

## WORDS FROM THE EDITORS

It is our great pleasure and honour to dedicate this special issue of the Nihonkai Mathematical Journal to Professor Kichi-Suke Saito in celebration of his retirement from Niigata University in March 31, 2015.

He was the first editor-in-chief of the Nihonkai Mathematical Journal, and has contributed greatly in first publication of it. After that he continues the editor of this journal and he has made significant contributions to the fields of Banach Spaces, Operator Algebras and Operator Theory.

In 1974, he was employed as an assistant professor at Department of Mathematics, Faculty of Science of Niigata University, and was promoted to a lecturer in 1979, an associate professor in 1982, and then to a professor in 1987. Finally, he became an emeritus professor upon his retirement in April of 2015.

His research topics consist of two main parts:

- (i) Analyticity in operator algebras,
- (ii) Geometry of Banach spaces.

For part (i), when he began his studies, the subject of operator algebras, spearheaded by the famous John von Neumann, was in full bloom. Notably, the basic theory of two classes of such algebras, called  $C^*$ -algebras and von Neumann algebras, had been worked out. The algebras in both classes possess a property called self-adjointness. Guided by his earlier studies of non-commutative Hardy spaces and the theory of invariant subspaces, he had the prescient insight that non-selfadjoint operator algebras are worthy of study. He set out to develop a theory of such algebras, and much of his work for the first 20 years focused on this project. In 1975–1976, he visited Tohoku University and started the study about non-commutative Hardy spaces (see [1, 2]).

In 1977, he visited Paul S. Muhly at University of Iowa and concentrated on the study of non-selfadjoint operator algebras. They introduced the notion of nonselfadjoint crossed product (it's called “analytic crossed product” later) and investigated its maximality, the structure of invariant subspaces and so on (see [3, 4, 5]).

After this, he kept studying non-commutative function algebra, subdiagonal algebra, triangular algebra, analytic subalgebra and analytic crossed product (see [6, 7, 8, 9, 10, 11, 12, 13]).

For part (ii), it was big turning point in 2000 for Professor Saito. He calculated and estimated von Neumann-Jordan constant of absolute norms on  $\mathbb{C}^2$  by means of their corresponding convex function on  $[0, 1]$ . Moreover, he proved that the set of all absolute norms on  $\mathbb{C}^n$  can be in a one-to-one correspondence with the set of all convex functions on some suitable convex subset of  $\mathbb{R}^{n-1}$  (with Takahashi and Kato). These results play important roles in the study of geometrical structure of Banach spaces. In particular, by using this correspondence, the notion of  $\psi$ -direct sum of Banach space was introduced as a generalization of  $\ell_p$ -direct sum and several results on the geometrical properties were obtained by many researchers. Moreover, other geometrical constants such as James constant for concrete spaces were determined and estimated in terms of corresponding convex functions, and some refinements of classical triangle inequality and Beckner's inequality were also studied (see [14, 15, 16, 17, 18, 19, 20]).



His work has been influential and it established him as a world leader in this area. Many researchers are being inspired by his work and are studying the geometrical properties of Banach spaces.

He is also a successful educator, as manifested by activities of researchers obtaining their PhD degrees under his supervision from Niigata University. Now he is a Fellow of the Institute of Science and Technology, Niigata University, and continues to work on mathematics (see [21, 22, 23]).

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