda(p)=\emptyset$ for all $p\in\mathscr{P}$.*

(iii): *a soft point if there are $p \in \mathscr{P}$ and $y \in \mathscr{Y}$ with $\lambda(p) = \{y\}$ and $\lambda(q) = \emptyset$ for all $q \in \mathscr{P} - \{p\}$. A soft point is briefly denoted by y_p. We write $y_p \in (\lambda, \mathscr{P})$ if $y \in \lambda(p)$.*

(iv): *pseudo constant if $\lambda(p) = \mathscr{Y}$ or $\emptyset$ for all $p \in \mathscr{P}$.*

Definition 2.3. [24] *We call $(\lambda, \mathscr{P})$ a soft subset of $(\delta, \mathscr{P})$ (or $(\delta, \mathscr{P})$ a soft superset of $(\delta, \mathscr{P})$), symbolized by $(\lambda, \mathscr{P}) \widetilde{\subseteq} (\delta, \mathscr{P})$ if $\lambda(p) \subseteq \delta(p)$ for each $p \in \mathscr{P}$.*

Definition 2.4. [4] *If $\delta(p) = \mathscr{Y} - \lambda(p)$ for all $p \in \mathscr{P}$, the we call $(\delta, \mathscr{P})$ a complement of $(\lambda, \mathscr{P})$. The complement of $(\lambda, \mathscr{P})$ is symbolized by $(\lambda, \mathscr{P})^c = (\lambda^c, \mathscr{P})$.*

Definition 2.5. *Let $(\lambda, \mathscr{P})$ and $(\delta, \mathscr{P})$ be soft sets. Then:*

(i): *$(\lambda, \mathscr{P}) \widetilde{\bigcup} (\delta, \mathscr{P}) = (\zeta, \mathscr{P})$, where $\zeta(p) = \lambda(p) \bigcup \delta(p)$ for all $p \in \mathscr{P}$* [29].

(ii): *$(\lambda, \mathscr{P}) \widetilde{\bigcap} (\delta, \mathscr{P}) = (\zeta, \mathscr{P})$, where $\zeta(p) = \lambda(p) \bigcap \delta(p)$ for all $p \in \mathscr{P}$* [4].

(iii): *$(\lambda, \mathscr{P}) \setminus (\delta, \mathscr{P}) = (\zeta, \mathscr{P})$, where $\zeta(p) = \lambda(p) \setminus \delta(p)$ for all $p \in \mathscr{P}$* [4].

(iv): *$(\lambda, \mathscr{P}) \times (\delta, \mathscr{P}) = (\zeta, \mathscr{P})$, where $\zeta(p_1, p_2) = \lambda(p_1) \times \delta(p_2)$ for all $(p_1, p_2) \in \mathscr{P} \times \mathscr{P}$* [18].

Definition 2.6. [23] *Let $(\lambda, \mathscr{P})$ be a soft set and $x \in \mathscr{Y}$. Then:*

(i): *$x \in (\lambda, \mathscr{P})$ if $x \in \lambda(p)$ for all $p \in \mathscr{P}$.*

(ii): *$x \Subset (\lambda, \mathscr{P})$ if $x \in \lambda(p)$ for some $p \in \mathscr{P}$.*

The adjusted version of the definition of soft mappings is given in the following.

Definition 2.7. [9] *Let $h : \mathscr{Y} \to \mathscr{Z}$ and $\pi : \mathscr{P} \to \mathscr{Q}$ be crisp mappings. A soft mapping h_π of $2^{\mathscr{Y}_{\mathscr{P}}}$ into $2^{\mathscr{Z}_{\mathscr{Q}}}$ is a relation such that each $y_p \in 2^{\mathscr{Y}_{\mathscr{P}}}$ is related to one and only one $z_q \in 2^{\mathscr{Z}_{\mathscr{Q}}}$ such that*

$$h_\pi(y_p) = h(y)_{\pi(p)} \text{ for all } y_p \in 2^{\mathscr{Y}_{\mathscr{P}}}.$$

In addition, $h_\pi^{-1}(z_q) = \widetilde{\bigcup\limits_{\substack{y \in h^{-1}(z) \\ p \in \pi^{-1}(q)}}} y_p$ *for each* $z_q \in 2^{\mathscr{Z}_{\mathscr{Q}}}$.

A soft mapping is described as surjective (resp., injective, bijective) if its two crisp mappings satisfy this description.

Proposition 2.8. [26] *Let $h_\pi : 2^{\mathscr{Y}_{\mathscr{P}}} \to 2^{\mathscr{Z}_{\mathscr{Q}}}$ be a soft mapping and let $(\lambda, \mathscr{P})$ and $(\delta, \mathscr{P})$ be soft subsets of $\widetilde{\mathscr{Y}}$ and $\widetilde{\mathscr{Z}}$, respectively. Then*

(i): *$(\lambda, \mathscr{P}) \widetilde{\subseteq} h_\pi^{-1}(h_\pi(\lambda, \mathscr{P}))$.*

(ii): *If h_π is injective, then $(\lambda, \mathscr{P}) = h_\pi^{-1}(h_\pi(\lambda, \mathscr{P}))$.*

(iii): *$h_\pi(h_\pi^{-1}(\delta, \mathscr{Q})) \widetilde{\subseteq} (\delta, \mathscr{Q})$.*

(iv): *If h_π is surjective, then $h_\pi(h_\pi^{-1}(\delta, \mathscr{Q})) = (\delta, \mathscr{Q})$*

Definition 2.9. [36] *A family $\mathscr{T}$ of soft sets defined over a universal set $\mathscr{Y}$ with a parameters set $\mathscr{P}$ which contains absolute soft set $\widetilde{\mathscr{Y}}$ and null soft set ϕ is said to be a soft topology on $\mathscr{Y}$ provided that it is closed under arbitrary soft union and finite soft intersection.*

Then $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$ is called a soft topological space (briefly, soft$_{TS}$). Each member in $\mathscr{T}$ is called soft open and its complement is called soft closed.

Definition 2.10. [14] *A soft$_{TS}$ $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is called full provided that every non-null soft open set has no empty component.*

Proposition 2.11. [36] *Let $(\mathscr{Y},\mathscr{T},\mathscr{P})$ be a soft$_{TS}$. Then $\mathscr{T}_p = \{\lambda(p) : (\lambda,\mathscr{P}) \in \mathscr{T}\}$ defines a classical topology on $\mathscr{Y}$ for each $p \in \mathscr{P}$. This topology is called a parametric topology.*

Definition 2.12. *(see,[15]) Let $(\lambda,\mathscr{P})$ be a soft subset of a soft$_{TS}$ $(\mathscr{Y},\mathscr{T},\mathscr{P})$. Then $(int(\lambda),\mathscr{P})$ and $(cl(\lambda),\mathscr{P})$ are respectively defined by $int(\lambda)(p) = int(\lambda(p))$ and $cl(\lambda)(p) = cl(\lambda(p))$, where $int(\lambda(p))$ and $cl(\lambda(p))$ are respectively the interior and closure of $\lambda(p)$ in $(\mathscr{Y},\mathscr{T}_p)$.*

Definition 2.13. [18, 34] *Let $(\mathscr{Y},\mathscr{T},\mathscr{P})$ be a soft$_{TS}$. Then $\mathscr{T}$ is said to be:*

(i): *an enriched soft topology provided that $\mathscr{T}$ contains all pseudo constant soft sets.*
(ii): *an extended soft topology provided that $(\lambda,\mathscr{P}) \in \mathscr{T}$ iff $\lambda(p) \in \mathscr{T}_p$ for each $p \in \mathscr{P}$.*

It was proved in [15] the identity between enriched soft topology and extended soft topology. To unite terminology, we call this type of soft topology an extended soft topology, and call $(\mathscr{Y},\mathscr{T},\mathscr{P})$ an extended soft$_{TS}$. The next theorem is a key point to discovering the behaviours of soft topological concepts and keeping them via classical and soft topologies.

Theorem 2.14. [15] *A soft subset of a soft$_{TS}$ $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is extended iff $(int(\lambda),\mathscr{P}) = int(\lambda,\mathscr{P})$ and $(cl(\lambda),\mathscr{P}) = cl(\lambda,\mathscr{P})$ for any soft subset $(\lambda,\mathscr{P})$.*

Definition 2.15. [16] *A soft subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is said to be soft somewhat open (briefly, soft sw-open) if $(\lambda,\mathscr{P}) = \phi$ or $int(\lambda,\mathscr{P}) \neq \phi$. The complement of a soft sw-open set is said to be soft somewhat closed (briefly, soft sw-closed).*

Theorem 2.16. [12] *A subset $(\lambda,\mathscr{P})$ of an extended soft$_{TS}$ $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is soft sw-open (resp., soft sw-closed) iff there is an sw-open (resp., sw-closed) subset G of $(E,\mathscr{T}_p)$ with $\lambda(p) = G$.*

Definition 2.17. [26] *A soft mapping $h_\pi : (\mathscr{Y},\mathscr{T}_{\mathscr{Y}},\mathscr{P}) \to (\mathscr{Z},\mathscr{T}_{\mathscr{Z}},\mathscr{P})$ is said to be soft continuous if $h_\pi^{-1}(\lambda,\mathscr{P})$ is a soft open set where $(\lambda,\mathscr{P})$ is soft open.*

Theorem 2.18. [15] *If $h_\pi : (\mathscr{Y},\mathscr{T},\mathscr{P}) \to (\mathscr{Z},\mathscr{S},\mathscr{Q})$ is soft continuous, then $h : (\mathscr{Y},\mathscr{T}_p) \to (\mathscr{Z},\mathscr{S}_{\pi(p)})$ is continuous for each $p \in \mathscr{P}$.*

3. Weakly soft *so*-sets and main features

In this section, we present the notion of weakly soft *so*-sets and study some basic properties of it. Elucidative examples and counterexamples are introduced.

Definition 3.1. *A soft subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is called a weakly soft so-set if $(\lambda,\mathscr{P}) = \phi$ or there is a component of it which is a nonempty sw-open set. That is, $\lambda(p) = \emptyset$ for all $p \in \mathscr{P}$ or $int(\lambda(p)) \neq \emptyset$ for some $p \in \mathscr{P}$. The complement of a weakly soft so-set is called a weakly soft sc-set.*

Note that there is a soft set which is neither a weakly soft *so*-subset or a weakly soft *sc*-subset.

Proposition 3.2. *A subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is a weakly soft sc-set iff $(\lambda,\mathscr{P}) = \widetilde{\mathscr{Y}}$ or $cl(\lambda(p)) \neq \mathscr{Y}$ for some $p \in \mathscr{P}$.*

Proof. "$\Rightarrow$": Let $(\lambda,\mathscr{P})$ be a weakly soft *sc*-set. Then, $(\lambda^c,\mathscr{P})=\phi$ or $int[(\lambda(p))^c]\neq\emptyset$ for some $p\in\mathscr{P}$. This means that $(\lambda,\mathscr{P})=\mathscr{Y}$ or $[int(\lambda^c(p))]^c\neq\emptyset\Leftrightarrow cl(\lambda(p))\neq\mathscr{Y}$ for some $p\in\mathscr{P}$, as required.
"$\Leftarrow$": Let $(\lambda,\mathscr{P})$ be a soft set such that $(\lambda,\mathscr{P})=\widetilde{\mathscr{Y}}$ or $cl(\lambda(p))\neq\mathscr{Y}$ for some $p\in\mathscr{P}$. Then, $(\lambda^c,\mathscr{P})=\phi$ or $[cl(\lambda(p))]^c\neq\emptyset\Leftrightarrow int(\lambda^c(p))\neq\emptyset$ for some $p\in\mathscr{P}$. This implies that $(\lambda^c,\mathscr{P})$ is a weakly soft *so*-set. This finishes the proof that $(\lambda,\mathscr{P})$ is a weakly soft *sc*-set. $\square$

Proposition 3.3. *Let $(\mathscr{Y},\mathscr{T},\mathscr{P})$ be a soft$_{TS}$. Every superset of a non-null weakly soft so-set is also a weakly soft so-set.*

Proof. Let $(\lambda,\mathscr{P})$,$(\delta,\mathscr{P})$ be soft sets with $\phi\neq(\delta,\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})$ and $(\delta,\mathscr{P})$ be a weakly soft *so*-set. Since $int(\delta(p))\neq\phi$ for some $p\in\mathscr{P}$ and $int(\delta(p))\subseteq int(\lambda(p))$ for all $p\in\mathscr{P}$ then $(\delta,\mathscr{P})$ is weakly soft *so*-set. $\square$

The proofs of the next corollaries follow immediately from the above proposition, so we omit their proofs.

Corollary 3.4. *Every subset of a proper weakly soft sc-set is also a weakly soft sc-set.*

Corollary 3.5. *The arbitrary union of weakly soft so-subsets of $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is a weakly soft so-set.*

Corollary 3.6. *The arbitrary intersection of weakly soft sc-subsets of $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is a weakly soft sc-set.*

By the next examples, we show that the finite intersection of a weakly soft *so*-sets need not be a weakly soft *so*-set as well as the finite union of a soft *sc*-sets need not be a soft *sc*-set.

Example 3.7. *Let $\mathbb{R}$ be the set of real numbers and $\mathscr{P}=\{p_1,p_2\}$ be a set of parameters. Let $\mathscr{T}$ be the soft topology on $\mathbb{R}$ generated by $\{(p_i,\lambda(p_i)):\lambda(p_i)=(a_i,b_i);\ a_i,b_i\in\mathbb{R};a_i\leq\ b_i$ and $i=1,2\}$. Set $(\lambda,\mathscr{P})=\{(p_1,[0,1]),(p_2,[0,1])\}$ and $(\delta,\mathscr{P})=\{(p_1,[1,2]),(p_2,[1,2])\}$ over $\mathbb{R}$. Since $int(\lambda(p_1))=(0,1)$ and $int(\delta(p_1))=(1,2)$ then both $(\lambda,\mathscr{P})$ and $(\delta,\mathscr{P})$ are weakly soft so-sets but $(\lambda,\mathscr{P})\sqcap(\delta,\mathscr{P})$ is not a weakly soft so-set.*

Example 3.8. *Let $\mathscr{Y}=\{x,y,t,s\}$ and $\mathscr{P}=\{p_1,p_2\}$. Define $\mathscr{T}=\{\phi,\widetilde{\mathscr{Y}},(\delta,\mathscr{P}),(\zeta,\mathscr{P}),(\gamma,\mathscr{P})\}$, where*
$(\delta,\mathscr{P})=\{(p_1,\{y,s\}),(p_2,\{x,y\})\}$
$(\zeta,\mathscr{P})=\{(p_1,\mathscr{Y}),(p_2,\{t,s\})\}$
$(\gamma,\mathscr{P})=\{(p_1,\{y,s\}),(p_2,\emptyset)\}$.

Now, $(\lambda,\mathscr{P})=\{(p_1,\mathscr{Y}),(p_2,\{x\})\}$ and $(\beta,\mathscr{P})=\{(p_1,\mathscr{Y}),(p_2,\{t\})\}$ are weakly soft sc-sets. But their soft union $\{(p_1,\mathscr{Y}),(p_2,\{x,t\})\}$ is not a weakly soft sc-set.

Proposition 3.9. *Let $(\mathscr{Y},\mathscr{T},\mathscr{P})$ be a full soft$_{TS}$. Then the soft intersection of soft open and weakly soft so-sets is a weakly soft so-set.*

Proof. Assume that $(\delta,\mathscr{P})$ and $(\lambda,\mathscr{P})$ are soft open and weakly soft *so*-sets, respectively. Then there exists $p\in\mathscr{P}$ such that $int(\lambda(p))\neq\emptyset$. Therefore, there exists a soft open set $(\zeta,\mathscr{P})$ with $\zeta(p)=int(\lambda(p))$. By the condition of a full soft topology, $int(\delta(p))=\delta(p)\neq\emptyset$ and $\delta(p)\cap\zeta(p)\neq\emptyset$.

By substituting, we get $\delta(p) \cap int(\lambda(p)) = int(\delta(p) \cap \lambda(p)) \neq \emptyset$. Hence, $(\delta, \mathscr{P}) \widetilde{\cap} (\lambda, \mathscr{P})$ is a weakly soft *so*-set. □

Remark 3.10. *Every pseudo constant soft subset* $(\lambda, \mathscr{P})$ *is a weakly soft so-subset because* $\lambda(p) = \emptyset$ *for all* $p \in \mathscr{P}$ *or* $int(\lambda(p)) = \mathscr{Y}$ *for some* $p \in \mathscr{P}$*. Moreover, the family of pseudo constant soft subsets of any soft*$_{TS}$ $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$ *forms a soft topology.*

Proposition 3.11. *Any soft subset* $(\lambda, \mathscr{P})$ *of* $(\mathscr{Y}, \mathrm{T}, \mathscr{P})$ *with* $\delta(p) = \mathscr{Y}$ *(resp.* $\delta(p) = \emptyset$*) is a weakly soft so-set (resp. weakly soft sc-set).*

Proof. Straightforward. □

According to Example 3.8, $\{(p_1, \{x\}), (p_2, \{x, s, t\})\}$ and $\{(p_1, \{s\}), (p_2, \{y\})\}$ are respectively weakly soft *so*-set and weakly soft *sc*-set. Hence, the converse of the above proposition is false.

Proposition 3.12. *Every soft sw-open set is a weakly soft so-set.*

Proof. Let $(\lambda, \mathscr{P})$ be a non-null soft *sw*-open set. Then $int(\lambda, \mathscr{P}) \neq \phi$ and so there is a non-null soft open $(\delta, \mathscr{P})$ with $(\delta, \mathscr{P}) \widetilde{\subseteq} (\lambda, \mathscr{P})$. Therefore, $int(\lambda(p)) \neq \emptyset$ for some $p \in \mathscr{P}$. Hence, $(\lambda, \mathscr{P})$ is a weakly soft *so*-set. □

Corollary 3.13. *Every soft open (soft* α*-open set, soft semi-open set) is a weakly soft so-set.*

Proof. Since any soft open (soft α-open set, soft semi-open set) is a soft *sw*-open set, the proof follows. □

By Example 3.8, note that a soft set $(\lambda, \mathscr{P}) = \{(p_1, \{y\}), (p_2, \{x, y\})\}$ is a weakly soft *so*-set but it is neither a soft *sw*-open set nor a soft semi-open set. Hence, the converses of Proposition 3.12 and Corollary 3.13 fail.

Proposition 3.14. *Let* $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$ *be an extended soft*$_{TS}$*. The the classes of soft sw-sets and weakly soft so-sets (soft sw-closed sets and weakly soft sc-sets) are identical.*

Proof. The proof follows directly from Theorem 2.14. □

Proposition 3.15. *The surjective soft continuous pre-image of a weakly soft so-set is also a weakly soft so-set*

Proof. To prove (i), let $h_\pi : (\mathscr{Y}, \mathscr{T}, \mathscr{P}) \to (\mathscr{Z}, \mathscr{S}, \mathscr{Q})$ be a soft continuous mapping and let $(\lambda, \mathscr{Q})$ be a weakly soft *so*-subset of $(\mathscr{Z}, \mathscr{S}, \mathscr{Q})$. Then, there exists $q \in \mathscr{Q}$ such that $int(\lambda(q)) \neq \emptyset$. Let $\pi(p) = q$. Then, by hypothesis of soft continuity of h_π, it follows from Theorem 2.18 that $h : (\mathscr{Y}, \mathscr{T}_p) \to (\mathscr{Z}, \mathscr{S}_{\pi(p)=q})$ is a continuous mapping. This means that $h^{-1}[int(\lambda(q))] \subseteq int[h^{-1}(\lambda(q))]$. Since h is surjective, $int[h^{-1}(\lambda(q))]$ is a nonempty open subset of $\mathscr{Z}$. By calculations obtained from Definition 2.17, we get that $h_\pi^{-1}(\lambda, \mathscr{Q})$ is a weakly soft *so*-subset of $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$. Hence, we obtain the desired result. □

Corollary 3.16. *The property of being a weakly soft so-set is a topological property.*

Proposition 3.17. *The product of two weakly soft so-sets is a weakly soft so-set.*

Proof. Suppose that $(\lambda, \mathscr{P})$ and $(\delta, \mathscr{P})$ are weakly soft *so*-sets and let $(\zeta, \mathscr{P}) = (\lambda, \mathscr{P}) \times (\delta, \mathscr{P})$. Then there are $p_1, p_2 \in \mathscr{P}$ such that $int(\lambda(p_1)) \neq \emptyset$ and $int(\delta(p_2)) \neq \emptyset$. Therefore, we have $(p_1, p_2) \in \mathscr{P} \times \mathscr{P}$ such that $\zeta(p_1, p_2) = \lambda(p_1) \times \delta(p_2)$. It is obvious that $int(\zeta(p_1, p_2)) = int(\lambda(p_1) \times \delta(p_2)) = int(\lambda(p_1)) \times int(\delta(p_2)) \neq \emptyset$. Hence, $(\zeta, \mathscr{P})$ is a weakly soft *so*-set, as required. □

4. *so*-interior and *so*-closure operators

In this section, depending on weakly soft *so*-sets and weakly soft *sc*-sets we will study the interior, closure, boundary, and limit soft points of any soft subset. Some relationships between these types of soft points will be illustrated with the aid of examples. Also, we will provide the formulas which determine the finite possible outcomes of *so*-interior, *so*-closure and *so*-boundary points of any soft subset.

Definition 4.1. *The so-interior points of a subset $(\lambda, \mathscr{P})$ of $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$, denoted by $int_{so}(\lambda, \mathscr{P})$, is defined as the union of all weakly soft so-sets contained in $(\lambda, \mathscr{P})$.*

Proposition 4.2. *The next properties are satisfied for a subset $(\lambda, \mathscr{P})$ of $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$.*

1. *$(\lambda, \mathscr{P})$ is a weakly soft so-set iff for* some *$y_p \in (\lambda, \mathscr{P})$ there is a weakly soft so-set $(\delta, \mathscr{P})$ with $y_p \in (\delta, \mathscr{P}) \tilde{\subseteq} (\lambda, \mathscr{P})$.*
2. *A soft set $(\lambda, \mathscr{P})$ is a weakly soft so-set iff $(\lambda, \mathscr{P}) = int_{so}(\lambda, \mathscr{P})$.*

Proof.

1. Let $(\lambda, \mathscr{P})$ be a non-null weakly soft *so*-set. Then there is $y_p \in (\lambda, \mathscr{P})$. Obviously, $y_p \in (\lambda, \mathscr{P}) \tilde{\subseteq} (\lambda, \mathscr{P})$. The sufficient part follows from Proposition 3.3.
2. If $(\lambda, \mathscr{P})$ is a weakly soft *so*-set, then $(\lambda, \mathscr{P})$ is the largest weakly soft *so*-set contained in itself. So that $(\lambda, \mathscr{P}) = int_{so}(\lambda, \mathscr{P})$. Conversely, let $(\lambda, \mathscr{P}) \neq \phi$ such that $(\lambda, \mathscr{P}) = int_{so}(\lambda, \mathscr{P})$. Then, it follows from Corollary 3.5 that $(\lambda, \mathscr{P})$ is a weakly soft *so*-set.

□

Proposition 4.3. *For any subset $(\lambda, \mathscr{P})$ of $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$, we have*

$$int_{so}(\lambda, \mathscr{P}) = \begin{cases} (\lambda, \mathscr{P}) & : \quad (int(\lambda), \mathscr{P}) \neq \phi \\ \phi & : \quad (int(\lambda), \mathscr{P}) = \phi \end{cases}$$

Proof. Let $(\lambda, \mathscr{P})$ be a non-null soft subset of $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$. Then, we have two cases: Either $(int(\lambda), \mathscr{P}) = \phi$ or $(int(\lambda), \mathscr{P}) \neq \phi$. If $(int(\lambda), \mathscr{P}) = \phi$, then $(\lambda, \mathscr{P})$ is not a weakly soft *so*-set. According to Proposition 3.3, there does not exists a non-null weakly soft *so*-set that is contained in $(\lambda, \mathscr{P})$. Thus, $int_{so}(\lambda, \mathscr{P}) = \phi$. Now, take the second case: $(int(\lambda), \mathscr{P}) \neq \phi$. Then $(int(\lambda), \mathscr{P})$ is a non-null weakly soft *so*-set. So, $int_{so}(\lambda, \mathscr{P}) = (\lambda, \mathscr{P})$. Hence, the proof is complete. □

Proposition 4.4. *For any two subsets $(\lambda, \mathscr{P})$, $(\delta, \mathscr{P})$ of $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$, if $(\lambda, \mathscr{P}) \tilde{\subseteq} (\delta, \mathscr{P})$, then $int_{so}(\lambda, \mathscr{P}) \tilde{\subseteq} int_{so}(\delta, \mathscr{P})$.*

Proof. Since for any weakly soft *so*-set contained in $(\lambda, \mathscr{P})$ is also contained in $(\delta, \mathscr{P})$, then the result is proved. □

Corollary 4.5. *For any two subsets $(\lambda, \mathscr{P})$, $(\delta, \mathscr{P})$ of $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$, we have the following results:*

1. $int_{so}[(\lambda, \mathscr{P}) \tilde{\bigcap} (\delta, \mathscr{P})] \tilde{\subseteq} int_{so}(\lambda, \mathscr{P}) \tilde{\bigcap} int_{so}(\delta, \mathscr{P})$.

(2) $int_{so}(\lambda,\mathscr{P})\widetilde{\bigcup} int_{so}(\delta,\mathscr{P})\widetilde{\subseteq} int_{so}[(\lambda,\mathscr{P})\widetilde{\bigcup}(\delta,\mathscr{P})]$.

Proof. The proof follows from the following facts and the above proposition:
1. $(\lambda,\mathscr{P})\widetilde{\bigcap}(\delta,\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})$ and $(\lambda,\mathscr{P})\widetilde{\bigcap}(\delta,\mathscr{P})\widetilde{\subseteq}(\delta,\mathscr{P})$.
2. $(\lambda,\mathscr{P})\widetilde{\subseteq}[(\lambda,\mathscr{P})\widetilde{\bigcup}(\delta,\mathscr{P})]$ and $(\delta,\mathscr{P})\widetilde{\subseteq}[(\lambda,\mathscr{P})\widetilde{\bigcup}(\delta,\mathscr{P})]$ □

The next example clarifies that the inclusion relations in the above proposition and corollary cannot be replaced by the equality relation.

Example 4.6. *Let $(\mathscr{Y},\mathscr{T},\mathscr{P})$ be as given in Example 3.8. Take the following soft subsets:*
$(\alpha_1,\mathscr{P})=\{(p_1,\{y,s,t\}),(p_2,\{t\})\}$;
$(\alpha_2,\mathscr{P})=\{(p_1,\{x,t\}),(p_2,\{t,s\})\}$;
$(\alpha_3,\mathscr{P})=\{(p_1,\{y\}),(p_2,\emptyset)\}$ *and*
$(\alpha_4,\mathscr{P})=\{(p_1,\{s\}),(p_2,\emptyset)\}$.

Then, $int_{so}(\alpha_3,\mathscr{P})=\phi\widetilde{\subseteq} int_{so}(\alpha_2,\mathscr{P})=(\alpha_2,\mathscr{P})$ but neither $(\alpha_3,\mathscr{P})\widetilde{\subseteq}(\alpha_2,\mathscr{P})$ nor $(\alpha_2,\mathscr{P})\widetilde{\subseteq}(\alpha_3,\mathscr{P})$. Also, $int_{so}(\alpha_1,\mathscr{P})\widetilde{\cap} int_{so}(\alpha_2,\mathscr{P})=\{(p_1,\{t\}),(p_2,\{t\})\}$, whereas $int_{so}[(\alpha_1,\mathscr{P})\widetilde{\cap}(\alpha_2,\mathscr{P})]=\phi$. Moreover, $int_{so}(\alpha_3,\mathscr{P})\widetilde{\cup} int_{so}(\alpha_4,\mathscr{P})=\phi$, whereas $int_{so}[(\alpha_1,\mathscr{P})\widetilde{\cup}(\alpha_2,\mathscr{P})]=\{(p_1,\{y,s\}),(p_2,\emptyset)\}$.

Definition 4.7. *The so-closure points of a subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$, denoted by $cl_{so}(\lambda,\mathscr{P})$, is defined as the intersection of all weakly soft sc-sets containing $(\lambda,\mathscr{P})$.*

Proposition 4.8. *Let $(\lambda,\mathscr{P})$ be a subset of $(\mathscr{Y},\mathscr{T},\mathscr{P})$ and $y_p\in\widetilde{\mathscr{Y}}$. Then $y_p\in cl_{so}(\lambda,\mathscr{P})$ iff $(\delta,\mathscr{P})\widetilde{\bigcap}(\lambda,\mathscr{P})\neq\phi$ for each weakly soft so-set $(\delta,\mathscr{P})$ contains y_p.*

Proof. [$\Rightarrow$] Let $y_p\in cl_{so}(\lambda,\mathscr{P})$. Suppose that there is a weakly soft *so*-set $(\delta,\mathscr{P})$ containing y_p with $(\delta,\mathscr{P})\widetilde{\bigcap}(\lambda,\mathscr{P})=\phi$. Then $(\lambda,\mathscr{P})\widetilde{\subseteq}(\delta^c,\mathscr{P})$. Therefore, $cl_{so}(\lambda,\mathscr{P})\widetilde{\subseteq}(\delta^c,\mathscr{P})$. Thus $y_p\notin cl_{so}(\lambda,\mathscr{P})$. But this contradicts that $y_p\in cl_{so}(\lambda,\mathscr{P})$. This means that the necessary part holds.
[$\Leftarrow$] Let $(\delta,\mathscr{P})\widetilde{\bigcap}(\lambda,\mathscr{P})\neq\phi$ for each weakly soft *so*-set $(\delta,\mathscr{P})$ contains y_p. Suppose that $y_p\notin cl_{so}(\lambda,\mathscr{P})$. Then there is a soft *sc*-set $(\zeta,\mathscr{P})$ containing $(\delta,\mathscr{P})$ with $y_p\notin(\zeta,\mathscr{P})$. So $y_p\in(\zeta^c,\mathscr{P})$ and $(\zeta^c,\mathscr{P})\widetilde{\bigcap}(\lambda,\mathscr{P})=\phi$. But this is a contradiction. Hence, the sufficient part holds. □

Corollary 4.9. *If $(\lambda,\mathscr{P})$ is a weakly soft so-set and $(\delta,\mathscr{P})$ is a soft sets in $(\mathscr{Y},\mathscr{T},\mathscr{P})$ such that $(\lambda,\mathscr{P})\widetilde{\bigcap}(\delta,\mathscr{P})=\phi$, then $(\lambda,\mathscr{P})\widetilde{\bigcap} cl_{so}(\delta,\mathscr{P})=\phi$.*

Proof. Obvious. □

Proposition 4.10. *The following properties hold for a subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$.*

(1) *A soft set $(\lambda,\mathscr{P})$ is a weakly soft sc-set iff $(\lambda,\mathscr{P})=cl_{so}(\lambda,\mathscr{P})$.*
(2) $[int_{so}(\lambda,\mathscr{P})]^c=cl_{so}(\lambda^c,\mathscr{P})$.
(3) $[cl_{so}(\lambda,\mathscr{P})]^c=int_{so}(\lambda^c,\mathscr{P})$.

Proof. 1. If $(\lambda,\mathscr{P})$ is a weakly soft *sc*-set, then $(\lambda,\mathscr{P})$ is the smallest weakly soft *sc*-set contained itself and hence $(\lambda,\mathscr{P})=cl_{so}(\lambda,\mathscr{P})$. Conversely, by Corollary 3.6, $(\lambda,\mathscr{P})$ is a weakly soft *sc*-set.
2. If $y_p\notin[int_{so}(\lambda,\mathscr{P})]^c$, then there is a weakly soft *so*-set $(\delta,\mathscr{P})$ with $y_p\in(\delta,\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})$. Therefore , $(\lambda^c,\mathscr{P})\widetilde{\bigcap}(\delta,\mathscr{P})=\phi$ and hence $y_p\notin cl_{so}(\lambda^c,\mathscr{P})$. Conversely, if $y_p\notin cl_{so}(\lambda^c,\mathscr{P})$ we can follow the previous steps to verify $y_p\notin[int_{so}(\lambda,\mathscr{P})]^c$.
3. Following similar approach given in 2. □

Proposition 4.11. *For any subset* $(\lambda,\mathscr{P})$ *of* $(\mathscr{Y},\mathscr{T},\mathscr{P})$*, we have*

$$cl_{so}(\lambda,\mathscr{P}) = \begin{cases} (\lambda,\mathscr{P}) & : \quad (cl(\lambda),\mathscr{P}) \neq \widetilde{\mathscr{Y}} \\ \widetilde{\mathscr{Y}} & : \quad (cl(\lambda),\mathscr{P}) = \widetilde{\mathscr{Y}} \end{cases}$$

Proof. Let $(\lambda,\mathscr{P})$ be a non-null soft subset of $(\mathscr{Y},\mathscr{T},\mathscr{P})$. Then, we have two cases: Either $(cl(\lambda),\mathscr{P}) = \mathscr{Y}$ or $(cl(\lambda),\mathscr{P}) \neq \mathscr{Y}$. If $(cl(\lambda),\mathscr{P}) = \mathscr{Y}$, then, according to Proposition 3.2, $(\lambda,\mathscr{P})$ is not a weakly soft *sc*-set. This implies that the only weakly soft *sc*-set containing $(\lambda,\mathscr{P})$ is the absolute soft set. Thus, $cl_{so}(\lambda,\mathscr{P}) = \mathscr{Y}$. Now, assume that $(cl(\lambda),\mathscr{P}) \neq \mathscr{Y}$. Then $(cl(\lambda),\mathscr{P})$ is a proper weakly soft *sc*-set. So, $cl_{so}(\lambda,\mathscr{P}) = (\lambda,\mathscr{P})$. Hence, we finish the proof. □

Proposition 4.12. *For any subset* $(\lambda,\mathscr{P})$ *of* $(\mathscr{Y},\mathscr{T},\mathscr{P})$*, we have if* $(\lambda,\mathscr{P})\tilde{\subseteq}(\delta,\mathscr{P})$*, then* $cl_{so}(\lambda,\mathscr{P})\tilde{\subseteq} cl_{so}(\delta,\mathscr{P})$*.*

Proof. Since any weakly soft *sc*-set contains $(\delta,\mathscr{P})$ also contains $(\lambda,\mathscr{P})$, the result is proved. □

Corollary 4.13. *The following results hold for any subsets* $(\lambda,\mathscr{P})$*,* $(\delta,\mathscr{P})$ *of* $(\mathscr{Y},\mathscr{T},\mathscr{P})$*.*

(1) $cl_{so}[(\lambda,\mathscr{P})\widetilde{\bigcap}(\delta,\mathscr{P})]\tilde{\subseteq} cl_{so}(\lambda,\mathscr{P})\widetilde{\bigcap} cl_{so}(\delta,\mathscr{P})$.
(2) $cl_{so}(\lambda,\mathscr{P})\widetilde{\bigcup} cl_{so}(\delta,\mathscr{P})\tilde{\subseteq} cl_{so}[(\lambda,\mathscr{P})\widetilde{\bigcup}(\delta,\mathscr{P})]$.

Proof. The proof follows from the following facts and the above proposition:
1. $(\lambda,\mathscr{P})\widetilde{\bigcap}(\delta,\mathscr{P})\tilde{\subseteq}(\lambda,\mathscr{P})$ and $(\lambda,\mathscr{P})\widetilde{\bigcap}(\delta,\mathscr{P})\tilde{\subseteq}(\delta,\mathscr{P})$.
2. $(\lambda,\mathscr{P})\tilde{\subseteq}[(\lambda,\mathscr{P})\widetilde{\bigcup}(\delta,\mathscr{P})]$ and $(\delta,\mathscr{P})\tilde{\subseteq}[(\lambda,\mathscr{P})\widetilde{\bigcup}(\delta,\mathscr{P})]$. □

The next example clarifies that the inclusion relations in the aforementioned proposition and corollary cannot be replaced by the equality relation.

Example 4.14. *Let* $(\mathbb{R},\mathscr{T},\mathscr{P})$ *be as given in Example 3.7 and* $\mathbb{Q}$ *be the set of rational numbers. Take the following soft subsets:*
$(\alpha_1,\mathscr{P}) = \{(p_1,\mathbb{Q}),(p_2,\mathbb{Q})\}$;
$(\alpha_2,\mathscr{P}) = \{(p_1,\mathbb{Q}^c),(p_2,\mathbb{Q}^c)\}$;
$(\alpha_3,\mathscr{P}) = \{(p_1,\{1\}),(p_2,\mathbb{Q})\}$ *and*
$(\alpha_4,\mathscr{P}) = \{(p_1,\mathbb{Q}),(p_2,\{1\})\}$.

Then, $cl_{so}(\alpha_1,\mathscr{P}) = cl_{so}(\alpha_4,\mathscr{P}) = \widetilde{\mathbb{R}}$ *but neither* $(\alpha_1,\mathscr{P})\tilde{\subseteq}(\alpha_2,\mathscr{P})$ *nor* $(\alpha_2,\mathscr{P})\tilde{\subseteq}(\alpha_1,\mathscr{P})$*. Also,* $cl_{so}(\alpha_1,\mathscr{P})\tilde{\cap} cl_{so}(\alpha,\mathscr{P}) = \widetilde{\mathbb{R}}$*, whereas* $cl_{so}[(\alpha_1,\mathscr{P})\tilde{\cap}(\alpha_2,\mathscr{P})] = \phi$*. Moreover,* $cl_{so}(\alpha_3,\mathscr{P})\tilde{\cup} cl_{so}(\alpha_4,\mathscr{P}) = \{(p_1,\mathbb{Q}),(p_2,\mathbb{Q})\}$*, whereas* $cl_{so}[(\alpha_3,\mathscr{P})\tilde{\cup}(\alpha_4,\mathscr{P})] = \widetilde{\mathbb{R}}$*.*

Definition 4.15. *A soft point* y_p *is said to be an so-boundary point of a subset* $(\lambda,\mathscr{P})$ *of* $(\mathscr{Y},\mathscr{T},\mathscr{P})$ *if* y_p *belongs to the complement of* $int_{so}(\lambda,\mathscr{P})\widetilde{\bigcup} int_{so}(\lambda^c,\mathscr{P})$*.*
All so-boundary points of $(\lambda,\mathscr{P})$*, denoted by* $b_{so}(\lambda,\mathscr{P})$*, is called an so-boundary set.*

Proposition 4.16. $b_{so}(\lambda,\mathscr{P}) = cl_{so}(\lambda,\mathscr{P})\widetilde{\bigcap} cl_{so}(\lambda^c,\mathscr{P})$ *for every subset* $(\lambda,\mathscr{P})$ *of* $(\mathscr{Y},\mathscr{T},\mathscr{P})$*.*

Proof. $b_{so}(\lambda,\mathscr{P}) = [int_{so}(\lambda,\mathscr{P})\widetilde{\bigcup} int_{so}(\lambda^c,\mathscr{P})]^c$
$= [int_{so}(\lambda,\mathscr{P})]^c\widetilde{\bigcap}[int_{so}(\lambda^c,\mathscr{P})]^c$ (De Morgan's law)
$= cl_{so}(\lambda^c,\mathscr{P})\widetilde{\bigcap} cl_{so}(\lambda,\mathscr{P})$ (Proposition 4.10(2)) □

Corollary 4.17. *For every subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$, the following properties hold.*

(1) $b_{so}(\lambda,\mathscr{P})$ *is a weakly soft sc-set.*
(2) $b_{so}(\lambda,\mathscr{P}) = b_{so}(\lambda^c,\mathscr{P})$.
(3) $b_{so}(\lambda,\mathscr{P}) = cl_{so}(\lambda,\mathscr{P}) \setminus int_{so}(\lambda,\mathscr{P})$.
(4) $cl_{so}(\lambda,\mathscr{P}) = int_{so}(\lambda,\mathscr{P}) \widetilde{\bigcup} b_{so}(\lambda,\mathscr{P})$.
(5) $int_{so}(\lambda,\mathscr{P}) = (\lambda,\mathscr{P}) \setminus b_{so}(\lambda,\mathscr{P})$.

Proof. 1. According to the above proposition $b_{so}(\lambda,\mathscr{P})$ is the soft intersection of two weakly soft sc-sets; so it follows from Corollary 3.6 that $b_{so}(\lambda,\mathscr{P})$ is a weakly soft sc-set.
2. Obvious.
3. $b_{so}(\lambda,\mathscr{P}) = cl_{so}(\lambda,\mathscr{P}) \widetilde{\bigcap} cl_{so}(\lambda^c,\mathscr{P}) = cl_{so}(\lambda,\mathscr{P}) \setminus [cl_{so}(\lambda^c,\mathscr{P})]^c$. By 3 of Proposition 4.10 we obtain the required relation.
4. $int_{so}(\lambda,\mathscr{P}) \widetilde{\bigcup} b_{so}(\lambda,\mathscr{P}) = int_{so}(\lambda,\mathscr{P}) \widetilde{\bigcup} [cl_{so}(\lambda,\mathscr{P}) \setminus int_{so}(\lambda,\mathscr{P})] = cl_{so}(\lambda,\mathscr{P})$.
5.
$$\begin{aligned}(\lambda,\mathscr{P}) \setminus b_{so}(\lambda,\mathscr{P}) &= (\lambda,\mathscr{P}) \setminus [cl_{so}(\lambda,\mathscr{P}) \setminus int_{so}(\lambda,\mathscr{P})]\\ &= (\lambda,\mathscr{P}) \widetilde{\bigcap} [cl_{so}(\lambda,\mathscr{P}) \widetilde{\bigcap} (int_{so}(\lambda,\mathscr{P}))^c]^c\\ &= (\lambda,\mathscr{P}) \widetilde{\bigcap} [(cl_{so}(\lambda,\mathscr{P}))^c \widetilde{\bigcup} int_{so}(\lambda,\mathscr{P})]\\ &= [(\lambda,\mathscr{P}) \widetilde{\bigcap} (cl_{so}(\lambda,\mathscr{P}))^c] \widetilde{\bigcup} [(\lambda,\mathscr{P}) \widetilde{\bigcap} int_{so}(\lambda,\mathscr{P})]\\ &= int_{so}(\lambda,\mathscr{P}).\end{aligned}$$
□

Proposition 4.18. *For every subsets $(\lambda,\mathscr{P}), (\delta,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$, the following properties hold.*

(1) $b_{so}(int_{so}(\lambda,\mathscr{P})) \widetilde{\subseteq} b_{so}(\lambda,\mathscr{P})$.
(2) $b_{so}(cl_{so}(\lambda,\mathscr{P})) \widetilde{\subseteq} b_{so}(\lambda,\mathscr{P})$.

Proof. By substituting in the formula No. 3 of Corollary 4.17, the proof follows. □

The inclusion relations in Proposition 4.18 can be proper as the next example elucidates.

Example 4.19. *Let $(\mathscr{Y},\mathscr{T},\mathscr{P})$ be as given in Example 3.8. By taking the soft sets $(\alpha,\mathscr{P}) = \{(p_1,\{y\}),(p_2,\{t\})\}$ and $(\beta,\mathscr{P}) = \{(p_1,\{y\}),(p_2,\{x,t\})\}$. Then, $int_{so}(\alpha,\mathscr{P}) = \phi$ and $cl_{so}(\beta,\mathscr{P}) = \widetilde{\mathscr{Y}}$. So, $b_{so}(int_{so}(\alpha,\mathscr{P})) = \phi$ whereas $b_{so}(\alpha,\mathscr{P}) = (\alpha,\mathscr{P})$, and $b_{so}(cl_{so}(\beta,\mathscr{P})) = \phi$ whereas $b_{so}(\beta,\mathscr{P}) = \widetilde{\mathscr{Y}}$.*

Proposition 4.20. *The following statements hold.*

(1) *A subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is a weakly soft so-set iff $b_{so}(\lambda,\mathscr{P}) \widetilde{\bigcap} (\lambda,\mathscr{P}) = \phi$.*
(2) *A subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is a weakly soft sc-set iff $b_{so}(\lambda,\mathscr{P}) \widetilde{\subseteq} (\lambda,\mathscr{P})$.*

Proof. 1. Suppose that $(\lambda,\mathscr{P})$ is weakly soft so-set. Then by 5 of Corollary 4.17, $(\lambda,\mathscr{P}) = int_{so}(\lambda,\mathscr{P}) = (\lambda,\mathscr{P}) \setminus b_{so}(\lambda,\mathscr{P})$ and hence $b_{so}(\lambda,\mathscr{P}) \widetilde{\bigcap} (\lambda,\mathscr{P}) = \phi$. Conversely, let $y_p \in (\lambda,\mathscr{P})$. Since $y_p \notin b_{so}(\lambda,\mathscr{P})$ and $y_p \in cl_{so}(\lambda,\mathscr{P})$, by 4 of Corollary 4.17, $y_p \in int_{so}(\lambda,\mathscr{P})$. Therefore, $int_{so}(\lambda,\mathscr{P}) = (\lambda,\mathscr{P})$, which proves that $(\lambda,\mathscr{P})$ is a weakly soft so-set.
2. Assume that $(\lambda,\mathscr{P})$ is weakly soft sc-set. Then $b_{so}(\lambda,\mathscr{P}) = cl_{so}(\lambda,\mathscr{P}) \widetilde{\bigcap} cl_{so}(\lambda^c,\mathscr{P}) \widetilde{\subseteq} cl_{so}(\lambda,\mathscr{P}) = (\lambda,\mathscr{P})$, as required. Conversely, if $b_{so}(\lambda,\mathscr{P}) \widetilde{\subseteq} (\lambda,\mathscr{P})$, then by 4 of Corollary 4.17, $cl_{so}(\lambda,\mathscr{P}) \widetilde{\subseteq} int_{so}(\lambda,\mathscr{P}) \widetilde{\bigcup} (\lambda,\mathscr{P}) = (\lambda,\mathscr{P})$ and hence $cl_{so}(\lambda,\mathscr{P}) = (\lambda,\mathscr{P})$, which proves that $(\lambda,\mathscr{P})$ is a weakly soft sc-set. □

Corollary 4.21. *A subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$ is both weakly soft so-set and weakly soft sc-set iff $b_{so}(\lambda,\mathscr{P})=\phi$.*

Proposition 4.22. *For any subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$, we have*

$$b_{so}(\lambda,\mathscr{P})=\begin{cases}\phi\\ (\lambda,\mathscr{P})\\ (\lambda^{c},\mathscr{P})\\ \widetilde{\mathscr{Y}}\end{cases}$$

Proof. By Proposition 4.3, Proposition 4.11 and 3 of Corollary 4.17, the proof follows. □

Definition 4.23. *A soft point y_p is said to be an so-limit point of a subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$ if $[(\delta,\mathscr{P})\backslash y_p]\bigcap(\lambda,\mathscr{P})\neq\phi$ for each weakly soft so-set $(\delta,\mathscr{P})$ containing y_p.*
All so-limit points of $(\lambda,\mathscr{P})$ is called an so-derived set and denoted by $l_{so}(\lambda,\mathscr{P})$.

Proposition 4.24. *Consider $(\lambda,\mathscr{P})$ and $(\delta,\mathscr{P})$ are subsets of $(\mathscr{Y},\mathscr{T},\mathscr{P})$. If $(\lambda,\mathscr{P})\widetilde{\subseteq}(\delta,\mathscr{P})$, then $l_{so}(\lambda,\mathscr{P})\widetilde{\subseteq}l_{so}(\delta,\mathscr{P})$.*

Proof. Straightforward by Definition 4.23. □

Corollary 4.25. *Consider $(\lambda,\mathscr{P})$ and $(\delta,\mathscr{P})$ are subsets of $(\mathscr{Y},\mathscr{T},\mathscr{P})$. Then:*

(1) $l_{so}[(\lambda,\mathscr{P})\widetilde{\bigcap}(\delta,\mathscr{P})]\widetilde{\subseteq}l_{so}(\lambda,\mathscr{P})\widetilde{\bigcap}\,l_{so}(\delta,\mathscr{P})$.
(2) $l_{so}(\lambda,\mathscr{P})\widetilde{\bigcup}\,l_{so}(\delta,\mathscr{P})\widetilde{\subseteq}l_{so}[(\lambda,\mathscr{P})\widetilde{\bigcup}(\delta,\mathscr{P})]$.

Theorem 4.26. *For any subset $(\lambda,\mathscr{P})$ of $(\mathscr{Y},\mathscr{T},\mathscr{P})$, we have the following results.*

(1) *$(\lambda,\mathscr{P})$ is a weakly soft sc-set iff $l_{so}(\lambda,\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})$.*
(2) *$(\lambda,\mathscr{P})\widetilde{\bigcup}\,l_{so}(\lambda,\mathscr{P})$ is a weakly soft sc-set.*
(3) *$cl_{so}(\lambda,\mathscr{P})=(\lambda,\mathscr{P})\widetilde{\bigcup}\,l_{so}(\lambda,\mathscr{P})$.*

Proof. 1. Assume that $(\lambda,\mathscr{P})$ is a weakly soft *sc*-set set and $y_p\notin(\lambda,\mathscr{P})$. Then $y_p\in(\lambda,\mathscr{P})^{c}$ which is a weakly soft *so*-set set. From the fact that $(\lambda,\mathscr{P})^{c}\widetilde{\bigcap}(\lambda,\mathscr{P})=\phi$, we obtain $y_p\notin l_{so}(\lambda,\mathscr{P})$. Thus $l_{so}(\lambda,\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})$. Conversely, let $y_p\in(\lambda,\mathscr{P})^{c}$ and $l_{so}(\lambda,\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})$. Then $y_p\notin l_{so}(\lambda,\mathscr{P})$. Therefore, there is a weakly soft *so*-set $(\delta_{y_p},\mathscr{P})$ containing y_p such that $[(\delta_{y_p},\mathscr{P})\setminus y_p]\widetilde{\bigcap}(\lambda,\mathscr{P})=\phi$. As $y_p\in(\lambda,\mathscr{P})^{c}$, then $(\delta_{y_p},\mathscr{P})\widetilde{\bigcap}(\lambda,\mathscr{P})=\phi$. Now, $(\delta_{y_p},\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})^{c}$. This implies that $(\lambda,\mathscr{P})^{c}$ is a weakly soft *so*-set. Thus $(\lambda,\mathscr{P})$ is a weakly soft *sc*-set.
2. Let $y_p\notin[(\lambda,\mathscr{P})\widetilde{\bigcup}l_{so}(\lambda,\mathscr{P})]$. Then $y_p\notin(\lambda,\mathscr{P})$ and $y_p\notin l_{so}(\lambda,\mathscr{P})$. Therefore, there is a weakly soft *so*-set $(\delta,\mathscr{P})$ containing y_p with

$$(\delta,\mathscr{P})\widetilde{\cap}(\lambda,\mathscr{P})=\phi \tag{1}$$

It follows from Corollary 4.9 that $(\delta,\mathscr{P})\widetilde{\bigcap}cl_{so}(\lambda,\mathscr{P})=\phi$. Since $l_{so}(\delta,\mathscr{P})\widetilde{\subseteq}cl_{so}(\lambda,\mathscr{P})$, we find the following:

$$(\delta,\mathscr{P})\widetilde{\cap}l_{so}(\lambda,\mathscr{P})=\phi \tag{2}$$

From 1 and 2, we obtain $(\delta,\mathscr{P})\widetilde{\bigcap}[(\lambda,\mathscr{P})\widetilde{\bigcup}\,l_{so}(\lambda,\mathscr{P})]=\phi$. This implies that $y_p\notin l_{so}[(\lambda,\mathscr{P})\widetilde{\bigcup}\,l_{so}(\lambda,\mathscr{P})]$. Hence, $l_{so}[(\lambda,\mathscr{P})\widetilde{\bigcup}\,l_{so}(\lambda,\mathscr{P})]\widetilde{\subseteq}(\lambda,\mathscr{P})\widetilde{\bigcup}\,l_{so}(\lambda,\mathscr{P})$. By 1, we get $(\lambda,\mathscr{P})\widetilde{\bigcup}l_{so}(\lambda,\mathscr{P})$ is a weakly

soft *sc*-set.
3. It is obvious that $(\lambda,\mathscr{P})\widetilde{\bigcup}\, l_{so}(\lambda,\mathscr{P})\widetilde{\subseteq} cl_{so}(\lambda,\mathscr{P})$. Conversely, $cl_{so}(\lambda,\mathscr{P})$ is the smallest weakly soft *sc*-set containing $(\lambda,\mathscr{P})$. By 2, we obtain $(\lambda,\mathscr{P})\widetilde{\bigcup} l_{so}(\lambda,\mathscr{P})$ is also a soft *sc*-set containing $(\lambda,\mathscr{P})$. This automatically leads to that $cl_{so}(\lambda,\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})\bigcup\, l_{so}(\lambda,\mathscr{P})$. Hence, we finish the proof that $cl_{so}(\lambda,\mathscr{P}) = (\lambda,\mathscr{P})\widetilde{\bigcup} l_{so}(\lambda,\mathscr{P})$. □

5. Soft *so*-continuous mappings

In this section, we introduce and discuss new sorts of soft continuity, irresolute, openness, closedness, and homeomorphism. We give several features of it and illustrate its relationships with some sorts of soft continuity.

Definition 5.1. *A soft mapping $h_\pi : (\mathscr{Y},\mathscr{T}_\mathscr{Y},\mathscr{P}) \to (\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ is said to be weakly soft so-continuous at $y_p \in \widetilde{\mathscr{Y}}$ if for any soft open set $(\lambda,\mathscr{P})$ containing $h_\pi(y_p)$, there is a weakly soft so-set $(\delta,\mathscr{P})$ containing y_p with $h_\pi(\delta,\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})$.*

Definition 5.2. *A soft mapping $h_\pi : (\mathscr{Y},\mathscr{T}_\mathscr{Y},\mathscr{P}) \to (\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ is said to be weakly soft so-continuous if it is weakly soft so-continuous for each $y_p \in \widetilde{\mathscr{Y}}$.*

Theorem 5.3. *A soft mapping $h_\pi : (\mathscr{Y},\mathscr{T}_\mathscr{Y},\mathscr{P}) \to (\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ is weakly soft so-continuous iff the inverse image of each soft open set is a weakly soft so-set.*

Proof. Let $(\lambda,\mathscr{P})$ be a soft open set in $(\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$. If $h_\pi^{-1}(\lambda,\mathscr{P}) \neq \phi$, then there is $y_p \in h_\pi^{-1}(\lambda,\mathscr{P})$. Since $h_\pi(y_p) \in (\lambda,\mathscr{P})$, there is a weakly soft *so*-set $(\delta,\mathscr{P})$ containing y_p with $h_\pi(\delta,\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})$. Therefore, $y_p \in (\delta,\mathscr{P})\widetilde{\subseteq} h_\pi^{-1}(\lambda,\mathscr{P})$ and hence $h_\pi^{-1}(\lambda,\mathscr{P})$ is a weakly soft *so*-set. Conversely, let $y_p \in \widetilde{\mathscr{Y}}$ and $(\lambda,\mathscr{P})$ be any soft open set containing $h_\pi(y_p)$. Since $h_\pi^{-1}(\lambda,\mathscr{P})$ is a weakly soft *so*-set with $y_p \in h_\pi^{-1}(\lambda,\mathscr{P})$, there is a weakly soft *so*-set $(\delta,\mathscr{P})$ containing y_p with $h_\pi(\delta,\mathscr{P})\widetilde{\subseteq}(\lambda,\mathscr{P})$. □

It is straightforward to prove the next result, so omit its proof.

Proposition 5.4. *If $h_\pi : (\mathscr{Y},\mathscr{T}_\mathscr{Y},\mathscr{P}) \to (\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ is a weakly soft so-continuous mapping and $h_\phi : (\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P}) \to (\mathscr{W},\mathscr{T}_\mathscr{W},\mathscr{P})$ is a soft continuous mapping, then $h_\phi \circ h_\pi$ is weakly soft so-continuous.*

Proposition 5.5. *Every soft continuous (soft α-continuous, soft semi-continuous) mapping is weakly soft so-continuous.*

Proof. It follows from Corollary 3.13 and Definition 5.1. □

The next result exhibits some equivalent descriptions for weakly soft *so*-continuity.

Theorem 5.6. *Let $h_\pi : (\mathscr{Y},\mathscr{T}_\mathscr{Y},\mathscr{P}) \to (\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ be a soft mapping. Then we have the next equivalent properties.*

(1) *h_π is weakly soft so-continuous.*
(2) *The inverse image of every soft closed subset of $(\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ is a weakly soft sc-set.*
(3) *$cl_{so}(h_\pi^{-1}(\lambda,\mathscr{P}))\widetilde{\subseteq} h_\pi^{-1}(cl(\lambda,\mathscr{P}))$ for each $(\lambda,\mathscr{P})\widetilde{\subseteq}\widetilde{\mathscr{Z}}$.*
(4) *$h_\pi(cl_{so}(\delta,\mathscr{P}))\widetilde{\subseteq} cl(h_\pi(\delta,\mathscr{P}))$ for each $(\delta,\mathscr{P})\widetilde{\subseteq}\widetilde{\mathscr{Y}}$.*
(5) *$h_\pi^{-1}(int(\lambda,\mathscr{P}))\widetilde{\subseteq} int_{so}(h_\pi^{-1}(\lambda,\mathscr{P}))$ for each $(\lambda,\mathscr{P})\widetilde{\subseteq}\widetilde{Z}$.*

Proof. $(1 \to 2)$: Suppose that $(\lambda, \mathscr{P})$ is a soft closed subset of $(\mathscr{Z}, \mathscr{T}_{\mathscr{Z}}, \mathscr{P})$. Then $(\lambda^c, \mathscr{P})$ is soft open. Therefore, $h_\pi^{-1}(\lambda^c, \mathscr{P}) = \widetilde{\mathscr{Y}} - h_\pi^{-1}(\lambda, \mathscr{P})$ is a weakly soft *so*-set. Thus, $h_\pi^{-1}(\lambda, \mathscr{P})$ is a weakly soft *sc*-set set.
$(2 \to 3)$: For any soft set $(\lambda, \mathscr{P}) \widetilde{\subseteq} \widetilde{\mathscr{Z}}$, $h_\pi^{-1}(cl(\lambda, \mathscr{P}))$ is a weakly soft *sc*-set. Then $cl_{so}(h_\pi^{-1}(\lambda, \mathscr{P})) \widetilde{\subseteq} cl_{so}(h_\pi^{-1}(cl(\lambda, \mathscr{P}))) = h_\pi^{-1}(cl(\lambda, \mathscr{P}))$.
$(3 \to 4)$: It is obvious that $cl_{so}(\delta, \mathscr{P}) \widetilde{\subseteq} cl_{so}(h_\pi^{-1}(h_\pi(\delta, \mathscr{P})))$ for each $(\delta, \mathscr{P}) \widetilde{\subseteq} \widetilde{\mathscr{Y}}$. By 3, we get $cl_{so}(h_\pi^{-1}(h_\pi(\delta, \mathscr{P}))) \widetilde{\subseteq} h_\pi^{-1}(cl(h_\pi(\delta, \mathscr{P})))$. Therefore, $h_\pi(cl_{so}(\delta, \mathscr{P}) \widetilde{\subseteq} h_\pi(h_\pi^{-1}(cl(h_\pi(\delta, \mathscr{P})))) \widetilde{\subseteq} cl(h_\pi(\delta, \mathscr{P}))$.
$(4 \to 5)$: Let $(\lambda, \mathscr{P})$ be an arbitrary soft set in $(\mathscr{Z}, \mathscr{T}_{\mathscr{Z}}, \mathscr{P})$. Then $h_\pi(cl_{so}(h_\pi^{-1}(\lambda^c, \mathscr{P})) \widetilde{\subseteq} cl(h_\pi(h_\pi^{-1}(\lambda^c, \mathscr{P})))$ $\widetilde{\subseteq} cl(\lambda^c, \mathscr{P})$. So that, $cl_{so}((h_\pi^{-1}(\lambda, \mathscr{P}))^c) \widetilde{\subseteq} h_\pi^{-1}((int(\lambda, \mathscr{P}))^c$. Hence, $h_\pi^{-1}(int(\lambda, \mathscr{P})) \widetilde{\subseteq} int_{so}(h_\pi^{-1}(\lambda, \mathscr{P}))$.
$(5 \to 1)$: Suppose that $(\lambda, \mathscr{P})$ is a soft open subset in $(\mathscr{Z}, \mathscr{T}_{\mathscr{Z}}, \mathscr{P})$. By 5, we obtain $h_\pi^{-1}(\lambda, \mathscr{P}) = h_\pi^{-1}(int(\lambda, \mathscr{P})) \widetilde{\subseteq} int_{so}(h_\pi^{-1}(\lambda, \mathscr{P}))$. But $int_{so}(h_\pi^{-1}(\lambda, \mathscr{P})) \widetilde{\subseteq} h_\pi^{-1}(\lambda, \mathscr{P})$, so $h_\pi^{-1}(\lambda, \mathscr{P}) = int_{so}(h_\pi^{-1}(\lambda, \mathscr{P}))$. Thus h_π is weakly soft *so*-continuous. □

Definition 5.7. *A soft mapping $h_\pi : (\mathscr{Y}, \mathscr{T}_{\mathscr{Y}}, \mathscr{P}) \to (\mathscr{Z}, \mathscr{T}_{\mathscr{Z}}, \mathscr{P})$ is said to be weakly soft so-irresolute if the pre-image of each weakly soft so-set is a weakly soft so-set.*

Proposition 5.8. *Every weakly soft so-irresolute mapping is weakly soft so-continuous.*

Proof. By the fact that every soft open set is a weakly soft *so*-set, The proof follows. □

Example below confirms that the converse of proposition above fails.

Example 5.9. *Let $\mathscr{Y} = \{a, b, c\}$ and $\mathscr{Z} = \{s, t\}$ with $\mathscr{P} = \{p_1, p_2\}$. Define $\mathscr{T} = \{\phi, \widetilde{\mathscr{Y}}, (\delta, \mathscr{P}), (\zeta, \mathscr{P})\}$ as a soft topology over $\mathscr{Y}$ with $\mathscr{P}$ and $\mathscr{S} = \{\phi, \widetilde{\mathscr{Y}}, (\gamma, \mathscr{P})\}$ as a soft topology over $\mathscr{Z}$ with $\mathscr{P}$, where*
$(\delta, \mathscr{P}) = \{(p_1, \{a, b\}), (p_2, \emptyset)\}$
$(\zeta, \mathscr{P}) = \{(p_1, \mathscr{Y}), (p_2, \{b, c\})\}$
$(\gamma, \mathscr{P}) = \{(p_1, \{s\}), (p_2, \{t\})\}$.

By defining the mappings $h : \mathscr{Y} \to \mathscr{Z}$ and $\pi : \mathscr{P} \to \mathscr{P}$ as follows

$$h(a) = h(b) = s \text{ and } h(c) = t;$$
$$\pi(p_i) = p_i \text{ for each } p_i \in \mathscr{P}.$$

Then a soft mapping $h_\pi : (\mathscr{Y}, \mathscr{T}, \mathscr{P}) \to (\mathscr{Z}, \mathscr{S}, \mathscr{P})$ is weakly soft so-continuous because $h_\pi^{-1}(\phi) = \phi$, $h_\pi^{-1}(\mathscr{Z}) = \mathscr{Y}$ and $h_\pi^{-1}(\gamma, \mathscr{P}) = \{(p_1, \{a, b\}), (p_2, \{c\})\}$. On the other hand, $(\lambda, \mathscr{P}) = \{(p_1, \{t\}), (p_2, \{t\})\}$ is a weakly soft so-subset of $(\mathscr{Z}, \mathscr{S}, \mathscr{P})$ whereas $h_\pi^{-1}(\lambda, \mathscr{P}) = \{(p_1, \{c\}), (p_2, \{c\})\}$ is not a weakly soft so-subset of $(\mathscr{Y}, \mathscr{T}, \mathscr{P})$. Hence, h_π is not a weakly soft so-irresolute.

We cancel the proof of the next finding because it can be obtained following similar approach of Theorem 5.6.

Theorem 5.10. *For a soft mapping $h_\pi : (\mathscr{Y}, \mathscr{T}_{\mathscr{Y}}, \mathscr{P}) \to (\mathscr{Z}, \mathscr{T}_{\mathscr{Z}}, \mathscr{P})$, the following statements are equivalent:*

(1) *h_π is weakly soft so-irresolute.*
(2) *The pre-image of each weakly soft sc-set subset of $(\mathscr{Z}, \mathscr{T}_{\mathscr{Z}}, \mathscr{P})$ is a weakly soft sc-set set.*
(3) *$cl_{so}(h_\pi^{-1}(\lambda, \mathscr{P})) \widetilde{\subseteq} h_\pi^{-1}(cl_{so}(\lambda, \mathscr{P}))$ for each $(\lambda, \mathscr{P}) \widetilde{\subseteq} \widetilde{\mathscr{Z}}$.*

(4) $h_\pi(cl_{so}(\delta,\mathscr{P}))\widetilde{\subseteq}cl_{so}(h_\pi(\delta,\mathscr{P}))$ *for each* $(\delta,\mathscr{P})\widetilde{\subseteq}\widetilde{\mathscr{Y}}$.
(5) $h_\pi^{-1}(int_{so}(\lambda,\mathscr{P}))\widetilde{\subseteq}int_{so}(h_\pi^{-1}(\lambda,\mathscr{P}))$ *for each* $(\lambda,\mathscr{P})\widetilde{\subseteq}\widetilde{\mathscr{Z}}$.

Theorem 5.11. *A soft mapping* $h_\pi:(\mathscr{Y},\mathscr{T}_\mathscr{Y},\mathscr{P})\to(\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ *is weakly soft so-irresolute provided that one of the next relations holds.*

(1) $cl(h_\pi^{-1}(\lambda,\mathscr{P}))\widetilde{\subseteq}h_\pi^{-1}(cl_{so}(\lambda,\mathscr{P}))$ *for each* $(\lambda,\mathscr{P})\widetilde{\subseteq}\widetilde{\mathscr{Z}}$.
(2) $h_\pi(cl(\delta,\mathscr{P}))\widetilde{\subseteq}cl_{so}(h_\pi(\delta,\mathscr{P}))$ *for each* $(\delta,\mathscr{P})\widetilde{\subseteq}\widetilde{\mathscr{Y}}$.
(3) $h_\pi^{-1}(int_{so}(\lambda,\mathscr{P}))\widetilde{\subseteq}int(h_\pi^{-1}(\lambda,\mathscr{P}))$ *for each* $(\lambda,\mathscr{P})\widetilde{\subseteq}\widetilde{\mathscr{Z}}$.

Proof. 1. It is obvious that $cl_{so}(\lambda,\mathscr{P})\widetilde{\subseteq}cl(\lambda,\mathscr{P})$ for each $(\lambda,\mathscr{P})\widetilde{\subseteq}\widetilde{\mathscr{Z}}$, so we obtain from condition 1 that $cl_{so}(h_\pi^{-1}(\lambda,\mathscr{P}))\widetilde{\subseteq}\ cl(h_\pi^{-1}(\lambda,\mathscr{P}))\widetilde{\subseteq}h_\pi^{-1}(cl_{so}(\lambda,\mathscr{P}))$. By Theorem 5.10, we have h_π is a weakly soft *so*-irresolute.
2. It is obvious that $cl_{so}(\delta,\mathscr{P})\widetilde{\subseteq}cl(\delta,\mathscr{P})$ for each $(\delta,\mathscr{P})\widetilde{\subseteq}\widetilde{\mathscr{Y}}$, so we obtain from condition 2 that $h_\pi(cl_{so}(\delta,\mathscr{P}))\widetilde{\subseteq}h_\pi(cl(\delta,\mathscr{P}))\ \widetilde{\subseteq}cl_{so}(h_\pi(\delta,\mathscr{P}))$. By Theorem 5.10, we have h_π is a weakly soft *so*-irresolute.
3. It is obvious that $int(\lambda,\mathscr{P})\widetilde{\subseteq}int_{so}(\lambda,\mathscr{P})$ for each $(\lambda,\mathscr{P})\widetilde{\subseteq}\widetilde{\mathscr{Z}}$, so we obtain from condition 3 that $h_\pi^{-1}(int_{so}(\lambda,\mathscr{P}))\widetilde{\subseteq}\ int(h_\pi^{-1}(\lambda,\mathscr{P}))\widetilde{\subseteq}int_{so}(h_\pi^{-1}(\lambda,\mathscr{P}))$. By Theorem 5.10, we have h_π is a weakly soft *so*-irresolute. □

Now, we introduce the concepts of weakly soft *so*-open, weakly soft *so*-closed and weakly soft *so*-homeomorphism mappings.

Definition 5.12. *A soft mapping* $h_\pi:(\mathscr{Y},\mathscr{T}_\mathscr{Y},\mathscr{P})\to(\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ *is called:*

(1) *weakly soft so-open provided that the image of each soft open set is a weakly soft so-set.*
(2) *weakly soft so-closed provided that the image of each soft closed set is r a soft sc-set.*

Theorem 5.13. *Let* $h_\pi:(\mathscr{Y},\mathscr{T}_\mathscr{Y},\mathscr{P})\to(\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ *be a soft mapping and* $(\lambda,\mathscr{P})$ *be any soft subset of* $\widetilde{\mathscr{Y}}$. *Then*

(1) h_π *is weakly soft so-open iff* $h_\pi(int(\lambda,\mathscr{P}))\widetilde{\subseteq}int_{so}(h_\pi(\lambda,\mathscr{P}))$.
(2) h_π *is weakly soft so-closed iff* $cl_{so}(h_\pi(\lambda,\mathscr{P}))\ \widetilde{\subseteq}h_\pi(cl(\lambda,\mathscr{P}))$.

Proof. 1. Let $(\lambda,\mathscr{P})$ be a soft subset of $\widetilde{\mathscr{Y}}$. Then $h_\pi(int(\lambda,\mathscr{P}))$ is a weakly soft *so*-subset of $(\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ and so $h_\pi(int(\lambda,\mathscr{P}))=int_{so}(h_\pi(int(\lambda,\mathscr{P})))\widetilde{\subseteq}int_{so}(h_\pi(\lambda,\mathscr{P}))$. Conversely, assume that $(\lambda,\mathscr{P})$ is a non-null soft open subset of $\widetilde{\mathscr{Y}}$. Since $h_\pi(\lambda,\mathscr{P})=h_\pi(int(\lambda,\mathscr{P}))\widetilde{\subseteq}int_{so}(h_\pi(\lambda,\mathscr{P}))$, $h_\pi(\lambda,\mathscr{P})$ is a weakly soft *so*-set, which confirms that h_π is weakly soft *so*-open.
2. The proof is similar to that of 1. □

Proposition 5.14. *A bijective soft mapping* $h_\pi:(\mathscr{Y},\mathscr{T}_\mathscr{Y},\mathscr{P})\to(\mathscr{Z},\mathscr{T}_\mathscr{Z},\mathscr{P})$ *is weakly soft so-open iff it is weakly soft so-closed.*

Proof. Necessity: Let $(\lambda,\mathscr{P})$ be a weakly soft *sc*-subset of $(\mathscr{Y},\mathscr{T}_\mathscr{Y},\mathscr{P})$. Since h_π is weakly soft *so*-open, $h_\pi(\lambda^c,\mathscr{P})$ is a weakly soft *so*-set. By bijectiveness of h_π, we obtain $h_\pi(\lambda^c,\mathscr{P})=(h_\pi(\lambda,\mathscr{P}))^c$. So that, $h_\pi(\lambda,\mathscr{P})$ is a weakly soft *sc*-set. Hence, h_π is weakly soft *so*-closed. To prove the sufficient, we follow similar approach. □

Proposition 5.15. *Let $h_\pi : (\mathscr{Y}, \mathscr{T}_\mathscr{Y}, \mathscr{P}) \to (\mathscr{Z}, \mathscr{T}_\mathscr{Z}, \mathscr{P})$ be a weakly soft so-closed mapping and $\widetilde{\mathscr{W}}$ be a soft closed subset of $\widetilde{\mathscr{Y}}$. Then $h_\pi |_\mathscr{W} : (\mathscr{W}, \mathscr{T}_\mathscr{W}, \mathscr{P}) \to (\mathscr{Z}, \mathscr{T}_\mathscr{Z}, \mathscr{P})$ is weakly soft so-closed.*

Proof. Suppose that $(\lambda, \mathscr{P})$ is a soft closed subset of $(\mathscr{W}, \mathscr{T}_\mathscr{W}, \mathscr{P})$. Then there is a soft closed subset $(\delta, \mathscr{P})$ of $(\mathscr{Y}, \mathscr{T}_\mathscr{Y}, \mathscr{P})$ with $(\lambda, \mathscr{P}) = (\delta, \mathscr{P}) \widetilde{\bigcap} \widetilde{\mathscr{W}}$. Since $\widetilde{\mathscr{W}}$ is a soft closed subset of $(\mathscr{Y}, \mathscr{T}_\mathscr{Y}, \mathscr{P})$, then $(\lambda, \mathscr{P})$ is also a soft closed subset of $(\mathscr{Y}, \mathscr{T}_\mathscr{Y}, \mathscr{P})$. Since $h_\pi |_\mathscr{W} (\lambda, \mathscr{P}) = h_\pi(\lambda, \mathscr{P})$, then $h_\pi |_\mathscr{W} (\lambda, \mathscr{P})$ is a weakly soft *sc*-set. Thus, $h_\pi |_\mathscr{W}$ is a weakly soft *so*-closed. □

Proposition 5.16. *The next four statements hold for soft mappings $h_\pi : (\mathscr{Y}, \mathscr{T}_\mathscr{Y}, \mathscr{P}) \to (\mathscr{Z}, \mathscr{T}_\mathscr{Z}, \mathscr{P})$ and $f_\varphi : (\mathscr{Z}, \mathscr{T}_\mathscr{Z}, \mathscr{P}) \to (\mathscr{W}, \mathscr{T}_\mathscr{W}, \mathscr{P})$.*

1. *If h_π is soft i-open and f_φ is soft j-open, then $f_\varphi \circ h_\pi$ is weakly soft so-open, where $i, j \in \{\alpha, semi\}$.*
2. *If $f_\varphi \circ h_\pi$ is weakly soft so-open and h_π is surjective soft continuous, then f_φ is weakly soft so-open.*
3. *If $f_\varphi \circ h_\pi$ is soft open and f_φ is injective weakly soft so-continuous, then h_π is weakly soft so-open.*
4. *If $f_\varphi \circ h_\pi$ is weakly soft so-open and f_φ is injective weakly soft so-irresolute mapping, then h_π is weakly soft so-open.*

Proof. 1. Without loss of generality, let $i = semi$ and $j = \alpha$. Then consider $(\lambda, \mathscr{P}) \neq \phi$ as a soft open subset of $\widetilde{\mathscr{Y}}$. So $h_\pi(\lambda, \mathscr{P}) \neq \phi$ is a soft semi open subset of $\widetilde{\mathscr{Z}}$. Thus, there is a non-null soft open subset $(\delta, \mathscr{P})$ of $\widetilde{\mathscr{Z}}$ with $(\delta, \mathscr{P}) \subseteq h_\pi(\lambda, \mathscr{P})$. Now, $f_\varphi(\delta, \mathscr{P}) \subseteq f_\varphi(h_\pi(\lambda, \mathscr{P}))$. Since f_φ is soft α-open, then $f_\varphi(\delta, \mathscr{P})$ is a non-null soft α-open subset of $\widetilde{\mathscr{W}}$. Therefore, $f_\varphi(\delta, \mathscr{P})$ is a weakly soft *so*-set. This automatically means that $f_\varphi(h_\pi(\lambda, \mathscr{P}))$ is a weakly soft *so*-set. Thus, $f_\varphi \circ h_\pi$ is weakly soft *so*-open.
2. Suppose that $(\lambda, \mathscr{P}) \neq \phi$ is a soft open subset of $\widetilde{\mathscr{Z}}$. Then $h_\pi^{-1}(\lambda, \mathscr{P}) \neq \phi$ is a soft open subset of $\widetilde{\mathscr{Y}}$. Therefore, $(f_\varphi \circ h_\pi)(h_\pi^{-1}(\lambda, \mathscr{P}))$ is a weakly soft *so*-subset of $\widetilde{\mathscr{W}}$. Since h_π is surjective, then $(f_\varphi \circ h_\pi)(h_\pi^{-1}(\lambda, \mathscr{P})) = f_\varphi(h_\pi(h_\pi^{-1}(\lambda, \mathscr{P}))) = f_\varphi(\lambda, \mathscr{P})$. Thus f_φ is weakly soft *so*-open.
3. Let $(\lambda, \mathscr{P}) \neq \phi$ be a soft open subset of $\widetilde{\mathscr{Y}}$. Then $(f_\varphi \circ h_\pi)(\lambda, \mathscr{P}) \neq \phi$ is a soft open subset of $\widetilde{\mathscr{W}}$. Therefore, $f_\varphi^{-1}(f_\varphi \circ h_\pi(\lambda, \mathscr{P}))$ is a weakly soft *so*-subset of $\widetilde{\mathscr{Z}}$. Since f_φ is injective, $f_\varphi^{-1}(f_\varphi \circ h_\pi(\lambda, \mathscr{P})) = (f_\varphi^{-1} f_\varphi)(h_\pi(\lambda, \mathscr{P})) = h_\pi(\lambda, \mathscr{P})$. Thus, h_π is weakly soft *so*-open.
4. Following similar approach of 3. □

We cancel the proof of the next finding because it can be obtained following similar approach of the above proposition.

Proposition 5.17. *The next four statements hold for soft mappings $h_\pi : (\mathscr{Y}, \mathscr{T}_\mathscr{Y}, \mathscr{P}) \to (\mathscr{Z}, \mathscr{T}_\mathscr{Z}, \mathscr{P})$ and $h_\varphi : (\mathscr{Z}, \mathscr{T}_\mathscr{Z}, \mathscr{P}) \to (\mathscr{W}, \mathscr{T}_\mathscr{W}, \mathscr{P})$.*

1. *If h_π is soft i-closed and f_φ is soft j-closed, then $f_\varphi \circ h_\pi$ is weakly soft so-closed, where $i, j \in \{\alpha, semi\}$..*
2. *If $f_\varphi \circ h_\pi$ is weakly soft so-closed and h_π is surjective soft continuous, then f_φ is weakly soft so-closed.*

(3) *If $f_\varphi \circ h_\pi$ is soft closed and f_φ is injective weakly soft so-continuous, then h_π is weakly soft so-closed.*
(4) *If $f_\varphi \circ h_\pi$ is weakly soft so-closed and f_φ is injective weakly soft so-irresolute mapping, then h_π is weakly soft so-closed.*

Definition 5.18. *A bijective soft mapping h_π in which is weakly soft so-continuous and weakly soft so-open is called a weakly soft so-homeomorphism.*

Theorem 5.19. *For a bijective soft mapping $h_\pi : (\mathscr{Y}, \mathscr{T}_{\mathscr{Y}}, \mathscr{P}) \to (\mathscr{Z}, \mathscr{T}_{\mathscr{Z}}, \mathscr{P})$, the following properties are equivalent:*

(1) *h_π is a weakly soft so-homeomorphism.*
(2) *h_π and h_π^{-1} is weakly soft so-continuous.*
(3) *h_π is weakly soft so-closed and weakly soft so-continuous.*

Proof. Straightforward. □

Theorem 5.20. *A bijective soft mapping $h_\pi : (\mathscr{Y}, \mathscr{T}_{\mathscr{Y}}, \mathscr{P}) \to (\mathscr{Z}, \mathscr{T}_{\mathscr{Z}}, \mathscr{P})$ is a weakly soft so-homeomorphism iff one of the following conditions holds.*

(1) *$h_\pi(cl_{so}(\lambda, \mathscr{P})) \widetilde{\subseteq} cl(h_\pi(\lambda, \mathscr{P}))$ and $cl_{so}(h_\pi(\lambda, \mathscr{P})) \widetilde{\subseteq} h_\pi(cl(\lambda, \mathscr{P}))$ for each $(\lambda, \mathscr{P}) \widetilde{\subseteq} \widetilde{\mathscr{Y}}$.*
(2) *$h_\pi^{-1}(int(\lambda, \mathscr{P})) \widetilde{\subseteq} int_{so}^{-1}(h_\pi(\lambda, \mathscr{P}))$ and $h_\pi(int(\delta, \mathscr{P})) \widetilde{\subseteq} int_{so}(h_\pi(\delta, \mathscr{P}))$ for each $(\lambda, \mathscr{P}) \widetilde{\subseteq} \widetilde{\mathscr{Z}}$ and $(\delta, \mathscr{P}) \widetilde{\subseteq} \widetilde{\mathscr{Y}}$.*

Proof. 1. Since $h_\pi(cl_{so}(\lambda, \mathscr{P})) \widetilde{\subseteq} cl(h_\pi(\lambda, \mathscr{P}))$, then by 4 of Theorem 5.6, h_π is weakly soft *so*-continuous and since $cl_{so}(h_\pi(\lambda, \mathscr{P})) \widetilde{\subseteq} h_\pi(cl(\lambda, \mathscr{P}))$, then by 2 of Theorem 5.6, h_π is weakly soft *so*-closed. From Theorem 5.19, we obtain h_π is a weakly soft *so*-homeomorphism.
2. Since $h_\pi^{-1}(int(\lambda, \mathscr{P})) \widetilde{\subseteq} int_{so}^{-1}(h_\pi(\lambda, \mathscr{P}))$, then by 5 of Theorem 5.6, h_π is weakly soft *so*-continuous and since $h_\pi(int(\delta, \mathscr{P})) \widetilde{\subseteq} int_{so}(h_\pi(\delta, \mathscr{P}))$, then by Theorem 5.13, h_π is weakly soft *so*-open. From Definition 5.18, we obtain h_π is a weakly soft *so*-homeomorphism. □

6. Conclusion

In this work, we have introduced the notion of weakly soft *so*-sets which is weaker than the class of soft somewhat open sets as well as we have provided a condition that guarantees the equivalence between them. Then, we have demonstrated the concepts of interior, closure, boundary, and limit soft points via weakly soft *so*-sets and weakly soft *sc*-sets. As unique properties of these soft points, we have successfully given closed formulas that determine soft sets produced by *so*-interior, *so*-closure, and *so*-boundary operators in Proposition 4.3, Proposition 4.11 and Proposition 4.22, respectively. Also, we have employed weakly soft *so*-sets and weakly soft *sc*-sets to prob new types of soft continuity, irresolute, openness, closeness, and homeomorphism.

In upcoming work, we intend to investigate a few topological ideas, including soft compactness, soft Lindelöfness, and soft connectedness utilizing weakly soft *so*-sets. Additionally, we will discuss soft relatively open sets in the supra soft topological content.

Competing interests

The authors declare that they have no competing interests.

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