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CONTRACTIBILITY OF TOPOLOGICAL SPACES ONTO METRIC SPACES

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A space X is *contractible onto a metric spce* proovided there exists a one-to-one and continuous map from X onto a metric space. A variety of diagonal conditions are used to establish necessary and sufficient conditions in order that a space be contractible onto a metric space, and some metrization theorems are also obtained, e.g.: (i) a separable space is contractible onto a metric space if and only if has a zero-set diagonal; (ii) a space X is metrizable if and only if X has a compatible semi-metric which is uniformly continuous with respect to some product uniformity; (iii) a space X is contractible onto a metric space if and only if X has a sequence $G_1, G_2, \cdots$ of open coverings which satisfies the following two conditions: (a) if $x \neq y$, then there exists n such that $y \notin \mathrm{St}(x, G_n)$; (b) if $A, B \in G_{n+1}$ and $A \cap B \neq \emptyset$, then there exists $C \in G_n$ with $A \cup B \subset C$; (iv) if Y is a perfectly normal space which is the pseudo-compact image of a space which is contractible onto a separable metric space, then Y is contractible onto a separable metric space.

1. Introduction. Terms not defined in the introduction will be defined as needed in the sections below or else may be found in [7]. All topological spaces in this note are assumed to be T_1-spaces.

A topological space X is said to be *contractible onto a metric space* if and only if X maps one-to-one and continuously onto a metric space. It is clear that a topological space (X, T) is contractible onto a metric space if and only if the set X has a topology M such that $M \subset T$ and (X, M) is metrizable. Throughout the paper we shall let E^1 denote the space of real numbers with the usual topology. Note that a topological space (X, T) is contractible onto a metric space if and only if there exists a continuous map $d: (X, T) \times (X, T) \rightarrow E^1$ where d is a metric, not necessarily compatible with T, on the set X. If a space X is contractible onto a metric space (X, d), then d is said to be a *weak metric* for the space X.

The *diagonal* Δ of a space X is the subset $\Delta = \{(x, x): x \in X\}$ of the product space $X \times X$. In §2, necessary and sufficient conditions for metrizability and for contractibility onto a metric space are given in terms of properties of the diagonal. In §3, conditions for metrizability and contractibility onto a metric space are given in terms of the existence of symmetrics having certain properties. In §4, the concept of a graded system of open covers is used to characterize contractibility onto metric spaces. Finally, in §5, the problem of the preservation of contractibility onto a metric space under compact quotient maps is studied.

2. Diagonal conditions. The fact that a space X is contractible onto a separable metric space if and only if $C(X)$ contains a countable subfamily of functions which separates points, which is the key to Theorem 2.1, was first pointed out to the author by F. Slaughter.

THEOREM 2.1. *A separable space is contractible onto a metric space if and only if it has a zero-set diagonal.*

Proof. Any space which is contractible onto a metric space clearly has a zero-set diagonal. Therefore, let X be a separable space which has a zero-set diagonal $\Delta = \{(x, x): x \in X\}$; let $f: X \rightarrow E^1$ be a real-valued continuous map on $X \times X$ such that $f^{-1}(0) = \Delta$ and let $\{x_n: n = 1, 2, \cdots\}$ be a countable, dense subset of X. For each n, define $f_n: X \rightarrow E^1$ by $f_n(x) = f(x_n, x)$. Then, define $d_n: X \times X \rightarrow E^1$ by $d_n(a, b) = |f_n(a) - f_n(b)|$ for all $a, b \in X$. Each function d_n is a continuous pseudo-metric for X, so that if we define $d(a, b) = \sum_{n=1}^{\infty} d_n(a, b)/2^n$, then d is also a continuous pseudo-metric for X. Since d is continuous, the space X is contractible onto the pseudo-metric space (X, d). It only remains to show that d is actually a metric. Let a and b be distinct elements of X. Since f is continuous and $f(a, a) \neq f(a, b)$, there exists an open neighborhood V of a and an open neighborhood W of b such that $f[V \times W] \cap f[V \times V] = \emptyset$. Choose a natural number n such that $x_n \in V$. Then, $(x_n, b) \in V \times W$ and $(x_n, a) \in V \times V$, so that $f(x_n, b) \neq f(x_n, a)$, that is, $f_n(a) \neq f_n(b)$. It follows that $d_n(a, b) > 0$, that is, $d(a, b) > 0$, completing the proof.

Theorem 2.1 raises the general question of whether any space having a zero-set diagonal is necessarily contractible onto a metric space. The next theorem partially answers this question.

If $\mathscr{U}$ is a uniformity for a set X, then let $\mathscr{U} \times \mathscr{U}$ denote the product uniformity. If $U, V \in \mathscr{U}$, let $U \times V = \{((a, b), (c, d)): (a, c) \in U$ and $(b, d) \in V\}$. If $\mathscr{B}$ is a base for the uniformity $\mathscr{U}$, then $\{U \times U: U \in \mathscr{B}\}$ is a base for $\mathscr{U} \times \mathscr{U}$. In all that follows, we let E^1 have its usual metric uniformity.

THEOREM 2.2 *A completely regular space X is contractible onto a metric space if and only if there exists a compatible uniformity $\mathscr{U}$ for X and a uniformly continuous map $f: (X \times X, \mathscr{U} \times \mathscr{U}) \rightarrow E^1$ such that $f^{-1}(0) = \Delta$.*

Proof. Let X be cotractible onto a metric space. Then X has a metric d such that X is contractible onto (X, d). Let $\mathscr{U}$ be any compatible uniformity for the space X which contains the metric uniformity induced by d. Then, $d\colon (X \times X, \mathscr{U} \times \mathscr{U}) \to E^1$ is uniformly continuous and $d^{-1}(0) = \varDelta$.

Conversely, let $\mathscr{U}$ be compatible uniformity for X and $f\colon X \times X, \mathscr{U} \times \mathscr{U}) \to E^1$ be a uniformly continuous map with $f^{-1}(0) = \varDelta$. Choose $V_1, V_2, \cdots$ to be a sequence of symmetric entourages of $\mathscr{U}$ such that $V_{n+1} \circ V_{n+1} \subset V_n$ and $f_2[V_n \times V_n] \subset \{(x, y)\colon |x - y| < 1/n\}$ for $n = 1, 2, \cdots$, where $f_2 = f \times f$. Let $\mathscr{M}$ be the uniformity on the set X generated by the family $\{V_1, V_2, \cdots\}$. Then $T_M \subset T_U$ and (X, T_M) is pseudo-metrizable, where T_M and T_U are the uniform topologies of $\mathscr{M}$ and $\mathscr{U}$ respectively. The proof shall be completed by showing that (X, T_M) is metrizable; it suffices to show that (X, T_M) is Hausdorff, which may be done by showing that $\varDelta = \bigcap_{n=1}^{\infty} V_n$. Let $(a, b) \in \bigcap_{n=1}^{\infty} V_n$. Then $((a, b), (b, b)) \in V_n \times V_n$ for $n = 1, 2, \cdots$, whence $(f(a, b), f(b, b)) \in \{(x, y)\colon |x-y| < 1/n\}$ for all n, that is $|f(a, b) - f(b, b)| = 0$ or $f(a, b) = 0$. But $f^{-1}(0) = \varDelta$, so that $a = b$, that is: $\varDelta = \bigcap_{n=1}^{\infty} V_n$, and the proof is complete.

It was once erroneously believed that every continuously semi-metrizable space(1) was metrizable. However, in 1967 H. Cook gave an example of nonmetrizable but continuously semi-metrizable space [2]. The following theorem is interesting not only as a criterion for metrizability in terms of continuous semi-metrics, but also as a metrization analogue to Theorem 2.2.

THEOREM 2.3. *A necessary and sufficient condition that a space X be metrizable is that X have a compatible uniformity $\mathscr{U}$ and a compatible semi-metric d such that $d\colon (X \times X, \mathscr{U} \times \mathscr{U}) \to E^1$ be uniformly continuous.*

Proof. The necessity of the condition is immediate. Therefore, let d be a compatible semi-metric for X and $\mathscr{U}$ be a compatible uniformity for X such that $d\colon (X \times X, \mathscr{U} \times \mathscr{U}) \to E^1$ is uniformly continuous. Just as in the proof of Theorem 2.2, we choose a sequence $\{V_1, V_2, \cdots\}$ of symmetric entourages in $\mathscr{U}$ such that $d_2[V_n \times V_n] \subset \{(x, y)\colon |x - y| < 1/n\}$ for $n = 1, 2, \cdots$ and note that X is contractible onto the space (X, T_M), where T_M is the uniform topology of the metrizable uniformity $\mathscr{M}$ induced on X by the sequence $\{V_1, V_2, \cdots\}$. We need only show that $T_U \subset T_M$ in order to complete the proof, where T_U is the uniform topology of $\mathscr{U}$. To this end, let A be a closed subset of (X, T_U). Suppose, contrary to what we want to show, that A is not closed in (X, T_M). Then, there exists $x \in X - A$

such that $V_n[x] \cap A \neq \varnothing$ for $n = 1, 2, \cdots$. Choose a sequence $\{a_n\}$ in A with $(x, a_n) \in V_n$. Since $(a_n, a_n) \in V_n$ and $d_2[V_n \times V_n] \subset \{(p, q) : |p - q| < 1/n\}$ we get $|d(x, a_n) - d(a_n, a_n)| < 1/n$, that is $d(x, a_n) \to 0$, which contradicts the original assumption that A is closed in (X, T_U), and completes the proof.

In [11], C. Pixley and P. Roy gave an example of completely regular nonseparable Moore space Λ in which every collection of mutually exclusive open sets is countable. S. Kenton proved the continuous semi-metrizability of Λ in order to show the existence of non-Cauchy completable continuous semi-metric spaces [8]. In [9], the author independently showed that Λ is continuously semi-metrizable thereby giving a negative answer to Question 5 of [2]. Another interesting feature of Λ is that Λ is contractible onto a metric space. Note that Λ is a nonseparable space which is contractible onto a metric space, each of whose weaker metric topologies is separable.

By definition, p is a point of Λ if and only if p is a nonempty finite subset of the real numbers. Given $p \in \Lambda$ and an open subset V of the real line with $p \subset V$, let $R(p, V) = \{q \in \Lambda : p \subset q \subset V\}$. The sets $R(p, V)$ form basic open sets for a topology for the space Λ. To see that Λ is contractible onto a metric space, let d be the Hausdorff metric on Λ defined as follows: $d(x, y) = \inf \{t : x \subset S(y, t)$ and, $y \subset S(x, t)\}$, where $S(z, t)$ denotes the sphere of radius t about the point z. It is easy to show that the space Λ is contractible onto the metric space (Λ, d).

3. Symmetrics.[1]

Let d be a symmetric for a space X. Define $L(d)$ to be the collection of all real-valued functions $f : X \to E^1$ such that $|f(x) - f(y)| \leqq d(x, y)$ whenever $x, y \in X$. Note that if $f \in L(d)$, then f is continuous; consequently, $L(d)$ induces the collection of all continuous pseudo-metrics p on X such that $p \leqq d$.

Let F be a family real-valued functions on a space X. The family F is said to *separate points* provided that whenever x and y are distinct points of X, there exists $f \in F$ such that $f(x) \neq f(y)$. Also, F is said to *separate points and closed sets* provided that whenever $x \notin A$ and A is a closed set, then there exists $f \in F$ such that $f(x) \notin \mathrm{cl}\,(f[A])$.

THEOREM 3.1. *A symmetrizable space X is contractible onto a*

[1] A space X is said to be *symmetrizable* provided that there exists a function d, called a *symmetric*, from $X \times X$ into the nonnegative real numbers such that: (i) $d(x, y) = d(y, x)$ for all $x, y \in X$; (ii) $d(x, y) = 0$ if and only if $x = y$; (iii) a subset A of X is closed if and only if $d(x, A) = \inf \{d(x, a) : a \in A\} > 0$ for every $x \in X - A$. A space X is said to be *semi-metrizable* provided that there exists a function d, called a *semi-metric*, from $X \times X$ into the nonnegative real numbers such that (i) and (ii) above both hold, as well as the following: (iv) let $A \subset X$; $x \in \mathrm{cl}\,(A)$ if and only if $d(x, A) = 0$.

metric space if and only if X has a symmetric d such that $L(d)$ separates points.

Proof. Assume that X has a weak metric q. Let p be any symmetric for X. Define $d = p + q$, that is, for all $a, b \in X$, $d(a, b) = p(a, b) + q(a, b)$. Then, d is a symmetric for X. Let a and b be distinct points of X; define $f: X \to E^1$ by $f(x) = q(a, x)$. Then, we have $|f(x) - f(y)| = |q(a, x) - q(a, y)| \leqq q(x, y)$ by the triangle inequality for q. But $q(x, y) \leqq d(x, y)$, so that $f \in L(d)$. Since $f(a) = 0$ and $f(b) \neq 0$, $L(d)$ separates points.

Conversely, let d be a symmetric on X such that $L(d)$ separates points. Given $a, b \in X$, define $r(a, b) = \sup \{|f(a) - f(b)|: f \in L(d)\}$. Since $L(d)$ separates points, $r(a, b) = 0$ if and only if $a = b$. Clearly, $r(a, b) = r(b, a)$ so that r is a symmetric for the space (X, T_r), where T_r is the topology induced on X by r, that is, $A \subset X$ is closed in (X, T_r) if and only if $r(x, A) > 0$ for every $x \in X - A$. Let the original topology on X be denoted by T_d, which is the topology induced on X by d. For each $f \in L(d)$, we have $|f(a) - f(b)| \leqq d(a, b)$; since $r(a, b) = \sup \{|f(a) - f(b)|: f \in L(d)\}$, we have $r(a, b) \leqq d(a, b)$ for all points a and b in X. It follows that $T_r \subset T_d$, that is, $X = (X, T_d)$ is contractible onto the space (X, T_r). The proof shall be completed by showing that r is a metric. To this end, let x, y, and z be arbitrary points of X. For any $f \in L(d)$, we have $r(x, y) + r(y, z) \geqq |f(x) - f(y)| + |f(y) - f(z)| \geqq |f(x) - f(z)|$. It follows that $r(x, y) + r(y, z) \geqq r(x, z)$, completing the proof.

As for metrizability, we have the following:

THEOREM 3.2. *A necessary and sufficient condition that a space X be metrizable is that X be symmetrizable via a symmetric d such that $L(d)$ separates points and closed sets.*

Proof. Suppose that X is metrizable. Let d be any compatible metric for X. Given $a \in X$, let $f_a: X \to E^1$ be defined by $f_a(x) = d(a, x)$ for all $x \in X$. The function f_a belongs to $L(d)$ for all $a \in X$ and the collection $\{f_a: a \in X\}$ separates points and closed sets.

Conversely, let d be a compatible symmetric for X such that $L(d)$ separates points and closed sets. Define $r(a, b) = \sup \{|f(a) - f(b)|: f \in L(d)\}$. As in the preceding proof, we have $T_r \subset T_d$ and (X, T_r) is a metrizable space. Therefore, we need only show that $T_d \subset T_r$. Suppose that $G \in T_d$, and let $x \in G$. There exists $f \in L(d)$ such that $f(x) \notin \mathrm{cl}\,(f[X - G])$, say $(f(x) - e, f(x) + e) \cap f[X - G] = \varnothing$ for some $e > 0$. Then, if $y \in X - G$, necessarily $f(y) \notin (f(x) - e, f(x) + e)$, that is $|f(x) - f(y)| > e$, whence $r(x, y) > e$. It follows that $G \in T_r$, and we may conclude that $T_d = T_r$, completing the proof.

4. Graded systems of open covers. A collection $\{g_n(x): x \in X; n = 1, 2, \cdots\}$ is said to be a *graded system of open covers* for a space X if and only if the following three conditions are satisfied: (i) $x \in g_n(x)$ and $g_n(x)$ is open for all natural n and all $x \in X$; (ii) $g_{n+1}(x) \subset g_n(x)$ for all natural n and all $x \in X$; (iii) if x and y are distinct points of X, there then exists a natural number n such that $y \notin g_n(x)$. Many kinds of generalized metric spaces have been characterized in terms of graded systems of open covers, e.g., see [3, 4, 5]. We now give such characterizations for spaces which are contractible onto metric spaces.

THEOREM 4.1. *A space X is contractible onto a metric space if and only if X has graded systems of open covers $\{g_n(x)\}$ and $\{V_n(x)\}$ such that the following two conditions are satisfied*:

(A) $y \in g_{n+1}(x)$ *implies that* $g_n + 1(y) \subset g_n(x)$.

(B) $y \in V_n(x)$ *implies that* $x \in g_n(y)$.

Proof. Let X have a weak metric d. Let $g_n(x) = \{y: d(x, y) < 1/2^n\}$. Then $\{g_n(x)\}$ is a graded system of open covers for X which satisfies condition A. Letting $V_n(x) = g_n(x)$, condition B is also satisfied.

To prove the converse, let $\{g_n(x)\}$ be a graded system of open covers which satisfies condition A and $\{V_n(x)\}$ be a graded system of open covers which, in conjunction with $\{g_n(x)\}$, satisfies condition B. Let $G_n = \{(a, b): b \in g_n(a)\}$ and $G_n^{-1} = \{(b, a): (a, b) \in G_n\}$. Let $H_n = G_n \cap G_n^{-1}$. We shall show that $\{H_n: n = 1, 2, \cdots\}$ is a base for a uniformity H on the set X. Note that for all n, we have $\varDelta \subset H_n$ and $H_n = H_n^{-1}$. Therefore, we need only show that $H_n \circ H_n \subset H_{n-1}$ for $n = 2, 3, \cdots$. Let $(a, b) \in H_n \circ H_n$. Then, there exists x such that (a, x) and (x, b) both belong to H_n, that is, $(a, x) \in G_n \cap G_n^{-1}$ and $(x, b) \in G_n \cap G_n^{-1}$. It follows that $a \in g_n(x)$, $x \in g_n(a)$, $b \in g_n(x)$ and $x \in g_n(b)$. By condition A this yields $g_n(x) \subset g_{n-1}(a)$ and $g_n(x) \subset g_{n-1}(b)$. Consequently, we have $b \in g_n(x) \subset g_{n-1}(a)$ and $a \in g_n(x) \subset g_{n-1}(b)$, that is, $(a, b) \in G_{n-1}$ and $(b, a) \in G_{n-1}$, that is, $(a, b) \in H_{n-1}$. It follows that $H_1, H_2, \cdots$ is a base for a uniformity H for the set X. Since H has a countable base, the space (X, T_H) is pseudo-metrizable, where T_H is the uniform topology of H. Let $(a, b) \in \bigcap_{n=1}^{\infty} H_n$. Then $b \in g_n(a)$ for all n so that $a = b$, that is, $\varDelta = \bigcap_{n=1}^{\infty} H_n$ and it follows that (X, T_H) is metrizable.

Let T denote the original topology on X. To complete the proof, we need only show that $T_H \subset T$. Let $G \in T_H$. If $x \in G$, then there exists H_n such that $H_n[x] \subset G$. It is easy to show that $y \in V_n(x) \cap g_n(x)$ implies that $(x, y) \in G_n$ and $(y, x) \in G_n$; as a consequence, $V_n(x) \cap g_n(x) \subset H_n[x]$. It follows that $G \in T$, i.e., $T_H \subset T$, completing the proof.

An almost immediate consequence of Theorem 4.1 is the following.

COROLLARY 4.2. *A necessary and sufficient condition that a space X be contractible onto a metric space is that X have a graded system of open covers $\{g_n(x)\}$ which satisfies condition A of Theorem* 4.1 *and the following symmetry condition*:

(C) $y \in g_n(x)$ *implies that* $x \in g_n(y)$.

If G is a collection of subsets of a space X and $x \in X$, then $\mathrm{St}\,(x, G) = \cup \{g \in G: x \in g\}$. As an easy consequence of Corollary 4.2, we have the following:

THEOREM 4.3. *A space X is contractible onto a metric space if and only if X has a sequence $G_1, G_2, \cdots$ of open coverings which satisfy the following two conditions*:

(D) $\{x\} = \cap \{\mathrm{St}\,(x, G_n): n = 1, 2, \cdots\}$ *for every* $x \in X$.

(E) *if* $A \cap B \neq \varnothing$ *where* $A, B \in G_{n+1}$, *then there exists* $G \in G_n$ *such that* $A \cup B \subset G$.

Proof. Suppose X is contractible onto a metric space. Then X has a weak metric, say d. Let $G_n = \{S(x, 1/2^n): x \in X\}$, where $S(x, 1/2^n) = \{y \in X: d(x, y) < 1/2^n\}$. The sequence $G_1, G_2, \cdots$ clearly satisfies conditions D and E. Conversely, suppose that $G_1, G_2, \cdots$ is a sequence of open coverings of a space X which satisfies conditions D and E. Let $H_n = \{S_1 \cap S_2 \cap \cdots \cap S_n: S_i \in G_i \text{ for } 1 \leqq i \leqq n\}$. Then, $H_1, H_2, \cdots$ is a sequence of open coverings of X which satisfies conditions D and E. Now define $g_n(x) = \mathrm{St}\,(x, H_n)$ for all $x \in X$ and all natural numbers n. Since H_{n+1} refines H_n for all n, we have $g_{n+1}(x) \subset g_n(x)$ and $\{g_n(x): x \in X; n = 1, 2, \cdots\}$ is a graded system of open coverings for the space X. It is easy to verify that $\{g_n(x)\}$ satisfies conditions A and C of Corollary 4.2, completing the proof that X is contractible onto a metric space.

With respect to Theorem 4.3, recall that a space X has a G_δ-diagonal if and only if X has a sequence $\{G_n\}$ of open covers which satisfies condition D, [1]. Theorem 4.3 is also interesting from the standpoint of being an analogue for contractibility onto a metric space of the following well-known metrization theorem of R. L. Moore, which is discussed in [6]; a space X is metrizable if and only if X has a development $G_1, G_2, \cdots$ which satisfies condition E.

5. Compact quotient maps. A map $f: X \to Y$ is said to be *compact* provided that $f^{-1}(y)$ is compact (i.e., *bi*-compact) for every $y \in Y$. A continuous, closed, compact map is said to be *perfect*. The perfect image of a metrizable space is metrizable [10, 13] However, F. Slaughter has shown that the perfect image of a space which is contractible onto a metric space need not be contractible onto a metric space [12]. In fact, Slaughter's example shows that a space may be paracompact and contractible onto the closed unit interval and yet be the domain of a perfect map onto a space which is not contractible onto a metric space. It appears that rather severe restrictions may have to be placed upon domain space, range or map in order to ensure the preservation of the property of being contractible onto a metric space under perfect maps. In fact, the following questions seem not to have been answered yet. If Y is the perfect image of a perfectly normal space which is contractible onto a metric space, must Y be contractible onto a metric space? Do perfect open maps preserve the property of being contractible onto a metric space? If X is contractible onto a metric space and $f: X \to Y$ is a perfect map such that the family of nontrivial fibers of f is discrete, must Y be contractible onto a metric space? A fiber $f^{-1}(y)$ is said to be *nontrivial* provided that the set $f^{-1}(y)$ contains more than one point.

Recall that a space X is contractible onto a separable metric space if and only if $C(X)$ contains a countable subfamily of functions which separates points. From this it follows that a perfectly normal space X is contractible onto a separable metric space if and only if X has a countable open covering $\mathscr{G}$ such that if x and y are distinct points of X, then there exists G and H in $\mathscr{G}$ for which $x \in G$, $y \notin G$, $x \notin H$, and $y \in H$. The following theorem is an easy consequence of this fact. A map $f: X \to Y$ is *pseudo-open* (or *hereditarily quotient*) provided that $y \in \operatorname{int}(f[V])$ whenever V is a neighborhood of $f^{-1}(y)$, where $y \in Y$.

THEOREM 5.1. *Let X be contractible onto a separable metric space. Every perfectly normal space which is the pseudo-open compact image of X is contractible onto a separable metric space.*

Proof. Let $i: X \to Z$ be a one-to-one continuous map from X onto a separable metric space Z and let $f: X \to Y$ be a pseudo-open compact map from X onto a perfectly normal space Y. Let $\mathscr{B}$ be a countable open basis for the space Z and let $G = \{G_1, G_2, \cdots\}$ denote the collection of all finite unions of members of $\mathscr{B}$. The collection $H = \{\operatorname{int}(f[i^{-1}[G_n]]): n = 1, 2, \cdots\}$ is a countable open covering of Y such that if a and b are distinct elements of Y, then there exists A

and B in H such that $a \in A$, $b \in B$, $a \notin B$ and $b \notin A$. Since Y is a perfectly normal space, it follows that Y is contractible onto a separable metric space, completing the proof.

Since perfect normality is preserved by closed maps, the following is an immediate consequence of Theorem 5.1.

COROLLARY 5.2. *Let X be a perfectly normal space which is contractible onto a separable metric space. Then, every perfect image of X is contractible onto a separable metric space.*

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