Commun. Math. Phys. 112, 335–341 (1987)

Communications in Mathematical Physics

On the Geometry of Dirac Determinant Bundles in Two Dimensions*

Jouko Mickelsson**

Center for Theoretical Physics, Laboratory for Nuclear Science, and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract. The gauge and diffeomorphism anomalies are used to define the determinant bundles for the left-handed Dirac operator on a two-dimensional Riemann surface. Three different moduli spaces are studied: (1) the space of vector potentials modulo gauge transformations; (2) the space of vector potentials modulo bundle automorphisms; and, (3) the space of Riemannian metrics modulo diffeomorphisms. Using the methods earlier developed for the studies of affine Kac-Moody groups, natural geometries are constructed for each of the three bundles.

The geometry of the determinant line bundle for the left-handed Dirac operator $\gamma^\mu(\nabla_\mu + P_- A_\mu)$ on a unit sphere S^2 (P_- is the projection in left-handed components of the spinor field and A_μ is a Lie algebra valued vector potential) is known to be closely related to the geometry of an affine Kac-Moody group, [M 1]. In fact, the determinant bundle Det is an associated bundle to a $U(1)$ bundle P over $\mathscr{A}/\mathscr{G}$ which in turn is a pull-back of the affine group $\hat{L}G$ with respect to a certain homotopy equivalence $\mathscr{A}/\mathscr{G} \to LG$; here, $\mathscr{A}$ is the space of vector potentials, $\mathscr{G}$ is the group of gauge transformations and LG is the loop group of the gauge group G. The affine group $\hat{L}G$ is a $U(1)$ bundle over LG. The connection form describing the geometry of P (and of Det) is a pull-back of the central projection of the Maurer-Cartan form on $\hat{L}G$, [M 2].

In this paper, I want to generalize the results of [M 1] and [M 2] to the case when S is an arbitrary compact connected oriented Riemann surface of genus $g \geqq 2$ (the case $g=1$ is left as an exercise to the reader). In addition, I shall discuss the geometry of the determinant bundle parametrized by the space $\mathscr{M}/\mathrm{Diff}\,S$, where $\mathscr{M}$ is the space of Riemannian metrics on S. The determinant bundle on $\mathscr{M}/\mathrm{Diff}\,S$ is

* This work was supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under contract # DE-AC02-76ER03069

** Permanent address: Department of Mathematics, University of Jyväskylä, Seminaarinkatu 15, SF-40100 Jyväskylä 10, Finland

obtained as a pull-back of the corresponding bundle on $\{GL(3,\mathbf{R})$ connections in a topologically trivial bundle Q over $S\}/\{$automorphisms of $Q\}$. To achieve this, we have to first generalize a slightly earlier setting: we started by considering bundles over $\mathscr{A}/\mathscr{G}$ which are determined by the non-Abelian gauge anomaly; however, one can use the gauge anomaly to produce bundles over $\mathscr{A}/\mathrm{Aut}\,Q$ as well. In the case $Q=S\times GL(3,\mathbf{R})$ the gauge anomaly in $\mathscr{A}/\mathrm{Aut}\,Q$ when pulled back to $\mathscr{M}/\mathrm{Diff}\,S$ produces the diffeomorphism anomaly in two dimensions. The pull-back will be determined by using an embedding of S into $\mathbf{R}^3$ and extending the geometry of S into a tubular neighborhood of S.

Let us start the construction of the determinant bundle parametrized by vector potentials by choosing a discrete subgroup $\Gamma\subset PSL(2,\mathbf{R})$ such that $\mathbf{C}_+/\Gamma\simeq S$, when $\mathbf{C}_+$ is the upper half plane $\{z=x+iy\,|\,y>0\}$ with the action

$$z\mapsto\frac{az+b}{cz+d}$$

of $PSL(2,\mathbf{R})$. It is known that any surface S with a given metric can be produced in this way by taking in $\mathbf{C}_+$ the Poincaré metric and choosing Γ in an appropriate way, [B], in the genus $\geqq 2$ case (S compact and oriented). However, at this stage it is not necessary to specify any metric. Let G be a finite-dimensional Lie group with Lie algebra $\underline{g}$ and let $\langle\cdot,\cdot\rangle$ be an invariant bilinear form on $\underline{g}$. Let $\mathscr{A}$ be the space of connections in the topologically trivial bundle $Q=S\times G$. Note that if G is simply connected, the any G bundle on S is a product bundle. We choose a base point $s_0\in S$ and define $\mathscr{G}=\{g:S\to G\,|\,g(s_0)=1\}$ as the group of smooth based gauge transformations. Since the bundle Q is trivial, a connection can be represented by a global $\underline{g}$ valued one-form $A\in\mathscr{A}$ on S. The right action of $\mathscr{G}$ in $\mathscr{A}$ is given by $A\mapsto A^g=g^{-1}Ag+g^{-1}dg$. The group $\mathrm{Aut}\,Q$ of automorphisms of Q is equal to the semidirect product $\mathrm{Diff}\times\mathscr{G}$; the action of $\mathrm{Diff}\,S$ on $\mathscr{G}$ is the natural action $g\mapsto g\circ h^{-1}$, $h\in\mathrm{Diff}\,S$.

Let $\theta^2=(\text{length})^2$ of the longest root of the maximal compact subgroup of G [let us assume for simplicity that G does not contain any $U(1)$ factors]. For $A\in\mathscr{A}$ and $g\in\mathscr{G}$ we define

$$\omega(A,g)=\frac{\theta^2}{16\pi^2}\int_S\langle A,dgg^{-1}\rangle-\frac{\theta^2}{48\pi^2}\int_B\langle dgg^{-1},\tfrac{1}{2}[dgg^{-1},dgg^{-1}]\rangle,\qquad(1)$$

where the second integral is taken over any compact three-space B with $\partial B=S$ and g has been extended in an arbitrarily smooth way to B. Taking another extension $\tilde{g}$ changes the value of ω at most by an integer since the integral

$$C(g)=\frac{\theta^2}{48\pi^2}\int\langle dgg^{-1},\tfrac{1}{2}[dgg^{-1},dgg^{-1}]\rangle\qquad(2)$$

is an integer when evaluated over a compact three-manifold without boundary, [W]. Thus, $\exp 2\pi i\omega(A,g)$ is single-valued; it is known as the non-abelian anomaly in physics literature, since the determinant (when properly regularized) of the left-handed Dirac operator $\gamma^\mu(\nabla_\mu+P_-A_\mu)$ changes by this phase when A is replaced by A^g, [Z]. The function $\exp 2\pi i\omega$ is a 1-cocycle,

$$\omega(A,g_1g_2)\equiv\omega(A^{g_1},g_2)+\omega(A,g_1)\bmod\mathbf{Z}.\qquad(3)$$

In fact, $\omega(A,g)$ defines a cocycle for the full automorphism group $\mathrm{Diff}S\times\mathscr{G}$. The group multiplication in $\mathrm{Aut}Q$ is given by

$$(h_1,g_1)(h_2,g_2)=(h_1\circ h_2, g_1 g_2^{h_1}), \tag{4}$$

where $g^h=g\circ h^{-1}$. We define $\omega(A,(h,g))=\omega(A,g)$. Then, by a simple computation,

$$\omega(A,(h_1,g_1)(h_2,g_2))=\omega(A^{(h_1,g_1)},(h_2,g_2))+\omega(A,(h_1,g_1)), \tag{5}$$

where $A^{(h,g)}=h^*(g^{-1}Ag+g^{-1}\,dg)$, with the natural action of $\mathrm{Diff}S$ on differential forms.

We can now define two principal $U(1)$ bundles $\mathrm{Det}=\mathrm{Det}(S,G)$ [respectively $\mathrm{Det}_0=\mathrm{Det}_0(S,G)$] on $\mathscr{A}/\mathscr{G}$ (respectively on $\mathscr{A}/\mathrm{Aut}Q$) as $\mathscr{A}\times U(1)/\sim$, where in the first case the equivalence relation "$\sim$" in $\mathscr{A}\times U(1)$ is defined by

$$(A,\lambda)\sim(A^g,\lambda e^{2\pi i\omega(A,g)})$$

for $g\in\mathscr{G}$ and in the second case the element g is replaced by an arbitrary element $(h,g)\in\mathrm{Aut}Q$. The bundle projection is defined by $[(A,\lambda)]\mapsto A \bmod \mathscr{G}$ (respectively $[(A,\lambda)]\mapsto A \bmod \mathrm{Aut}Q$). The action of $U(1)$ in the total space of the bundles is the right multiplication in the second component.

I shall now describe the geometry of the bundle Det in terms of a natural connection. Let us fix a fundamental domain $D\subset\mathbf{C}_+$ for the projection $\mathbf{C}_+\to S$. The interior of D is mapped bijectively to a dense contractible domain in S and the image of D is S. The action of Γ is $\mathbf{C}_+$ defines a set of identifications on the boundary ∂D. If we think of D as a polygon with 4g sides, then S is obtained by identifying the boundary a_i with a_i^{-1} and b_i with b_i^{-1} as in Fig. 1 (when $g=2$). Fix a point $z_0\in\mathbf{C}_+$ covering $s_0\in S$. For any $A\in\mathscr{A}$ there exists a unique gauge transformation $f_A:\mathbf{C}_+\to G$ such that $f_A(z_0)=1$ and $\tilde{A}=f_A^{-1}(\pi^*A)f_A+f_A^{-1}\,df_A$ is in the radial gauge for rays starting from z_0; that is, $\tilde{A}_r=0$ in the polar coordinates (r,φ) with origin at z_0; π^*A is the pull-back of A under $\pi:\mathbf{C}_+\to\mathbf{C}_3/\Gamma=S$.

Let $DG=\{f:D\to G\,|\,f(z_0)=1,\ f \text{ smooth}\}$. Here, "smooth" means that f can be extended to a smooth map in an open set containing the closed set D. The gauge

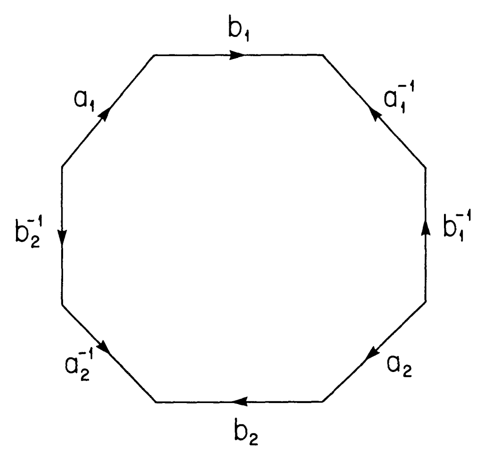

Fig. 1

group $\mathscr{G}$ can be thought of as the subgroup of DG consisting of maps $g: D \to G$ which obtain equal values at those points on the boundary ∂D which are identified under the projection $D \to S$. We can define a $U(1)$ bundle Det′ on $DG/\mathscr{G}$ by the cocycle

$$\omega'(f,g) = \frac{\theta^2}{16\pi^2} \int_D \langle f^{-1} df, dg g^{-1} \rangle - C(g). \tag{6}$$

The action of $\mathscr{G}$ on DG is the point-wise right multiplication. The bundle Det is a pull-back of Det′ with respect to the mapping $A \mapsto f_A$; note that $f_{A^g} = f_A \cdot g$. The cocycle $(A,g) \mapsto \omega'(f_A, g)$ represents the same cohomology class as $\omega(A,g)$, since

$$\omega'(f_A, g) = \omega(A,g) + F(A^g) - F(A), \tag{7}$$

where

$$F(A) = \frac{\theta^2}{16\pi^2} \int_D \langle A, df_A f_A^{-1} \rangle. \tag{8}$$

In the genus $=0$ case (D is a disc; ∂D identified with one point), [M2], it was possible to define a connection in the bundle Det′ by pushing the central projection $pr_c\, dkk^{-1}$ of the Maurer-Cartan form on $DG \times U(1)$ to $\mathrm{Det}' = DG \times U(1)/\mathscr{G}$; the group multiplication in $DG \times U(1)$ is given by

$$(f,\lambda)(f',\lambda') = (ff', \lambda\lambda' \exp 2\pi i \gamma(f,f')), \tag{9}$$

where

$$\gamma(f,f') = \frac{\theta^2}{16\pi^2} \int_D \langle f^{-1} df, df' f'^{-1} \rangle. \tag{10}$$

The group structure in $DG \times U(1)$ is well-defined also in the higher genus case but now $\mathscr{G}$ cannot be embedded in $DG \times U(1)$ as a normal subgroup, and for this reason it is not possible to push $pr_c\, dkk^{-1}$ to Det′. However, there is a slight modification of $pr_c\, dkk^{-1}$ which will give a connection on Det. I shall describe this connection directly in terms of parallel transport as follows. Let $t \mapsto A(t) \bmod \mathscr{G}$ be a path in $\mathscr{A}/\mathscr{G}$, $t_0 \leqq t \leqq t_1$. Denote $f(t,\cdot) = f_{A(t)}$. Let $\varrho_0 = [(A(t_0), \lambda(t_0))] \in \mathrm{Det}$ be any point in the fiber over $A(t_0)$. Denote by $\varrho_1 = [(A(t_1), \lambda(t_1))]$ the parallel transport of ϱ_0 along $A(t) \bmod \mathscr{G}$ at $A(t_1)$. We define $\lambda(t_1) = \lambda(t_0) \exp 2\pi i J$, where

$$\begin{aligned} J = {} & \frac{\theta^2}{16\pi^2} \int_{t_0}^{t_1} \int_D \langle f^{-1} df, d(f^{-1} \dot{f}) \rangle \, dt \\ & + \frac{\theta^2}{16\pi^2} \int_D \langle \pi^* A, f^{-1} df \rangle_{t=t_1} - \frac{\theta^2}{16\pi^2} \int_D \langle \pi^* A, f^{-1} df \rangle_{t=t_0} \\ & - \frac{\theta^2}{8\pi^2} \int_{t_0}^{t_1} \int_{\partial D} \langle \pi^* A, f^{-1} \dot{f} \rangle \, dt . \end{aligned} \tag{11}$$

We have to show that the class $[(A(t_1), \lambda(t_1))]$ is well-defined. Let $t \mapsto \tilde{A}(t)$ be another path with $\tilde{A}(t) \equiv A(t) \bmod \mathscr{G}$. Denote $\tilde{f}(t,\cdot) = f_{\tilde{A}(t)} = f(t,\cdot)\, g(t,\cdot)$; here g is a gauge transformation such that $\tilde{A}(t) = A(t)^{g(t)}$. Let us first rewrite (11) using partial integration in the form

$$\begin{aligned} J = C(f) &+ \frac{\theta^2}{16\pi^2} \int_{t_0}^{t_1} \int_{\partial D} \langle f^{-1} df, f^{-1} \dot{f} \rangle \, dt \\ &+ \frac{\theta^2}{16\pi^2} \int_D \langle \pi^* A, f^{-1} df \rangle_{t=t_1} \\ &- \frac{\theta^2}{16\pi^2} \int_D \langle \pi^* A, f^{-1} df \rangle_{t=t_0} \\ &- \frac{\theta^2}{8\pi^2} \int_{t_0}^{t_1} \int_{\partial D} \langle \pi^* A, f^{-1} \dot{f} \rangle \, dt \,, \end{aligned} \tag{12}$$

where the integral defining $C(f)$ is evaluated over $D \times [t_0, t_1]$. The term C has the basic property

$$C(ff') = C(f) + C(f') - \frac{\theta^2}{16\pi^2} \int \langle f^{-1} df, df' f'^{-1} \rangle \,, \tag{13}$$

verified by a simple computation. Using (13) we get from (12),

$$\begin{aligned} \tilde{J} - J = C(g) &+ \frac{\theta^2}{16\pi^2} \int_D \langle \pi^* A, dg g^{-1} \rangle_{t=t_1} \\ &- \frac{\theta^2}{16\pi^2} \int_D \langle \pi^* A, dg g^{-1} \rangle_{t=t_0} \\ &- \frac{\theta^2}{8\pi^2} \int_{t_0}^{t_1} \int_{\partial D} \langle \pi^* A, \dot{g} g^{-1} \rangle \, dt \\ &- \frac{\theta^2}{8\pi^2} \int_{t_0}^{t_1} \int_{\partial D} \langle g^{-1} dg, g^{-1} \dot{g} \rangle \, dt \,. \end{aligned} \tag{14}$$

The last two terms are zero, since g and A were defined on S, and therefore the pieces obtained by integrating along a_i and b_i cancel (for any fixed t) with the pieces along a_i^{-1} and b_i^{-1}. Let g_i be an extension of $g(t_i, \cdot)$ to the three-dimensional manifold B, $i = 0, 1$. Then $C(g) \equiv C(g_1) - C(g_0) \bmod \mathbf{Z}$, and therefore

$$\tilde{J} - J \equiv \omega(A(t_1), g(t_1)) - \omega(A(t_0), g(t_0)) \bmod \mathbf{Z} \,, \tag{15}$$

which shows that $\tilde{\lambda}(t_1) = \lambda(t_1) \exp 2\pi i \omega(A(t_1), g(t_1))$ and thus the class ϱ_1 is well-defined.

The curvature of the connection is evaluated by taking the parallel transport around an infinitesimal parallelogram; the result is

$$\begin{aligned} F(\delta A, \delta B) = \frac{1}{4\pi} \int_{\partial D} \langle X, dY \rangle &+ \frac{1}{4\pi} \int_{\partial D} \langle Y, \delta A \rangle \\ &- \frac{1}{4\pi} \int_{\partial D} \langle X, \delta B \rangle - \frac{1}{4\pi} \int_{\partial D} \langle [X, Y], A \rangle \,, \end{aligned} \tag{16}$$

where X (respectively Y) is the image of δA (respectively δB) under the derivative of the mapping $A \mapsto f_A$; δA and δB are tangent vectors at $A \in \mathscr{A}$.

Next I want to relate the geometry of the bundle $\mathrm{Det}^{\mathcal{M}}$ to that of the bundle $\mathrm{Det}_0(S, GL(3,\mathbf{R}))$. The bundle $\mathrm{Det}^{\mathcal{M}}$ will be defined below using the diffeomorphism anomaly of the Dirac operator. The group $\mathrm{Diff}S$ can be taken either the full diffeomorphism group of S or the connected component of the identity in the full group. However, one should bear in mind that in the former case $\mathcal{M}/\mathrm{Diff}S$ has singularities and it is not a manifold in the usual sense. In the latter case the quotient is contractible and therefore any bundle over that space is topologically trivial. Let us first fix an embedding $S \subset \mathbf{R}^3$. Choose a tubular neighborhood $\tilde{S} = S \times \mathcal{J}$ of S in $\mathbf{R}^3$; $\mathcal{J} \subset \mathbf{R}$ is an open interval. Using the natural metric on $\mathcal{J}$ and setting $\mathcal{J} \perp S$, we can uniquely extend any metric $g_{\mu\nu}$ on S to a metric $\tilde{g}_{\mu\nu}$ on $\tilde{S}$. Also, if $h: S \to S$ is any diffeomorphism we have a natural extension $\tilde{h}: \tilde{S} \to \tilde{S}$. Using the Cartesian coordinates of $\mathbf{R}^3$, we can represent the Levi-Civita connection Γ of the metric $\tilde{g}_{\mu\nu}$ by a $\underline{gl}(3,\mathbf{R})$ valued one-form on $\tilde{S}$. Here $\underline{gl}(3,\mathbf{R})$ is the Lie algebra of the general linear group $GL(3,\mathbf{R})$ in $\mathbf{R}^3$. Similarly, the derivative of the diffeomorphism $\tilde{h}$ gives a $GL(3,\mathbf{R})$ valued function H on S; extending H to B we can define

$$\omega_1(g, h) = \omega(\Gamma, H), \tag{17}$$

where $\omega(\Gamma, H)$ is as before, with the gauge group $G = GL(3,\mathbf{R})$. Since $h \mapsto (h, H)$ is a homomorphism from $\mathrm{Diff}S$ into $\mathrm{Diff}S \times \mathcal{G}$, ω_1 is a 1-cocycle for the right action of $\mathrm{Diff}S$ on $\mathcal{M}$. By definition, the determinant bundle $\mathrm{Det}^{\mathcal{M}}$ over $\mathcal{M}/\mathrm{Diff}S$ is

$$\mathcal{M} \times U(1)/\sim\,,$$

where the equivalence "$\sim$" is defined by

$$(g_{\mu\nu}, \lambda) \sim (g^h_{\mu\nu}, \lambda e^{2\pi i \omega_1(g,h)}). \tag{18}$$

In fact, by (18) the bundle $\mathrm{Det}^{\mathcal{M}}$ is a pull-back of the bundle $\mathrm{Det}_0(S, GL(3,\mathbf{R}))$ under the mapping $\mathcal{M}/\mathrm{Diff}S \to \mathcal{A}/\mathrm{Aut}Q$ given by $g_{\mu\nu} \mapsto \Gamma$ [here $Q = S \times GL(3,\mathbf{R})$].

The construction of $\mathrm{Det}^{\mathcal{M}}$ does not depend on the choice of the embedding $S \to \mathbf{R}^3$. The reason is that any two embeddings are related by a diffeomorphism (defined in the respective tubular neighborhoods of the embedded surfaces) and the anomaly ω_1 defining the bundle $\mathrm{Det}^{\mathcal{M}}$ is invariant under diffeomorphisms. The bundle Det_0 will be useful when relating the geometry of $\mathrm{Det}^{\mathcal{M}}$ to the bundle Det; no other (physical) significance will be assigned to Det_0.

Note that the infinitesimal version $\left(\text{evaluate } \frac{d}{dt}\omega_1(g, h_t)|_{t=0} \text{ for a one-parameter subgroup of diffeomorphisms } h_t \text{ generated by a vector field } \vartheta_\mu \text{ on } S\right)$ of (17) is the diffeomorphism anomaly

$$\begin{aligned} \Delta\omega_1(g, \vartheta) &= \frac{1}{32\pi^2} \int_S \mathrm{tr}\, d\Gamma \frac{d\vartheta}{\partial x} \\ &= \frac{1}{32\pi^2} \int_S (\partial_\mu \Gamma^\alpha_{\nu\beta}\, \partial^\beta \vartheta_\alpha - \partial_\nu \Gamma^\alpha_{\mu\beta}\, \partial^\beta \vartheta_\alpha). \end{aligned} \tag{19}$$

[Note that $\theta^2 = \frac{1}{2}$ when we use as $\langle \cdot, \cdot \rangle$ the trace form in the defining representation of $GL(3,\mathbf{R})$.]

One can define a connection in $\mathrm{Det}^{\mathscr{M}}$ by pulling back any connection in the bundle $\mathrm{Det}_0(S, GL(3, \mathbf{R}))$. However, one cannot push the simple geometry of Det described by the formulas (11) and (16) to the bundle Det_0. This can be seen from the curvature formula (16): the right-hand side is not invariant under the group Diff$S$. On the other hand, the connection defined by Atiyah and Singer [AS], is reparametrization invariant. The curvature form of the AS connection is

$$\int_S \mathrm{tr}(D_A^* D_A)^{-1} [\delta A_\mu, \delta B^\mu] d^2x . \tag{20}$$

The tangent vectors δA and δB are taken to be in the background gauge $D_A^* \delta A = D_A^* \delta B = 0$. One must keep in mind that the metric is transformed along with the one-forms A, δA and δB under a diffeomorphism of S. The metric is needed to define the adjoint D_A^* of the covariant derivative and the product $[\delta A_\mu, \delta B^\mu]$.

Finally, I want to point out that the pull-back of the connection in Det to the topologically trivial bundle over the space $\mathscr{A}$ is not directly related to the Kähler geometry studied by Quillen [Q] (and extended to the determinant bundles over Teichmüller spaces by Belavin and Knizhnik [BK]). The reason is that the curvature in [Q] has non-zero components even to the vertical directions of the canonical projection $\mathscr{A} \to \mathscr{A}/\mathscr{G}$. The holomorphic geometries have also been studied recently by Bismut and Freed using families index theory, [BF].

References

[AS] Atiyah, M.F., Singer, I.M.: Dirac operators coupled to vector potentials. Proc. Natl. Acad. Sci. USA **81**, 2597 (1984)

[BK] Belavin, A.A., Knizhnik, V.A.: Complex geometry and quantum string theory. Landau Institute preprint No. 32, 1986

[B] Bers, L.: Finite dimensional Teichmüller spaces and generalizations. Bull. Am. Math. Soc. **5**, [NS] 131 (1981)

[BF] Bismut, J.-M., Freed, D.S.: The analysis of elliptic families. I. Metrics and connections on determinant bundles. Commun. Math. Phys. **106**, 159 (1986)

[M1] Mickelsson, J.: Kac-Moody groups topology of the Dirac determinant bundle and fermionization. University of Helsinki preprint HU-TFT-85-50 (to be published in CMP); Kac-Moody groups and the Dirac determinant bundle. Proceedings of the symposium on topological and geometrical methods in field theory. Espoo 1986. Hietarinta, J., Westerholm, J. (eds.). Singapore: World Scientific (1986)

[M2] Mickelsson, J.: Strings on a group manifold, Kac-Moody groups and anomaly cancellation. Phys. Rev. Lett. **57** (20), 2493 (1986)

[Q] Quillen, D.: Determinants of Cauchy-Riemann operators over a Riemann surface. J. Funct. Anal. Appl. **19** (1), 31 (1985)

[W] Witten, E.: Non-Abelian bosonization in two dimensions. Commun. Math. Phys. **92**, 445 (1984)

[Z] Zumino, B.: Chiral anomalies and differential geometry. In: Les Houches Proceedings 1983. DeWitt, B., Stora, R. (eds.). Amsterdam: North-Holland 1985

Communicated by S.-T. Yau

Received October 27, 1986; in revised form February 20, 1987