ERRATA

VOLUME 29

Book Review by Georg Kreisel of *Kurt Gödel, Collected Works, Volume I, Publications 1929–1936*

Page 168, lines 11/12	*Read* Sterling Hayden *for* George C. Scott
Page 173, line 18	*Read* $\vec{P} \mapsto D_{\vec{P}}$ *for* $\vec{P} \vdash D_{\vec{P}}$.
Page 179, line 9	*Read* Σ_1^0 *for* Σ_0^1.
Page 180, line 8	*Read* AI *for* AL.

VOLUME 28

Correction to 'Survey of generalizations of Urquhart semantics', by R. A. Bull, *Notre Dame Journal of Formal Logic*, vol. 28 (1987), pp. 220–237.

My survey of generalizations of Urquhart semantics gave a summary of A. Q. Abraham's unpublished "Completeness of quantified classical relevant logic". Abraham's paper was discovered to have a subtle but apparently fatal flaw and my survey was hastily revised to avoid this flaw. That revision, while correct in principle, was badly botched in detail. This note gives further details of the necessary correction, together with a description of the original version, lest my botches be attributed to Abraham.

In Abraham's original version, the theory T introduced on p. 234 of my survey is not the set of theses, but any regular, prime, consistent-and-complete theory which extends the set of theses. Further, $\mathbf{P}_T$ is the set of principal T-theories which are *consistent*. To prove that $\mathbf{P}_T$ is closed under $\cdot$ requires

$$\vdash (A \to F) \vee ((A \to F) \to F)$$

and the primeness of T. To prove that the condition $a \cdot a^* \leq 0$ holds on $\mathbf{P}_T$ requires

$$\vdash (\overline{A \to (\neg A)}) \vee (A \to B)$$

and the primeness of T. Deriving these theses requires the Contraction Axiom W, $\vdash (A \to (A \to B)) \to (A \to B)$, via

$$\vdash (A \to B) \to (\bar{A} \vee B).$$

To prove that the condition

if $a \cdot b \leq 0$ then, for some c, $a \leq c$ & $b \leq c^*$

holds on $\mathbf{P}_T$ requires that T be closed under the rule

if $\vdash (A \wedge B) \to (\neg C)$ then $\vdash (A \wedge C) \to (\neg B)$

of 'classical' relevant logic. Alas, there is no reason to believe that the set of theses can be extended to a consistent prime theory T which satisfies this condition.

To avoid this flaw, it is necessary to take T to be the set of theses of *CR*-

W or CRQ-W. Since T is then not prime and W is not available in my paper, the condition of consistency is dropped from the theories in $\mathbf{P}_T$ and $\mathbf{N}_T$. N.B: I should have deleted the clause "the primeness of T and various theses of CR-W" (p. 234) in the revision of my paper. In this context of 'classical' logic we are only adding the set of all formulas, call it 1, to $\mathbf{P}_T$ and $\mathbf{N}_T$, with conditions

$$a \leq 1 \text{ and } a \cdot 1 = 1 \text{ for all } a \in \mathbf{P}_T.$$

N.B: I should have modified the conditions on valuations V for $\neg$ and $-$ to:

$$V(\neg A, a) = T \text{ iff, for each } b \text{ such that } a \leq b < 1,\ V(A,b) = F,$$
$$V(\bar{A}, a) = T \text{ iff, for each } b \text{ such that } a^* \leq b < 1,\ V(A,b) = F.$$

The condition on $\mathbf{P}_T$ that $a \cdot a^* \leq 0$ cannot hold with the revised identification of T as the set of theses. However, this condition and the other mentioned above are only used in the verification of

$$\vdash (A \to \bar{B}) \to (B \to \bar{A}).$$

The main purpose of this note is to replace both conditions by

$$\text{if } a \cdot b \leq c^* < 1 \text{ then, for some } d,$$
$$a \cdot c, b^* \leq d < 1.$$

The new proofs are slight modifications of Abraham's. This condition does hold in $\mathbf{P}_T$, taking d to be the principal T-theory generated by $(A \circ C) \wedge (\neg \bar{B})$, where a, b, c are generated by A, B, C. For if it is not the case that $d < 1$ then

$$\vdash ((A \circ C) \wedge (\neg \bar{B})) \to F$$

implies

$$\vdash (A \circ C) \to \bar{B}$$

(using the 'classical' rule mentioned above) implies

$$\vdash (A \circ B) \to \bar{C}$$

(using contraposition for $\bar{\ }$) implies

$$\vdash (A \circ B) \to F$$

(using $\vdash (A \circ B) \to \neg \bar{C}$ from $a \cdot b \leq c^*$) – contrary to $a \cdot b < 1$. Now, to verify that

$$\vdash (A \to \bar{B}) \to (B \to \bar{A}),$$

suppose that

$$V(A \to \bar{B}, a) = T, V(B, b) = T,$$

and show that $V(\bar{A}, a \cdot b) = T$. For each c with $(a \cdot b)^* \leq c < 1$, we have that $a \cdot b \leq c^* < 1$ in Abraham's semantics, and hence there is some $d < 1$ with

$$a \cdot c \leq d \text{ and } b \leq d^*.$$

Therefore

$$V(B, d^*) = T \text{ and } (a \cdot c)^* \leq d^*$$

implies

$$V(A \to \bar{B}, a) = T, V(\bar{B}, a \cdot c) = F$$

implies

$$V(A, c) = F$$

for each c with $(a \cdot b)^* \le c < 1$ – so that $V(\bar{A}, a \cdot b) = T$ as required.

Department of Mathematics
University of Canterbury
Christchurch 1
New Zealand

VOLUME 14

Corrigendum to 'Diagonalization and the recursion theorem', by James C. Owings, Jr., *Notre Dame Journal of Formal Logic*, vol. 14 (1973), pp. 95–99.

It has been recently pointed out to me by Maurizio Negri that Application 4 of the abovementioned paper contains a serious error. It is the purpose of this note to rectify this mistake. I sincerely thank Professor Negri for bringing this matter to my attention. In the original treatment it was falsely claimed that there existed a formula $\delta(v)$ of elementary number theory such that, for any $n \in N$, $\vdash \delta(\mathbf{n}) \leftrightarrow \Phi_n(\mathbf{n})$. However, if there were such a formula, then, letting $\neg\delta$ be Φ_k, we would have $\vdash \delta(\mathbf{k}) \leftrightarrow \neg\delta(\mathbf{k})$, implying that number theory was inconsistent. A corrected version follows.

Application 4 (Feferman's fixed-point theorem for elementary number theory). Let $S = N$, let $\Phi_0, \Phi_1, \Phi_2, \ldots$ be the customary enumeration of all formulas of elementary number theory with at most one free variable v, and, if Ψ is such a formula, let $\ulcorner\Psi\urcorner = e$, where $\Psi = \Phi_e$. Also, let $\phi_0, \phi_1, \phi_2, \ldots$ be a standard enumeration of all partial recursive functions of one variable. If $p, q \in N$, let $p \Box q = \phi_p(q), p * q = p \cdot q = \ulcorner\Phi_p(\mathbf{q})\urcorner, p \circ q = \ulcorner\exists z(\Phi_p(z) \wedge \theta_q(v, z))\urcorner$, where θ_q is a formula which strongly represents the partial recursive function ϕ_q (i.e., for all $m, n \in N$, $\phi_q(m) = n \Leftrightarrow \vdash \theta_q(\mathbf{m}, \mathbf{n})$ and, for all $m \in N$, $\vdash \forall y \forall z((\theta_q(\mathbf{m}, y) \wedge \theta_q(\mathbf{m}, z)) \to y = z)$). Let δ be any number such that, for all $p, \phi_\delta(p) = \ulcorner\Phi_p(\mathbf{p})\urcorner$ and let $p \equiv q$ mean $\vdash \Phi_p \leftrightarrow \Phi_q$.

By definition of δ, $\delta \Box p = p * p$. We have that $(p \circ q) * r) = \ulcorner\Phi_{p \circ q}(\mathbf{r})\urcorner = \ulcorner\exists z(\Phi_p(z) \wedge \theta_q(\mathbf{r}, z))\urcorner$; so $\Phi_{(p \circ q) * r} = \exists z(\Phi_p(z) \wedge \theta_q(\mathbf{r}, z))$. On the other hand, $p \cdot (q \Box r) = \ulcorner\Phi_p(\underline{q \Box r})\urcorner = \ulcorner\Phi_p(\underline{\phi_q(r)})\urcorner$, so $\Phi_{p \cdot (q \Box r)} = \Phi_p(\phi_q(r))$. One now easily shows that $\vdash \Phi_{(p \circ q) * r} \leftrightarrow \Phi_{p \cdot (q \Box r)}$; i.e., $(p \circ q) * r \equiv p \cdot (q \Box r)$. So, by Theorem 1 of this paper, given any formula $\Psi(v)$ there exists a sentence θ such that $\vdash \Psi(\underline{\ulcorner\theta\urcorner}) \leftrightarrow \theta$, namely $\theta = \exists z(\Psi(z) \wedge \theta_\delta(\underline{\ulcorner\exists z(\Psi(z) \wedge \theta_\delta(z, v))\urcorner}, z))$.

Department of Mathematics
The University of Maryland
College Park, Maryland 20742