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A TERNARY CHARACTERIZATION OF AUTOMORPHISMS OF $\mathbb{B}(\mathscr{H})$

ALI TAGHAVI JELODAR, MOHAMMAD SAL MOSLEHIAN, AND ABOLFAZL SANAMI

Abstract. If $\mathscr{H}$ is a Hilbert space, φ is a (not necessary linear) $*$-surjective mapping on $\mathbb{B}(\mathscr{H})$ and φ preserves the spectrum of operators of the form ABA^*, then φ is either an algebra automorphism or an algebra anti-automorphism.

1. Introduction

The preserver problems deal with mappings on specific subsets of algebras that preserve certain relations, sets and so on. The theory of spectrum preserving linear mapping originated from Hua's theorem on fields. Later, Kaplansky [11] asked "If φ is linear, satisfies $\varphi(1) = 1$ and maps invertible elements into invertible elements, then is it true that it is a Jordan morphism, that is, $\varphi(x^2) = \varphi(x)^2$ for every x?". By Lemma 4 of [3] one can observe that this question is equivalent to the study of linear mappings preserving the spectrum. Jafarian and Sourour [8] proved that if $\varphi : \mathbb{B}(\mathscr{X}) \to \mathbb{B}(\mathscr{Y})$ is a linear surjective spectrum preserving linear mapping, then either there exists a bounded invertible linear operator A from $\mathscr{X}$ into $\mathscr{Y}$ such that $\varphi(A) = TAT^{-1}$ for every $T \in \mathbb{B}(\mathscr{X})$, or there exists a bounded invertible operator B from the dual $\mathscr{X}^*$ of $\mathscr{X}$ into $\mathscr{Y}$ such that $\varphi(A) = TA^*T^{-1}$ for every $T \in \mathbb{B}(\mathscr{X})$.

Molnár [15] proved that if $\mathscr{H}$ is an infinite dimensional Hilbert space, and $\varphi : \mathbb{B}(\mathscr{H}) \to \mathbb{B}(\mathscr{H})$ is a surjective function with the property that $\mathrm{sp}(\varphi(A)\varphi(B)) = \mathrm{sp}(AB)$ for every A and B in $\mathbb{B}(\mathscr{H})$, then φ is either an algebra automorphism or the negative of an algebra automorphism. Furthermore, if $\psi : \mathbb{B}(\mathscr{H}) \to \mathbb{B}(\mathscr{H})$ is a surjective function with the property that $\mathrm{sp}(\psi(A)^*\psi(B)) = \mathrm{sp}(A^*B)$ for every$A$ and B in $\mathbb{B}(\mathscr{H})$, then ψ is an algebra $*$-automorphism of $\mathbb{B}(\mathscr{H})$ multiplied by a fixed unitary element, see also [14, 21].

There are some related works in the literature dealing with ternary structures; cf. [9, 19] and references therein. Molnár and Šemrl [17] proved that if $\mathscr{A}$, $\mathscr{A}'$

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are (not necessarily uniformly closed) subalgebras of $\mathbb{B}(\mathscr{H})$ containing the finite rank operators and ϕ is an order isomorphism, then either there is a linear bijection $T\colon \mathscr{H} \to \mathscr{H}$ satisfying $\phi(x \otimes y) = Tx \otimes Ty$ $(x, y \in \mathscr{H})$ or there is a bijective conjugate-linear map $T' : \mathscr{H} \to \mathscr{H}$ satisfying $\phi(x \otimes y) = T'y \otimes T'x (x, y \in \mathscr{H})$. Furthermore, if $\mathscr{A}$ contains an ideal of $\mathbb{B}(\mathscr{H})$ different from the ideal of compact operators and $\mathscr{A}$ is generated by its positive elements, then the above T and T' are bounded and $\phi(A) = TAT^*$, respectively $\phi(A) = T'A^*T'^*$, for $A \in \mathscr{A}$. If ϕ is a triple isomorphism, then either there are unitary operators U, V on $\mathscr{H}$ such that $\phi(A) = UAV(A \in \mathscr{A})$ or anti-unitary operators U', V' on $\mathscr{H}$ such that $\phi(A) = U'A^*V'(A \in \mathscr{A})$. Lu [12] showed that if $\mathscr{A}$ is a standard operator algebra (i.e., an algebra containing all finite rank operators) on a Banach space of dimension > 1, and if $\mathscr{B}$ is an arbitrary $\mathbb{Q}$-algebra, then a bijective mapping $\phi : \mathscr{A} \to \mathscr{B}$ which satisfies $\phi(kABA) = k\phi(A)\phi(B)\phi(A)$ for all $A, B \in \mathscr{A}$, where k is a fixed nonzero rational number, is additive; see also [16].

In addition, many mathematicians have obtained valuable results in this topic; see [4, 5, 7, 20, 22] and references therein.

Following some ideas of [15], we prove that if $\mathscr{H}$ is a Hilbert space, φ is a $*$-surjective (not necessary linear) mapping on $\mathbb{B}(\mathscr{H})$ and

$$\mathrm{sp}(\varphi(A)\varphi(B)\varphi(A)^*) \quad = \quad \mathrm{sp}(ABA^*) \tag{1.1}$$

then φ is either an algebra automorphism or an algebra anti-automorphism. We also show that if $\varphi : \mathbb{B}(\mathscr{H}) \to \mathbb{B}(\mathscr{H})$ is a $*$-surjective mapping with the property that

$$\mathrm{sp}(|\varphi(A)|^2\varphi(B)) \quad = \quad \mathrm{sp}(|A|^2B), \tag{1.2}$$

where $A, B \in \mathbb{B}(\mathscr{H})$, then φ is either an algebra automorphism or an algebra anti-automorphism. In the case where $\mathscr{H}$ is a one dimensional Hilbert space we have $\mathscr{H} \simeq \mathbb{C} \simeq \mathbb{B}(\mathscr{H})$ and it is easy to see that $\varphi(I) = I$ and φ is the identity map. From now on, we assume that $\mathscr{H}$ is of the dimension greater than one.

2. Preliminaries

Throughout the paper $\mathscr{H}$ stands for a complex Hilbert space with dimension greater than one. The spectrum of an operator A is denoted by $\mathrm{sp}(A)$. If x and y are in $\mathscr{H}$, then $x \otimes y$ stands for the rank one operator defined by

$$(x \otimes y)(z) \quad = \quad \langle z, y\rangle x = y^*(z)x. \tag{2.1}$$

Note that

$$\mathrm{sp}_{\mathrm{p}}(x \otimes y) \quad = \quad \{0, y^*(x)\}, \tag{2.2}$$

where $\mathrm{sp_p}$ means the point spectrum. Recall that the point spectrum $\mathrm{sp_p}(A)$ of an operator A is $\{\lambda \in \mathbb{C} : Ax = \lambda x \text{ for some } x \neq 0\}$. Note that by the Fredholm alternative theorem [18, P.25], for any compact operator $A \in \mathbb{B}(\mathscr{H})$ we have

$$\mathrm{sp_P}(A) - \{0\} \quad = \quad \mathrm{sp}(A) - \{0\}. \tag{2.3}$$

Moreover recall that $\mathrm{sp}(AB) - \{0\} = \mathrm{sp}(BA) - \{0\}$ for every A and B in $\mathbb{B}(\mathscr{H})$.

Obviously, every finite rank operator $A \in \mathbb{B}(\mathscr{H})$ is a finite linear combination of such operators and the ideal of compact operators is the closed linear span of finite rank operators. On the finite rank elements of $\mathbb{B}(\mathscr{H})$, one can define the trace functional tr by $\mathrm{tr}(A) = \sum_{j=1}^{n} y_j^*(x_j)$, where $A = \sum_{j=1}^{n}(x_j \otimes y_j)$. Then tr is a well-defined linear functional with the property $\mathrm{tr}(TA) = \mathrm{tr}(AT)$ for any finite rank operator $A \in \mathbb{B}(\mathscr{H})$ and for any $T \in \mathbb{B}(\mathscr{H})$.

If $B \in \mathbb{B}(\mathscr{H})$ and $\mathrm{tr}(ABA^*) = 0$ for every rank one operator A, then

$$B = 0. \tag{2.4}$$

In fact, for any nonzero $x, y \in \mathscr{H}$ by considering $A = x \otimes y$ we have

$$\begin{aligned} \mathrm{sp}(ABA^*) \quad &= \quad \mathrm{sp}(\langle By, y\rangle(x \otimes x)) &(2.5)\\ &= \quad \{0, \langle By, y\rangle \|x\|^2\}. &(2.6) \end{aligned}$$

Hence $0 = \mathrm{tr}(ABA^*) = \langle By, y\rangle \|x\|^2$, whence $B = 0$. Furthermore recall that (see [15]):

$$A \text{ is rank one} \iff A \neq 0,\ 0 \in \mathrm{sp_p}(AB),\ \mathrm{Card}\left(\mathrm{sp_p}(AB)\right) \leq 2\ (B \in \mathbb{B}(\mathscr{H})). \tag{2.7}$$

In this matter, Brits, Lenore, and Raubenheimer in [6, Corollary 2.4] proved that

$$A \text{ is rank one} \iff A \neq 0, \text{ and } \mathrm{Card}\left(\mathrm{sp}(AB) - \{0\}\right) \leq 1\ (B \in \mathbb{B}(\mathscr{H})). \tag{2.8}$$

3. Main result

We begin our work with some lemmas.

Lemma 3.1. *Suppose $A \in \mathbb{B}(\mathscr{H})$. Then A is rank one if and only if so is A^*A.*

Proof. If A is rank one, then it is clear that A^*A is also rank one.

Conversely, if $\dim \mathrm{ran}(A) \geq 2$, where $\mathrm{ran}(A)$ denotes the range of A, then we can find at least two linearly independent elements z_1, z_2 in $\mathrm{ran}(A)$. Since $\dim \mathrm{ran}(A^*A) = 1$, there exist scalars $\alpha, \beta \in \mathbb{C}$ with $\alpha\beta \neq 0$ such that

$$\alpha A^* z_1 + \beta A^* z_2 \quad = \quad 0. \tag{3.1}$$

Now, if we assume that $\alpha z_1 = y_1$ and $-\beta z_2 = y_2$, then the elements $y_1,\ y_2$ are in $\mathrm{ran}(A)$ and are linearly independent. Now, from (3.1) we have $y_1 - y_2 \in \ker(A^*) =$

$\overline{\operatorname{ran}(A)}^{\perp}$. On the other hand, since $y_1, y_2 \in \operatorname{ran}(A)$, we infer that $y_1 = y_2$. This is a contradiction. $\square$

Lemma 3.2. *If $\varphi : \mathbb{B}(\mathscr{H}) \to \mathbb{B}(\mathscr{H})$ is a surjective mapping satisfying* (1.1)*, then $ker(\varphi) = \{0\}$.*

Proof. Let $\varphi(A) = 0$. Then $\operatorname{sp}(\varphi(A)\varphi(B)\varphi(A)^*) = 0$. It follows from property (1.1) that $\operatorname{sp}(ABA^*) = 0$ for every $B \in \mathbb{B}(\mathscr{H})$. In particular, $\operatorname{sp}(A^*A) = 0$, whence $A = 0$. Furthermore, since φ is surjective, then there exists an operator A such that $\varphi(A) = 0$. It follows from the first part of the proof that $A = 0$. Note that φ is not necessarily linear. $\square$

Remark 3.3. Under conditions of Lemma 3.2 we conclude that

$$A^*A = 0 \Leftrightarrow A = 0 \Leftrightarrow \varphi(A) = 0 \Leftrightarrow \varphi(A)^*\varphi(A) = 0$$

for any operator $A \in \mathbb{B}(\mathscr{H})$.

Lemma 3.4. *Suppose that $\varphi : \mathbb{B}(\mathscr{H}) \to \mathbb{B}(\mathscr{H})$ is a surjective mapping satisfying* (1.1)*. Then*

(i) *φ is injective,*

(ii) *φ preserves rank one operators in both directions.*

Proof. (i) If $\varphi(B) = \varphi(B')$, then $\operatorname{sp}(\varphi(A)\varphi(B)\varphi(A)^*) = \operatorname{sp}(\varphi(A)\varphi(B')\varphi(A)^*)$ for every A in $\mathbb{B}(\mathscr{H})$. So, we have $\operatorname{sp}(ABA^*) = \operatorname{sp}(AB'A^*)$ for every A in $\mathbb{B}(\mathscr{H})$. If we put $A = x \otimes y$, where $\|x\| = 1$, then by using (2.5), $\langle By, y \rangle = \langle B'y, y \rangle$ $(y \in \mathscr{H})$. Therefore $B = B'$.

(ii) If A is rank one operator, then by Lemma 3.1 A^*A is rank one operator and by (2.7), we have

$$A^*A \neq 0, \quad 0 \in \operatorname{sp}_p(A^*AB), \quad \operatorname{Card} \operatorname{sp}_P(A^*AB) \leq 2 \qquad (B \in \mathbb{B}(\mathscr{H})).$$

It follows from the Fredholm alternative theorem and conditions (1.1), (2.2), (2.3) and (2.7) that

$$\operatorname{Card} \operatorname{sp}_P(A^*AB) = \operatorname{Card} \operatorname{sp}(\varphi(A)^*\varphi(A)\varphi(B)) \leq 2 \tag{3.2}$$

$$\operatorname{sp}_P(A^*AB) - \{0\} = \operatorname{sp}(\varphi(A)^*\varphi(A)\varphi(B)) - \{0\}. \tag{3.3}$$

Remark 3.3 implies that $\varphi(A)^*\varphi(A) \neq 0$ and we deduce from (3.2) and (3.3) that

$$\varphi(A)^*\varphi(A) \neq 0 \quad \text{and} \quad \operatorname{Card} \operatorname{sp}(\varphi(A)^*\varphi(A)\varphi(B)) - \{0\} \leq 1 \qquad (B \in \mathbb{B}(\mathscr{H})).$$

By (2.8), $\varphi(A)^*\varphi(A)$ is rank one operator and again by applying Lemma 3.1, we deduce that $\varphi(A)$ is rank one. Due to φ is bijective, φ preserves rank one operators in both directions. $\square$

Lemma 3.5. *Suppose that $\varphi : \mathbb{B}(\mathscr{H}) \to \mathbb{B}(\mathscr{H})$ is a surjective mapping satisfying* (1.1)*. Then*

(i) *if A is a rank one operator, then tr$(\varphi(A)\varphi(B)\varphi(A)^*)$=tr$(ABA^*)$ for every $B \in \mathbb{B}(\mathscr{H})$,*

(ii) *φ is a linear mapping.*

Proof. (i) Let $B \in \mathbb{B}(\mathscr{H})$. Since A is rank one operator, there exist elements x and y such that $A = x \otimes y$. By Lemma 3.4, $\varphi(A)$ is also rank one. Due to $\mathrm{sp}(\varphi(A)\varphi(B)\varphi(A)^*) = \mathrm{sp}(ABA^*)$ $(B \in \mathbb{B}(\mathscr{H}))$ we have $\mathrm{tr}(\varphi(A)\varphi(B)\varphi(A)^*) = \mathrm{tr}(ABA^*)$.

(ii) Taking (i) into account, if A is rank one operator, then

$$\begin{aligned}
\mathrm{tr}(\varphi(A)(\varphi(B)+\varphi(B'))\varphi(A)^*) &= \mathrm{tr}(\varphi(A)\varphi(B)\varphi(A)^* + \varphi(A)\varphi(B')\varphi(A)^*) \\
&= \mathrm{tr}(\varphi(A)\varphi(B)\varphi(A)^*) + \mathrm{tr}(\varphi(A)\varphi(B')\varphi(A)^*) \\
&= \mathrm{tr}(ABA^*) + \mathrm{tr}(AB'A^*) \\
&= \mathrm{tr}(A(B+B')A^*) \\
&= \mathrm{tr}(\varphi(A)\varphi(B+B')\varphi(A)^*).
\end{aligned}$$

Hence $\mathrm{tr}(C\,(\varphi(B+B')-\varphi(B)-\varphi(B'))\,C^*) = 0$ for every rank one operator C, since φ preserves rank one operators in both directions. By (2.4), $\varphi(B)+\varphi(B') = \varphi(B+B')$. One can check that φ is homogeneous in a similar way. □

Before we prove our main result, we notice a remark. If A is a positive operator in $\mathbb{B}(\mathscr{H})$, that is an operator of the form T^*T for some $T \in \mathbb{B}(\mathscr{H})$, then $\mathrm{sp}(\varphi(A)) \subseteq [0,\infty)$. Indeed, since φ is a surjective map, there exists an operator $B \in \mathbb{B}(\mathscr{H})$ such that $\varphi(B) = I$. Thus

$$\begin{aligned}
\mathrm{sp}(\varphi(A)) &= \mathrm{sp}(\varphi(B)\varphi(A)\varphi(B)^*) \\
&= \mathrm{sp}(BAB^*) && \text{by (1.1)} \\
&= \mathrm{sp}((BT^*)(BT^*)^*) \\
&\subseteq [0,\infty).
\end{aligned}$$

We now ready to state the main result.

Theorem 3.6. *If $\varphi : \mathbb{B}(\mathscr{H}) \to \mathbb{B}(\mathscr{H})$ is a $*$- surjective mapping satisfying* (1.1)*, then φ is either an algebra automorphism or an algebra anti-automorphism.*

Proof. By Lemmas 3.4 and 3.5 φ is a linear bijective mapping preserving rank-one operators. There are some characterizations of such mappings. It follows from the arguments in [8, Theorem 2] that

(i) there exist bijective linear operators $T : \mathscr{H} \to \mathscr{H}$ and $S : \mathscr{H} \to \mathscr{H}$ such that $\varphi\,(x \otimes y) = Tx \otimes Sy$ where $x, y \in \mathscr{H}$;

or

(ii) there exist bijective linear operators $T' : \mathscr{H} \to \mathscr{H}$ and $S' : \mathscr{H} \to \mathscr{H}$ such that $\varphi(x \otimes y) = T'y \otimes S'x$, where $x, y \in \mathscr{H}$.

Assume (i) holds. Let $x \in \mathscr{H}$. According to Lemma 3.5 (i), if we put $A = B = x \otimes x$ with $\|x\| = 1$, then

$$\langle x, x\rangle\langle x, x\rangle\langle x, x\rangle = \langle Tx, Sx\rangle\langle Tx, Sx\rangle\langle Tx, Sx\rangle.$$

Now, since $A = x \otimes x$ is positive by the remark above, $\operatorname{sp}(\varphi(A)) = \operatorname{sp}(Tx \otimes Sx) = \{0, \langle Tx, Sx\rangle\} \subseteq [0, \infty)$. So $\langle Tx, Sx\rangle = \langle x, x\rangle$, whence $T^*S = I$ that is $S = (T^{-1})^*$. Hence

$$\begin{aligned} \varphi(x \otimes y) &= Tx \otimes Sy \\ &= Tx \otimes (T^{-1})^* y \\ &= T(x \otimes y)T^{-1}. \end{aligned}$$

Therefore $\varphi(A) = TAT^{-1}$ for every rank one operator A.

Next, let B be an operator in $\mathbb{B}(\mathscr{H})$ and A be a rank one operator. By Lemma 3.5 (i),

$$\begin{aligned} \operatorname{tr}(\varphi(A)\,\varphi(B)\,\varphi(A)^*) &= \operatorname{tr}(ABA^*) \\ &= \operatorname{tr}\left(TAT^{-1}TBT^{-1}TA^*T^{-1}\right) \\ &= \operatorname{tr}\left(\varphi(A)\,TBT^{-1}\varphi(A)^*\right). \end{aligned}$$

Hence $\varphi(B) = TBT^{-1}$ by (2.4).

Assume now that (ii) holds. By the same argument as in (i) one can prove that $T' : \mathscr{H} \to \mathscr{H}$ is a bounded invertible linear operator and

$$\varphi(x \otimes y) = \lambda T'(y \otimes x)T'^{-1},$$

where $x, y \in \mathscr{H}$. We can similarly show that φ is of form $\varphi(A) = \lambda T' A^* T'^{-1}$ where A is a rank one operator. Just as above, we infer that $\lambda = 1$, and A can be assumed to be an arbitrary operator in $\mathbb{B}(\mathscr{H})$. $\square$

Theorem 3.7. *If $\varphi : \mathbb{B}(\mathscr{H}) \to \mathbb{B}(\mathscr{H})$ is a $*$-surjective mapping satisfying* (1.2), *then φ is either an algebra automorphism or an algebra anti-automorphism.*

Proof. By modifying our arguments we can show that Lemmas 3.4 and 3.5 are true. Let A be a positive operator. Due to φ is a surjective map, there exists an operator

$B \in \mathbb{B}(\mathscr{H})$ such that $\varphi(B) = I$. Hence

$$\begin{aligned} \mathrm{sp}(\varphi(A)) - \{0\} &= \mathrm{sp}(\varphi(B)^*\varphi(B)\varphi(A)) - \{0\} \\ &= \mathrm{sp}(B^*BA) - \{0\} && \text{by (1.2)} \\ &= \mathrm{sp}(BAB^*) - \{0\} \\ &= \mathrm{sp}((BT^*)(BT^*)^*) - \{0\} \\ &\subseteq [0, \infty) - \{0\}. \end{aligned}$$

Thus $\mathrm{sp}(\varphi(A)) \subseteq [0, \infty)$. By the same argument as in Theorem 3.6, in case (i) we can conclude $\varphi(A) = TAT^{-1}$ for every rank one operator and can easily check that equality $\varphi(AS) = \varphi(A)\varphi(S)$ holds for any rank one operator A and arbitrary operator $S \in \mathbb{B}(\mathscr{H})$. Hence if A is rank one operator, then

$$\begin{aligned} \mathrm{tr}(\varphi(A)\varphi(SUS^*)\varphi(A)^*) &= \mathrm{tr}(ASUS^*A^*) \\ &= \mathrm{tr}((AS)U(AS)^*) \\ &= \mathrm{tr}(\varphi(AS)\varphi(U)\varphi(AS)^*) \\ &= \mathrm{tr}(\varphi(A)\varphi(S)\varphi(U)\varphi(S)^*\varphi(A)^*). \end{aligned}$$

Thus, $\varphi(SUS^*) = \varphi(S)\varphi(U)\varphi(S)^*$ by Lemma 3.5 and (2.4). Now if we set $S = U = I$, then we can deduce that φ is unital and by (1.2) $\mathrm{sp}(\varphi(B)) = \mathrm{sp}(B)$ for every $B \in \mathbb{B}(\mathscr{H})$. Therefore, $0 \in \mathrm{sp}(ABA^*) \Leftrightarrow 0 \in \mathrm{sp}(\varphi(ABA^*)) = \mathrm{sp}(\varphi(A)\varphi(B)\varphi(A)^*)$. If (ii) holds by utilizing a similar argument we can conclude the result. $\square$

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(Ali Taghavi Jelodar) Department of Mathematics, Faculty of Basic Sciences, University of Mazandaran, P.O. Box 47416-1468, Babolsar, Iran
E-mail address: `taghavi@nit.ac.ir`

(Mohammad Sal Moslehian) Department of Pure Mathematics, Center of Excellence in Analysis on Algebraic Structures (CEAAS), Ferdowsi University of Mashhad, P. O. Box 1159, Mashhad 91775, Iran
E-mail address: `moslehian@ferdowsi.um.ac.ir and moslehian@ams.org`

(Abolfazl Sanami) Department of Mathematics, Faculty of Basic Sciences, University of Mazandaran, P.O. Box 47416-1468, Babolsar, Iran; Tusi Mathematical Research Group (TMRS), Mashhad, Iran
E-mail address: `a.sanami@yahoo.com`

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