Nihonkai Math. J.
Vol.26(2015), 71–74

# CORRIGENDUM TO "BI-UNIQUE RANGE SETS FOR MEROMORPHIC FUNCTIONS" [NIHONKAI MATH. J. 24 (2013) 121-134]

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## 1. Corrigendum of the paper

There is a gap in the analysis in **Subcase 1.2.1** of the proof of **Theorem 1.1** in **page number 130** line numbers 15–22 from top. In **Subcase 1.2.1** in the case of $\frac{A}{C} = \frac{1}{c}$ using the second fundamental theorem we wrote

$$\begin{aligned}
& (n-1)T(r,f) \\
\leq\ & \overline{N}(r,0;f) + \overline{N}(r,1;f) + \overline{N}(r,\infty;f) + \overline{N}\left(r,\frac{1}{c};F\right) + S(r,f) \\
\leq\ & \ldots .
\end{aligned}$$

Here in the very beginning, at the time of using the second fundamental theorem we counted distinct 1-points of $f$ twice once in $\overline{N}(r,1;f)$ and other in $\overline{N}\left(r,\frac{1}{c};F\right)$. This is the violation of the second fundamental theorem.

So **Page number 130 line numbers 15–22 from top** will be replaced by the following arguments :-

Next suppose $\frac{A}{C} = \frac{1}{c}$. Then

$$F - \frac{A}{C} \equiv \frac{BC - AD}{C(CG + D)}$$

i.e.,

$$(f-1)^3 Q_{n-3}(f) \equiv \frac{BC - AD}{C(CG + D)}.$$

If there are some 1 points of $f$ then the above expression implies that those 1-points of $f$ will be poles of $g$ which is a contradiction to the fact that $f$ and $g$ share the

2010 *Mathematics Subject Classification.* Primary 30D35.
*Key words and phrases.* Meromorphic functions, uniqueness, weighted sharing, shared set.

set $S_1$. Therefore let 1 be an e.v.p of $f$. Now,

$$Q_{n-3}(f) = (f-\alpha_1)(f-\alpha_2)\ldots(f-\alpha_{n-3}),$$

where $\alpha_i$'s $i = 1, 2, \ldots, n-3$ are distinct. Let any $\alpha_i$-pt of $f$ of order $p$ be a pole of order $q$ of $g$ then we have

$$p = nq \geq n.$$

Now by the second fundamental theorem we have

$$\begin{aligned}
& (n-2)T(r,f) \\
\leq\ & \overline{N}(r,0;f) + \overline{N}(r,1;f) + \overline{N}(r,\infty;f) + \sum_{i=1}^{n-3} \overline{N}\,(r,\alpha_i;f) + S(r,f) \\
\leq\ & \overline{N}(r,0;f) + \overline{N}(r,\infty;f) + \frac{(n-3)}{n} T(r,f) + S(r,f) \\
\leq\ & \left(2 + \frac{n-3}{n}\right) T(r,f) + S(r,f),
\end{aligned}$$

which is a contradiction for $n \geq 5$.

**Page number 131 the lastline before Subcase 1.2.3**

Since we have proved that $F \equiv G$ and this is under $\Phi \not\equiv 0$. So $F \equiv G$ implies $\Phi \equiv 0$, we do not have to use Lemma 2.7. So line number 9 from bottom in Page number 131 i.e., "So by Lemma 2.7 we get $f \equiv g$." will be replaced by :-

So we have $\Phi \equiv 0$, a contradiction to the initial assumption.

Next in **Page number 132 Subcase 2.2** there should be more subcases to be considered. Its elaborative form will be as follows :-

**Subcase 2.2.** Next suppose that $f$, $g$ do not share $(0,0)$, $(1,0)$. We now consider the following subcases.

**Subcase 2.2.1.** Suppose there exist $z_0$, $z_1$ such that

$$\begin{aligned}
f(z_0) = 0, &\quad g(z_0) = 1 \\
f(z_1) = 1, &\quad g(z_1) = 0.
\end{aligned}$$

i.e., none of 0 and 1 is an e.v.P. of $f$ and $g$. We note that from $(F-1) \equiv A(G-1)$ we get $P(f) - c(1-A) \equiv AP(g)$. If $A \neq 1$, then $c(1-A) \neq 0$. If $c(1-A) = 1$, then $A = \frac{c-1}{c}$. So $F - \frac{1}{c} \equiv \frac{c-1}{c} G$. At the point $z_0$, we have $F(z_0) = 0$ and $G(z_0) = \frac{1}{c}$. Putting this values we obtain $\frac{-1}{c} = \frac{c-1}{c^2}$ which implies $c = \frac{1}{2}$, a contradiction. So $c(1-A) \neq 0, 1$. Hence $P(f) - c(1-A)$ has simple zeros and consequently we have

$$(f-\omega_1)(f-\omega_2)\ldots(f-\omega_n) \equiv A\frac{(n-1)(n-2)}{2} g^{n-2}(g-\gamma)(g-\delta),$$

where $\omega_i$, $(i = 1, 2, \ldots, n)$ be the distinct zeros of $P(f) - c(1 - A)$. Since $f$, $g$ share the set $S_1$, from above we get 0 is an e.v.P. of $g$, a contradiction.
**Subcase 2.2.2.** If no such $z_0$ exists i.e., if 0 is an e.v.P. of $f$ and 1 is an e.v.P. of $g$, then again as above from $\Phi \equiv 0$ we get

$$F \equiv AG + 1 - A \tag{1}$$

i.e.,

$$\frac{P(f)}{A} \equiv P(g) - \frac{c(A-1)}{A}. \tag{2}$$

Clearly, $\frac{c(A-1)}{A} \neq 0$ as $c \neq 0$ and $A \neq 1$. Now if $\frac{c(A-1)}{A} = 1$ then $A = \frac{c}{c-1}$. Since any 1-point of $f$ is 0-point of $g$, so from (1) we have $\frac{1}{c} = 1 - A$ i.e., $A = \frac{c-1}{c}$. Therefore we get

$$\frac{c-1}{c} = \frac{c}{c-1},$$

which implies $c = \frac{1}{2}$, a contradiction. This implies $\frac{c(A-1)}{A} \neq 1$ and so $P(g) - \frac{c(A-1)}{A}$ has $n$ distinct zeros $\beta_j^{'}$, say $(j = 1, 2, \ldots, n)$. Hence from (2) we have

$$\frac{(n-1)(n-2)}{2A} f^{n-2}(f-\gamma)(f-\delta) \equiv (g-\beta_1^{'})(g-\beta_2^{'})\cdots(g-\beta_n^{'}).$$

Now by the second fundamental theorem and noting that $T(r, g) = T(r, f) + O(1)$ we get

$$\begin{aligned} nT(r,g) &\leq \overline{N}(r,0;g) + \overline{N}(r,1;g) + \sum_{j=1}^{n} \overline{N}(r,\beta_j^{'};g) + S(r,g) \\ &\leq \overline{N}(r,0;g) + \overline{N}(r,\gamma;f) + \overline{N}(r,\delta;f) + S(r,g) \\ &\leq 3T(r,g) + S(r,g), \end{aligned}$$

which is a contradiction for $n \geq 4$.
**Subcase 2.2.3.** If no such $z_0$, $z_1$ exist at all i.e., 0 and 1 both are Picard exceptional values of $f$ and $g$ then again as above we can obtain either (2) or

$$P(f) - c(1-A) \equiv AP(g). \tag{3}$$

We prove that either the right hand side expression of (2) or the left hand side expression of (3) will have $n$ distinct factors. Now if $\frac{c(A-1)}{A} = 1$ i.e., the right hand side expression of (2) does not have $n$ distinct factors, then $A = \frac{c}{c-1}$ and hence $c(1-A) = -A = \frac{c}{1-c} \neq 1$ as $c \neq \frac{1}{2}$. So $P(f) - c(1-A)$ has simple zeros and consequently we have $(f-\omega_1)(f-\omega_2)\cdots(f-\omega_n) \equiv A\frac{(n-1)(n-2)}{2}g^{n-2}(g-\gamma)(g-\delta)$. Therefore by the second fundamental theorem and again noting that $T(r, g) =$

$T(r, f) + O(1)$ we get

$$\begin{aligned} nT(r,f) &\leq \sum_{i=1}^{n} \overline{N}(r,\omega_i;f) + \overline{N}(r,0;f) + \overline{N}(r,1;f) + S(r,f) \\ &\leq \overline{N}(r,\gamma;g) + \overline{N}(r,\delta;g) + S(r,f), \end{aligned}$$

which is a contradiction for $n \geq 3$.

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Received 3 February, 2015