Commun. Math. Phys. 116, 175–176 (1988)

Communications in
Mathematical
Physics

Erratum

Convergence of Local Charges and Continuity Properties of $W^\star$-Inclusions

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Commun. Math. Phys. **110**, 325–348 (1987)

The bound implied by Corollary B.2 in the appendix, while being correct, is not, as stated there, optimal in general, but may be considerably improved. The optimal bound is given in the following proposition:

Proposition. *Under the assumptions of Corollary B.2 the optimal bound in Proposition B.1 is*

$$|f(0)| \leqq \begin{cases} \exp\{F_{\lambda(d)}\} , & d \leqq d_0 \\ \exp\{F_{\lambda_0} - \lambda_0(d-d_0)\} , & d > d_0 , \end{cases}$$

where $F_\lambda = \frac{2}{\pi}\int\limits_0^\infty ds \frac{\log N(h_\lambda(s))}{\cosh s}$, $h_\lambda(s) = ((\log N)')^{-1}(-\lambda \cosh s)$, $d_0 = \lim\limits_{\lambda \downarrow \lambda_0} \frac{2}{\pi}\int\limits_0^\infty ds h_\lambda(s)$,

$\lambda_0 = -\sup(\log N)'$, *and* $\lambda(d)$, $d \leqq d_0$ *is determined by* $\frac{2}{\pi}\int\limits_0^\infty ds\, h_{\lambda(d)}(s) = d$.

Proof. If $\lambda_0 = 0$ then $d_0 = \infty$, and one obtains the bound already described in Corollary B.2. The same is true in the general case if $d \leqq d_0$.

So assume $d_0 < \infty$ and $d > d_0$. Then condition (B.19) cannot be satisfied by h_λ for $\lambda > \lambda_0$. From (B.27) one finds the lower bound

$$\inf_{\int k = \frac{\pi}{2} d} F(k) \geqq \lim_{\lambda \downarrow \lambda_0} \sup F(h_\lambda) - \lambda_0(d-d_0) . \tag{1}$$

An upper bound can be obtained by inserting the functions $h_\lambda^{(n)}$ into (B.2) with

$$h_\lambda^{(n)}(s) = h_\lambda(s) + \frac{\pi}{2}(d-d_0)k_n(s) , \tag{2}$$

where k_n is a δ-sequence ($\int k_n = 1$, $k_n > 0$, $k_n \to \delta$).

Then $\lim_{\lambda \downarrow \lambda_0} \int h_\lambda^{(n)} = \frac{\pi}{2} d$, and from the mean value theorem

$$\log N(h_\lambda^{(n)}(s)) \leqq \log N(h_\lambda(s)) - \lambda_0 \frac{\pi}{2} (d - d_0) k_n(s) \ , \tag{3}$$

hence

$$\inf_{\lambda > \lambda_0} \lim_{n \to \infty} F(h_\lambda^{(n)}) \leqq \lim_{\lambda \downarrow \lambda_0} \inf F(h_\lambda) - \lambda_0 (d - d_0) \ . \tag{4}$$

Thus $\lim_{\lambda \downarrow \lambda_0} F(h_\lambda) \equiv F_{\lambda_0}$ exists and

$$\inf_{\int k = \frac{\pi}{2} d} F(k) = F_{\lambda_0} - \lambda_0 (d - d_0) \ . \tag{5}$$

The Proposition follows now from Proposition B.1. q.e.d.

Acknowledgement. We thank Peter Junglas for pointing out that the method of Corollary B.2 does not always lead to the optimal bound.

Communicated by R. Haag

Received October 25, 1987