COMMON LIMIT IN THE RANGE OF $\mathcal{G}-$PROPERTY IN GENERALIZED INTUITIONISTIC FUZZY METRIC SPACE

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Abstract. In this article, we broaden the idea of the $\mathcal{CLR_G}-$property in generalized intuitionistic fuzzy metric space in the setting of $\mathcal{V}-$ fuzzy metric space. Further, we have given the existence theorems for weakly compatible mappings satisfying $\mathcal{CLR_G}-$ property. An illustrative example is also given to justify that our results are new and unique, which will be useful for future researches.

1. Introduction and Preliminaries

Atanassov [2] presented and examined the idea of intuitionistic fuzzy sets. Park [9] utilizing the possibility of intuitionistic fuzzy sets characterized the idea of intuitionistic fuzzy metric spaces with the assistance of continuous t-norm and continuous t-conorm. Gupta and Kanwar[10] defined the concept of $\mathcal{V}-$ fuzzy metric spaces. Jeyaraman and Malligadevi [5] generalized the idia of intuitionistic fuzzy metric spaces and discussed about their properties. Roldan and Martinez Moreno [7] have explored multidimensional coincidence point results utilizing $\mathcal{CLR_G}-$ property in ordered fuzzy metric spaces.

Let p be a positive integer and let $\digamma_p = \{1, 2, \ldots, p\}$. Let $\mathcal{F} : \mathcal{X}^p \to \mathcal{X}$ and $\mathcal{G} : \mathcal{X} \to \mathcal{X}$ be two mappings. Henceforth, let $\varkappa_1, \varkappa_2, \ldots, \varkappa_p : \digamma_p \to \digamma_p$ be p mappings from $\digamma_p$ into itself and let φ be the p- tuple $(\varkappa_1, \varkappa_2, \ldots, \varkappa_p)$.

1.1. **Definition [5].** Let $\mathcal{X}$ be a non-empty set. A 5-tuple $(\mathcal{X}, \mathcal{V}, \mathcal{W}, *, \diamond)$ is said to be generalized intuitionistic fuzzy metric space(compactly $IGFMS$)where $*$ is a

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continuous t-norm, $\diamond$ is a continuous co-norm and $\mathcal{V}, \mathcal{W}$ is a fuzzy set on $\mathcal{X}^n \times (0, \infty)$ satisfying the following conditions for all $t, s > 0$:

($\mathcal{V}$-1) $\mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t) + \mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t) \leq 1$;
($\mathcal{V}$-2) $\mathcal{V}(\vartheta, \vartheta, \dots, \vartheta, \omega, t) > 0$ for all $\vartheta, \omega \in \mathcal{X}$ with $\vartheta \neq \omega$;
($\mathcal{V}$-3) $\mathcal{V}(\vartheta_1, \vartheta_1, \dots, \vartheta_1, \vartheta_2, t) \geq \mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t)$ for all $\vartheta_1, \vartheta_2, \dots, \vartheta_n \in \mathcal{X}$ with $\vartheta_2 \neq \vartheta_3 \neq \dots \neq \vartheta_n$;
($\mathcal{V}$-4) $\mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t) = 1 \Leftrightarrow \vartheta_1 = \vartheta_2 = \vartheta_3 = \dots = \vartheta_n$;
($\mathcal{V}$-5) $\mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t) = \mathcal{V}(p(\vartheta_1, \vartheta_2, \dots, \vartheta_n), t)$, where p is a permutation function;
($\mathcal{V}$-6) $\mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t+s) \geq \mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_{n-1}, \nu, t) * \mathcal{V}(\nu, \nu, \dots, \nu, \vartheta_n, s)$;
($\mathcal{V}$-7) $\mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, .) : (0, \infty) \to [0, 1]$ is continuous;
($\mathcal{V}$-8) $\mathcal{V}$ is a non-decreasing function on $\mathbb{R}^+$, $\lim_{t\to\infty} \mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t) = 1$ and $\lim_{t\to 0} \mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t) = 0$, for all $\vartheta_1, \vartheta_2, \dots, \vartheta_n \in \mathcal{X}, t > 0$;
($\mathcal{V}$-9) $\mathcal{W}(\vartheta, \vartheta, \dots, \vartheta, \omega, t) < 1$ for all $\vartheta, \omega \in \mathcal{X}$ with $\vartheta \neq \omega$;
($\mathcal{V}$-10) $\mathcal{W}(\vartheta_1, \vartheta_1, \dots, \vartheta_1, \vartheta_2, t) \leq \mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t)$ for all $\vartheta_1, \vartheta_2, \dots, \vartheta_n \in \mathcal{X}$ with $\vartheta_2 \neq \vartheta_3 \neq \dots \neq \vartheta_n$;
($\mathcal{V}$-11) $\mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t) = 0 \Leftrightarrow \vartheta_1 = \vartheta_2 = \vartheta_3 = \dots = \vartheta_n$;
($\mathcal{V}$-12) $\mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t) = \mathcal{W}(p(\vartheta_1, \vartheta_2, \dots, \vartheta_n), t)$, where p is a permutation function;
($\mathcal{V}$-13) $\mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t+s) \leq \mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_{n-1}, \nu, t) \diamond \mathcal{W}(\nu, \nu, \dots, \nu, \vartheta_n, s)$;
($\mathcal{V}$-14) $\mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, .) : (0, \infty) \to [0, 1]$ is continuous;
($\mathcal{V}$-15) $\mathcal{W}$ is a non-increasing function on $\mathbb{R}^+$, $\lim_{t\to\infty} \mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t) = 0$ and $\lim_{t\to 0} \mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t) = 1$, for all $\vartheta_1, \vartheta_2, \dots, \vartheta_n \in \mathcal{X}, t > 0$.

1.2. **Lemma [5].** Let $(\mathcal{X}, \mathcal{V}, \mathcal{W}, *, \diamond)$ be a $IGFMS$ such that $\mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, kt) \geq \mathcal{V}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t)$ and $\mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, kt) \leq \mathcal{W}(\vartheta_1, \vartheta_2, \dots, \vartheta_n, t)$ with
$k \in (0, 1)$. Then $\vartheta_1 = \vartheta_2 = \vartheta_3 = \dots = \vartheta_n$.

1.3. **Definition [5].** Let $(\mathcal{X}, \mathcal{V}, \mathcal{W}, *, \diamond)$ be a $IGFMS$.

- A sequence $\{\vartheta_r\}$ is said to be convergent to ϑ if $\lim\limits_{r\to\infty} \mathcal{V}(\vartheta_r, \vartheta_r, \dots, \vartheta_r, \vartheta, t) = 1$ and $\lim\limits_{r\to\infty} \mathcal{W}(\vartheta_r, \vartheta_r, \dots, \vartheta_r, \vartheta, t) = 0$.
- A sequence $\{\vartheta_r\}$ is said to be a Cauchy sequence if $\lim\limits_{r,q\to\infty} \mathcal{V}(\vartheta_r, \vartheta_r, \dots, \vartheta_r, \vartheta_q, t) = 1$ and $\lim\limits_{r,q\to\infty} \mathcal{W}(\vartheta_r, \vartheta_r, \dots, \vartheta_r, \vartheta_q, t) = 0$ as $r, q, \to \infty$ for all $t > 0$.
- The $IGFMS$ $(\mathcal{X}, \mathcal{V}, \mathcal{W}, *, \diamond)$ is said to be complete if every Cauchy sequence in $\mathcal{X}$ is convergent.

1.4. **Definition.** The mapping $\mathcal{F} : \mathcal{X}^p \to \mathcal{X}$ and $\mathcal{G} : \mathcal{X} \to \mathcal{X}$ are said to be $\varphi-$weakly compatible on $IGFMS$ if

$$\mathcal{V}\Big(\mathcal{G}\mathcal{F}(\vartheta_{\varkappa_i(1)}, \vartheta_{\varkappa_i(2)}, \dots \vartheta_{\varkappa_i(p)}), \dots, \mathcal{G}\mathcal{F}(\vartheta_{\varkappa_i(1)}, \vartheta_{\varkappa_i(2)}, \dots \vartheta_{\varkappa_i(p)}), \mathcal{F}(\mathcal{G}(\vartheta_{\varkappa_i(1)}), \mathcal{G}(\vartheta_{\varkappa_i(2)}) \dots, \mathcal{G}(\vartheta_{\varkappa_i(p)}), t\Big) = 1$$

and

$$\mathcal{W}\Big(\mathcal{G}\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\dots\vartheta_{\varkappa_i(p)}),\dots,\mathcal{G}\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\dots\vartheta_{\varkappa_i(p)}),$$
$$\mathcal{F}(\mathcal{G}(\vartheta_{\varkappa_i(1)}),\mathcal{G}(\vartheta_{\varkappa_i(2)})\dots,\mathcal{G}(\vartheta_{\varkappa_i(p)}),t\Big)=0$$

whenever $\mathcal{G}\vartheta_i=\mathcal{G}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\dots\vartheta_{\varkappa_i(p)})$, for all i and some $(\vartheta_1,\vartheta_2,\dots,\vartheta_p,)\in\mathcal{X}^p$.

1.5. **Definition.** Let $(\mathcal{X},\mathcal{V},*)$ be a $IGFMS$. The mapping $\mathcal{F}:\mathcal{X}^p\to\mathcal{X}$ and $\mathcal{G}:\mathcal{X}\to\mathcal{X}$ satisfy $\mathcal{CLR}_{\mathcal{G}}$ - property if there exists $\{\vartheta_r^1\},\{\vartheta_r^2\},\dots,\{\vartheta_r^p\}\in\mathcal{X}$ such that

$$\lim_{r\to\infty}\mathcal{V}\Big(\mathcal{G}(\vartheta_r^i),\dots,\mathcal{G}(\vartheta_r^i),\mathcal{G}(\vartheta_i),t\Big)=1=\lim_{r\to\infty}\mathcal{V}\Big(\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\dots\vartheta_r^{\varkappa_i(p)}),\dots,$$
$$\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\dots\vartheta_r^{\varkappa_i(p)}),\mathcal{G}(\vartheta_i),t\Big)$$

and

$$\lim_{r\to\infty}\mathcal{W}\Big(\mathcal{G}(\vartheta_r^i),\dots,\mathcal{G}(\vartheta_r^i),\mathcal{G}(\vartheta_i),t\Big)=0=\lim_{r\to\infty}\mathcal{W}\Big(\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\dots\vartheta_r^{\varkappa_i(p)}),\dots,$$
$$\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\dots\vartheta_r^{\varkappa_i(p)}),\mathcal{G}(\vartheta_i),t\Big)$$

for some $\vartheta_1,\vartheta_2,\dots,\vartheta_p\in\mathcal{X}$.

2. Main Results

2.1. **Theorem.** Let $(\mathcal{X},\mathcal{V},\mathcal{W},*,\diamond)$ be a $IGFMS$ and $\varphi=(\varkappa_1,\varkappa_2,\dots,\varkappa_p)$ be an p -tuple of mappings from $\digamma_p$ to itself. Consider the functions $\mathcal{F}:\mathcal{X}^p\to\mathcal{X}$ and $\mathcal{G}:\mathcal{X}\to\mathcal{X}$ such that

(3.1.1) $\mathcal{F}$ and $\mathcal{G}$ are $\varphi-$ weakly compatible,
(3.2.2) The pair $(\mathcal{F},\mathcal{G})$ holds $\mathcal{CLR}_{\mathcal{G}}$ - property,
(3.3.3) Assume that there exists $k\in(0,1)$ such that

$$\mathcal{V}\Big(\mathcal{F}(\vartheta_1,\vartheta_2,\dots,\vartheta_p),\dots,\mathcal{F}(\vartheta_1,\vartheta_2,\dots,\vartheta_p),\mathcal{F}(\omega_1.\omega_2,\dots,\omega_p),kt\Big)$$
$$\geq\chi\Big(*_{i=1}^p\mathcal{V}(\mathcal{G}\vartheta_i,\dots,\mathcal{G}\vartheta_i,\mathcal{G}\omega_i,t)\Big)$$

and

$$\mathcal{W}\Big(\mathcal{F}(\vartheta_1,\vartheta_2,\dots,\vartheta_p),\dots,\mathcal{F}(\vartheta_1,\vartheta_2,\dots,\vartheta_p),\mathcal{F}(\omega_1.\omega_2,\dots,\omega_p),kt\Big)$$
$$\leq\chi\Big(\diamond_{i=1}^p\mathcal{W}(\mathcal{G}\vartheta_i,\dots,\mathcal{G}\vartheta_i,\mathcal{G}\omega_i,t)\Big)$$

for all $t>0$, $\vartheta_1,\vartheta_2,\dots,\vartheta_p,\omega_1.\omega_2,\dots,\omega_p\in\mathcal{X}$, where $\chi:[0,1]\to[0,1]$ is a continuous mapping such that $*^p\chi(a)\geq a$ for each $a\in[0,1]$. Suppose that $\chi\Big(*_{i=1}^p\mathcal{V}(\mathcal{G}\vartheta_{\varkappa_j(i)},\dots,\mathcal{G}\vartheta_{\varkappa_j(i)},\mathcal{G}\omega_{\varkappa_j(i)},t)\Big)\geq\chi\Big(*_{i=1}^p\mathcal{V}(\mathcal{G}\vartheta_i,\dots,\mathcal{G}\vartheta_i,\mathcal{G}\omega_i,t)\Big)$
and

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$$\chi\left(\diamond_{i=1}^{p}\mathcal{W}(\mathcal{G}\vartheta_{\varkappa_j(i)},\ldots,\mathcal{G}\vartheta_{\varkappa_j(i)},\mathcal{G}\omega_{\varkappa_j(i)},t)\right)\leq\chi\left(\diamond_{i=1}^{p}\mathcal{W}(\mathcal{G}\vartheta_i,\ldots,\mathcal{G}\vartheta_i,\mathcal{G}\omega_i,t)\right)$$

for all $i,j\in\{1,2,\ldots,p\}$ and $\vartheta_1,\vartheta_2,\ldots,\vartheta_p,\omega_1.\omega_2,\ldots,\omega_p\in\mathcal{X}$.

Then $\mathcal{F}$ and $\mathcal{G}$ have a unique $\varphi-$ common fixed point.

Proof:

The $\mathcal{CLR}_{\mathcal{G}}$ - property for the pair $(\mathcal{F},\mathcal{G})$ implies that

$$\lim_{r\to\infty}\mathcal{V}\left(\mathcal{G}(\vartheta_r^i),\ldots,\mathcal{G}(\vartheta_r^i),\mathcal{G}(\vartheta_i),t\right)=1=\lim_{r\to\infty}\mathcal{V}\left(\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\ldots,\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\mathcal{G}(\vartheta_i),t\right)$$

and

(3.1.4)

$$\lim_{r\to\infty}\mathcal{W}\left(\mathcal{G}(\vartheta_r^i),\ldots,\mathcal{G}(\vartheta_r^i),\mathcal{G}(\vartheta_i),t\right)=0=\lim_{r\to\infty}\mathcal{W}\left(\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\ldots,\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\mathcal{G}(\vartheta_i),t\right)$$

for sequences $\{\vartheta_r^1\},\{\vartheta_r^2\},\ldots,\{\vartheta_r^p\}$ in $\mathcal{X}$ and for some $\vartheta_1,\vartheta_2,\ldots,\vartheta_p\in\mathcal{X}$.

By using (3.1.1) and (3.1.4), we get

$$\mathcal{V}\left(\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\ldots,\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\ldots\vartheta_{\varkappa_i(p)},kt\right)\geq\chi\left(*_{j=1}^{p}\mathcal{V}(\mathcal{G}\vartheta_r^{\varkappa_i(j)},\ldots,\mathcal{G}\vartheta_r^{\varkappa_i(j)},\mathcal{G}\vartheta_{\varkappa_i(j)},t)\right)$$

and

$$\mathcal{W}\left(\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\ldots,\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\ldots\vartheta_{\varkappa_i(p)},kt\right)\leq\chi\left(\diamond_{j=1}^{p}\mathcal{W}(\mathcal{G}\vartheta_r^{\varkappa_i(j)},\ldots,\mathcal{G}\vartheta_r^{\varkappa_i(j)},\mathcal{G}\vartheta_{\varkappa_i(j)},t)\right).$$

Let $r\to\infty$, we get

$$\mathcal{V}\left(\mathcal{G}\vartheta_i,\ldots,\mathcal{G}\vartheta_i,\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\ldots,\vartheta_{\varkappa_i(p)}),kt\right)\geq\chi\left(*_{j=1}^{p}\mathcal{V}(\mathcal{G}\vartheta_{\varkappa_i(j)},\ldots,\mathcal{G}\vartheta_{\varkappa_i(j)},\mathcal{G}\vartheta_{\varkappa_i(j)},t)\right)\geq\chi(1)$$

and

$$\mathcal{W}\left(\mathcal{G}\vartheta_i,\ldots,\mathcal{G}\vartheta_i,\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\ldots,\vartheta_{\varkappa_i(p)}),kt\right)\leq\chi\left(\diamond_{j=1}^{p}\mathcal{W}(\mathcal{G}\vartheta_{\varkappa_i(j)},\ldots,\mathcal{G}\vartheta_{\varkappa_i(j)},\mathcal{G}\vartheta_{\varkappa_i(j)},t)\right)\leq\chi(0).$$

Since, χ verifies $1\leq *^p\chi(1)\leq min(\chi(1),\chi(1),\ldots,\chi(1))=\chi(1)$, then we have $\chi(1)=1$.

Similarly $\chi(0)=0$. Therefore,

(3.1.5) $$\mathcal{G}\vartheta_i=\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\ldots,\vartheta_{\varkappa_i(p)}).$$

Suppose that

(3.1.6) $$\mathcal{G}\vartheta_i=\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\ldots,\vartheta_{\varkappa_i(p)})=\omega_i.$$

Since $\mathcal{F}$ and $\mathcal{G}$ are $\varphi-$ weakly compatible mappings, we have

$$\begin{aligned}&\mathcal{V}\Big(\mathcal{G}\omega_i,\ldots,\mathcal{G}\omega_i,\mathcal{F}(\omega_{\varkappa_i(1)},\omega_{\varkappa_i(2)},\ldots,\omega_{\varkappa_i(p)}),t\Big)\\&\quad=\mathcal{V}\Big(\mathcal{G}\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\ldots,\vartheta_{\varkappa_i(p)}),\ldots,\mathcal{G}\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\ldots,\vartheta_{\varkappa_i(p)}),\\&\qquad\qquad\mathcal{F}(\mathcal{G}\vartheta_{\varkappa_i(1)},\mathcal{G}\vartheta_{\varkappa_i(2)}\ldots,\mathcal{G}\vartheta_{\varkappa_i(p)},t\Big)=1\end{aligned}$$

and

$$\begin{aligned}&\mathcal{W}\Big(\mathcal{G}\omega_i,\ldots,\mathcal{G}\omega_i,\mathcal{F}(\omega_{\varkappa_i(1)},\omega_{\varkappa_i(2)},\ldots,\omega_{\varkappa_i(p)}),t\Big)\\&\quad=\mathcal{W}\Big(\mathcal{G}\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\ldots,\vartheta_{\varkappa_i(p)}),\ldots,\mathcal{G}\mathcal{F}(\vartheta_{\varkappa_i(1)},\vartheta_{\varkappa_i(2)},\ldots,\vartheta_{\varkappa_i(p)}),\\&\qquad\qquad\mathcal{F}(\mathcal{G}\vartheta_{\varkappa_i(1)},\mathcal{G}\vartheta_{\varkappa_i(2)}\ldots,\mathcal{G}\vartheta_{\varkappa_i(p)},t\Big)=0.\end{aligned}$$

This gives,

$$\mathcal{G}\omega_i=\mathcal{F}(\omega_{\varkappa_i(1)},\omega_{\varkappa_i(2)},\ldots,\omega_{\varkappa_i(p)}). \tag{3.1.7}$$

From (3.1.5) and (3.1.7) we get
$(\vartheta_1,\vartheta_2,\ldots,\vartheta_p),(\omega_1,\omega_2,\ldots,\omega_p)\in\mathcal{X}^p$ are two $\varphi-$ coincidence points of $\mathcal{F}$ and $\mathcal{G}$.
For all t and q , define

$\Upsilon_q(t)=*_{j=1}^{q}\mathcal{V}(\mathcal{G}\omega_q^j,\ldots,\mathcal{G}\omega_q^j,\mathcal{G}\vartheta_j,t)\quad\text{and}\quad\Psi_q(t)=\diamond_{j=1}^{q}\mathcal{W}(\mathcal{G}\omega_q^j,\ldots,\mathcal{G}\omega_q^j,\mathcal{G}\vartheta_j,t).$

We guarantee that $\Upsilon_{q+1}(kt)\geq\Upsilon_q(t)\quad and\quad\Psi_{q+1}(kt)\leq\Psi_q(t)$ for all $q,t>0$.
For all q,t and j,

$$\begin{aligned}\mathcal{V}(\mathcal{G}\omega_{q+1}^j,\ldots,\mathcal{G}\omega_{q+1}^j,\mathcal{G}\vartheta_j,kt)&=\mathcal{V}\Big(\mathcal{F}(\omega_q^{\varkappa_j(1)},\omega_q^{\varkappa_j(2)},\ldots,\omega_q^{\varkappa_j(p)}),\ldots,\mathcal{F}(\omega_q^{\varkappa_j(1)},\omega_q^{\varkappa_j(2)},\ldots,\omega_q^{\varkappa_j(p)}),\\&\qquad\qquad\mathcal{F}(\vartheta_{\varkappa_j(1)},\vartheta_{\varkappa_j(2)},\ldots,\vartheta_{\varkappa_j(p)}),kt\Big)\\&\geq\chi\Big(*_{i=1}^{p}\mathcal{V}(\mathcal{G}\omega_q^{\varkappa_j(i)},\ldots,\mathcal{G}\omega_q^{\varkappa_j(i)},\mathcal{G}\vartheta_{\varkappa_j(i)},t)\Big)\\&\geq\chi\Big(*_{i=1}^{p}\mathcal{V}(\mathcal{G}\omega_q^{i},\ldots,\mathcal{G}\omega_q^{i},\mathcal{G}\vartheta_i,t)\Big)\\&=\chi(\Upsilon_q(t))\end{aligned}$$

and

$$\begin{aligned}\mathcal{W}(\mathcal{G}\omega_{q+1}^j,\ldots,\mathcal{G}\omega_{q+1}^j,\mathcal{G}\vartheta_j,kt)&=\mathcal{W}\Big(\mathcal{F}(\omega_q^{\varkappa_j(1)},\omega_q^{\varkappa_j(2)},\ldots,\omega_q^{\varkappa_j(p)}),\ldots,\mathcal{F}(\omega_q^{\varkappa_j(1)},\omega_q^{\varkappa_j(2)},\ldots,\omega_q^{\varkappa_j(p)}),\\&\qquad\qquad\mathcal{F}(\vartheta_{\varkappa_j(1)},\vartheta_{\varkappa_j(2)},\ldots,\vartheta_{\varkappa_j(p)}),kt\Big)\\&\leq\chi\Big(\diamond_{i=1}^{p}\mathcal{W}(\mathcal{G}\omega_q^{\varkappa_j(i)},\ldots,\mathcal{G}\omega_q^{\varkappa_j(i)},\mathcal{G}\vartheta_{\varkappa_j(i)},t)\Big)\\&\leq\chi\Big(\diamond_{i=1}^{p}\mathcal{W}(\mathcal{G}\omega_q^{i},\ldots,\mathcal{G}\omega_q^{i},\mathcal{G}\vartheta_i,t)\Big)\\&=\chi(\Psi_q(t)).\end{aligned}$$

Therefore,

$$\Upsilon_{q+1}(kt) = *_{i=1}^{p}\mathcal{V}(\mathcal{G}\omega_{q+1}^{j},\dots,\mathcal{G}\omega_{q+1}^{j},\mathcal{G}\vartheta_j,kt) \geq *_{i=1}^{p}\chi(\Upsilon_q(t)) \geq \Upsilon_q(t) \quad \text{and}$$
$$\chi_{q+1}(kt) = \diamond_{i=1}^{p}\mathcal{W}(\mathcal{G}\omega_{q+1}^{j},\dots,\mathcal{G}\omega_{q+1}^{j},\mathcal{G}\vartheta_j,kt) \leq \diamond_{i=1}^{p}\chi(\Psi_q(t)) \leq \Psi_q(t).$$

That is, $\Upsilon_{q+1}(kt) \geq \Upsilon_q(t) \geq \cdots \geq \Upsilon_0(\frac{t}{k^q})$ *and* $\Psi_{q+1}(kt) \leq \Psi_q(t) \leq \cdots \leq \Psi_0(\frac{t}{k^q})$ and as a consequence $\lim_{q\to\infty}\Upsilon_q(t) = 1$ and $\lim_{q\to\infty}\Psi_q(t) = 0$ for all $t > 0$.
For all i and t

$$\mathcal{V}(\mathcal{G}\omega_q^i,\dots,\mathcal{G}\omega_q^i,\mathcal{G}\vartheta_i,t) \geq *_{j=1}^{p}\mathcal{V}(\mathcal{G}\omega_q^j,\dots,\mathcal{G}\omega_q^j,\mathcal{G}\vartheta_j,t) \geq \Upsilon_q(t) \quad \text{and}$$
$$\mathcal{W}(\mathcal{G}\omega_q^i,\dots,\mathcal{G}\omega_q^i,\mathcal{G}\vartheta_i,t) \leq \diamond_{j=1}^{p}\mathcal{W}(\mathcal{G}\omega_q^j,\dots,\mathcal{G}\omega_q^j,\mathcal{G}\vartheta_j,t) \leq \Psi_q(t).$$

Which means that $\lim_{q\to\infty}\mathcal{V}(\mathcal{G}\omega_q^i,\dots,\mathcal{G}\omega_q^i,\mathcal{G}\vartheta_i,t) = 1$ and $\lim_{q\to\infty}\mathcal{W}(\mathcal{G}\omega_q^i,\dots,\mathcal{G}\omega_q^i,\mathcal{G}\vartheta_i,t) = 0$ for all $t > 0$. That is, $\lim_{q\to\infty}\mathcal{G}\omega_q^i = \mathcal{G}\vartheta_i$. Thus, $\mathcal{G}\vartheta_i = \mathcal{G}\omega_i$.
From (3.1.6) and (3.1.7), we get

$$\mathcal{G}\omega_i = \mathcal{F}(\omega_{\varkappa_i(1)},\omega_{\varkappa_i(2)},\dots,\omega_{\varkappa_i(p)}) = \omega_i.$$

Hence, $(\omega_1,\omega_2,\dots,\omega_p) \in \mathcal{X}^p$ be a $\varphi-$ common fixed point of $\mathcal{F}$ and $\mathcal{G}$.
Suppose that $(\vartheta_1,\vartheta_2,\dots,\vartheta_p),(\omega_1,\omega_2,\dots,\omega_p) \in \mathcal{X}^p$ are two $\varphi-$ common fixed points of $\mathcal{F}$ and $\mathcal{G}$ with $\vartheta_i \neq \omega_i$. Clearly, $\vartheta_i = \mathcal{G}\vartheta_i = \mathcal{G}\omega_i = \omega_i$, which proves uniqueness.

2.2. **Theorem.** Let $(\mathcal{X},\mathcal{V},\mathcal{W},*,\diamond)$ be a $IGFMS$ and $\varphi = (\varkappa_1,\varkappa_2,\dots,\varkappa_p)$ be an p -tupleof mappings from $\digamma_p$ to itself. Consider the functions $\mathcal{F}:\mathcal{X}^p \to \mathcal{X}$ and $\mathcal{G}:\mathcal{X}\to\mathcal{X}$ such that

(3.2.1) $\mathcal{F}$ and $\mathcal{G}$ are $\varphi-$ weakly compatible,
(3.2.2) The pair $(\mathcal{F},\mathcal{G})$ holds $E.A$ - property,
(3.2.3) Assume that there exists $k \in (0,1)$ such that

$$\mathcal{V}\Bigg(\mathcal{F}(\vartheta_1,\vartheta_2,\dots,\vartheta_p),\dots,\mathcal{F}(\vartheta_1,\vartheta_2,\dots,\vartheta_p),\mathcal{F}(\omega_1.\omega_2,\dots,\omega_p),kt\Bigg) \geq \chi\Bigg(*_{i=1}^{p}\mathcal{V}(\mathcal{G}\vartheta_i,\dots,\mathcal{G}\vartheta_i,\mathcal{G}\omega_i,t)\Bigg)$$

and

$$\mathcal{W}\Bigg(\mathcal{F}(\vartheta_1,\vartheta_2,\dots,\vartheta_p),\dots,\mathcal{F}(\vartheta_1,\vartheta_2,\dots,\vartheta_p),\mathcal{F}(\omega_1.\omega_2,\dots,\omega_p),kt\Bigg) \leq \chi\Bigg(\diamond_{i=1}^{p}\mathcal{W}(\mathcal{G}\vartheta_i,\dots,\mathcal{G}\vartheta_i,\mathcal{G}\omega_i,t)\Bigg)$$

for all $t > 0$, $\vartheta_1,\vartheta_2,\dots,\vartheta_p,\omega_1.\omega_2,\dots,\omega_p \in \mathcal{X}$, where $\chi:[0,1]\to[0,1]$ is a continuous mapping such that $*^p\chi(a) \geq a$ for each $a \in [0,1]$. Suppose that
$\chi\Bigg(*_{i=1}^{p}\mathcal{V}(\mathcal{G}\vartheta_{\varkappa_j(i)},\dots,\mathcal{G}\vartheta_{\varkappa_j(i)},\mathcal{G}\omega_{\varkappa_j(i)},t)\Bigg) \geq \chi\Bigg(*_{i=1}^{p}\mathcal{V}(\mathcal{G}\vartheta_i,\dots,\mathcal{G}\vartheta_i,\mathcal{G}\omega_i,t)\Bigg)$
and
$\chi\Bigg(\diamond_{i=1}^{p}\mathcal{W}(\mathcal{G}\vartheta_{\varkappa_j(i)},\dots,\mathcal{G}\vartheta_{\varkappa_j(i)},\mathcal{G}\omega_{\varkappa_j(i)},t)\Bigg) \leq \chi\Bigg(\diamond_{i=1}^{p}\mathcal{W}(\mathcal{G}\vartheta_i,\dots,\mathcal{G}\vartheta_i,\mathcal{G}\omega_i,t)\Bigg)$
for all $i,j \in \{1,2,\dots,p\}$ and $\vartheta_1,\vartheta_2,\dots,\vartheta_p,\omega_1.\omega_2,\dots,\omega_p \in \mathcal{X}$,
(3.2.4) The Range of $\mathcal{G}$ is a closed subspace of $\mathcal{X}$.

Then $\mathcal{F}$ and $\mathcal{G}$ have a unique $\varphi-$ common fixed point.
Proof:
The Pair of the functions $(\mathcal{F},\mathcal{G})$ holds $E.A$ - property, there exists p sequences $\{\vartheta_r^1\},\{\vartheta_r^2\},\ldots,\{\vartheta_r^p\}\in\mathcal{X}$ such that

$$\lim_{r\to\infty}\mathcal{V}\bigg(\mathcal{G}(\vartheta_r^i),\ldots,\mathcal{G}(\vartheta_r^i),\vartheta_i,t\bigg)=1=\lim_{r\to\infty}\mathcal{V}\bigg(\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\ldots,\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\vartheta_i,t\bigg)$$

and

$$\lim_{r\to\infty}\mathcal{W}\bigg(\mathcal{G}(\vartheta_r^i),\ldots,\mathcal{G}(\vartheta_r^i),\vartheta_i,t\bigg)=0=\lim_{r\to\infty}\mathcal{W}\bigg(\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\ldots,\mathcal{F}(\vartheta_r^{\varkappa_i(1)},\vartheta_r^{\varkappa_i(2)},\ldots\vartheta_r^{\varkappa_i(p)}),\vartheta_i,t\bigg)$$

for all i and some $\vartheta_1,\vartheta_2,\ldots,\vartheta_p\in\mathcal{X}$.
Since the range of $\mathcal{G}$ is a closed subspace of $\mathcal{X}$. That is, $\vartheta_i\in\mathcal{G}(\mathcal{X})$ which implies that $\vartheta_i=\mathcal{G}(\omega_i)$ for some $\omega_i\in\mathcal{X}$ and for all i. Hence $\mathcal{F}$ and $\mathcal{G}$ are fulfill the $\mathcal{CLR}_{\mathcal{G}}$ - property. The result follows from the previous theorem.

2.3. **Example.** Let $(\mathcal{X},\mathcal{V},\mathcal{W},*.\diamond)$ be a $IGFMS$ with $\mathcal{X}=[0,1]$ where

$$\mathcal{V}(\vartheta_1,\vartheta_2,\ldots,\vartheta_n,t)=\frac{t}{t+\sum_{i=1}^n\sum_{i<j}|\vartheta_i-\vartheta_j|}.$$

and

$$\mathcal{W}(\vartheta_1,\vartheta_2,\ldots,\vartheta_n,t)=\frac{\sum_{i=1}^n\sum_{i<j}|\vartheta_i-\vartheta_j|}{t+\sum_{i=1}^n\sum_{i<j}|\vartheta_i-\vartheta_j|}.$$

Suppose $\mathcal{F}:\mathcal{X}^p\to\mathcal{X}$ and $\mathcal{G}:\mathcal{X}\to\mathcal{X}$ defined by

$$\mathcal{F}(\vartheta_1,\vartheta_2,\ldots,\vartheta_p)=\frac{\sum_{i=1}^p\vartheta_i^2}{p},\qquad\mathcal{G}(\vartheta)=\frac{\vartheta}{p}.$$

Consider the sequences for $i=1,2,\ldots,p$.

$$\{\vartheta_q^i\}=\left\{\frac{1}{p}+\frac{(-1)^{i+1}}{q}\right\}.$$

Then we have,

$$\lim_{q\to\infty}\mathcal{F}(\vartheta_q^1,\vartheta_q^2,\ldots,\vartheta_q^p)=\lim_{q\to\infty}\mathcal{G}(\vartheta_q^i)=\mathcal{G}(\frac{1}{p})=\frac{1}{p^2}.$$

This implies that $(\mathcal{F},\mathcal{G})$ holds $\mathcal{CLR}_{\mathcal{G}}$ - property and $E.A$ - property.
And also, the pair $(\mathcal{F},\mathcal{G})$ is $\varphi-$ weakly compatible. All the states of Theorem 3.1 are satisfied. Hence, $\mathcal{F}$ and $\mathcal{G}$ have $\varphi-$ common fixed point.

3. Conclusion

In this article, we presented multidimensional common fixed point theorems for weakly compatible mappings in ordered generalized intuitionistic fuzzy metric spaces. Based on the results in this paper, interesting future researches may be prospective. In the future study, one can establish the integral version of fixed point theorem in the multiplicative sense and can also think of establishing some new fixed point results in different spaces. The work presented here is likely to provide a ground to the researchers to do work in different structures by using these conditions. Moreover, the technique used in this paper is suggestive to discuss the related problem in the general case.

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