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A Classification of SU_3 Magnetic Monopoles

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Abstract. A classification scheme is proposed for SU_3 magnetic monopoles when the Higgs fields lie in the adjoint representation. The scheme is based on a study of the second homotopy groups of the orbit spaces.

I. Introduction

Consider an octet of self-interacting scalar fields transforming according to the adjoint representation of SU_3, minimally coupled to a set of Yang-Mills fields. Suppose that the scalar self-interaction potential is generic—that it depends explicitly on both invariants of the adjoint representation. Then at infinity the little group is U_2. See, for example [1, 2]. Spontaneous symmetry breaking leaves massless the Yang-Mills fields corresponding to U_2, that is, those that commute with the octet of scalar fields. If we wish to discuss the electromagnetic properties of a solution to the field equations, one of these must be chosen as the electromagnetic field. That is, we must choose a U_1 subgroup of U_2. U_2 contains also SU_2 as a subgroup. There are three ways to choose U_1 according to how it intersects SU_2:

(i) $U_1 \cap SU_2 = \{1\}$,

(ii) $U_1 \cap SU_2 = \mathbb{Z}_2$,

(iii) $U_1 \subset SU_2$.

We are interested in everywhere regular, finite energy, static solutions to the field equations and we wish to show that they may be classified by two integers s and t which satisfy the following conditions, according to the three possible choices of U_1:

(i) s is even or odd, $t=0$,

(ii) s is even, $t=0$,

(iii) $s=0$, t is even or odd.

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Cases (i) and (ii) constitute a refinement of a classification proposed by Tyupkin et al. [3] and by Monastyrskii and Perelomov [4]. Case (iii) has also been considered by Corregan et al. [5] and by Corregan and Olive [6]. The solutions generally have magnetic charge in the sense of Dirac [7] (see also Schwinger [8]). Let e be the electric charge and g the magnetic charge; they are connected by the relation $2eg=s+t$ (i, iii), $4eg=s$ (ii).

As in the SU_2 case [9, 10, 11, 12], the classification is incomplete in that we cannot calculate the dimension of the solution space for every possible value for the pair (s, t).

Section II contains a discussion of the field equations and the topological implications of a choice of boundary conditions at infinity.

Section III contains a review of the SU_2 magnetic monopoles. The SU_3 magnetic monopoles are discussed in Section IV.

II. Global Problems

Suppose that G is the symmetry group of a given Lagrangian defined on a manifold M, which we shall define later as Minkowski space with a boundary at space-like infinity. Let P be the trivial bundle over $M: P=M\times G$, and H a subgroup of G. We shall say that the symmetry is broken if there is given a reduction of P to an H-subbundle Q. The reduction determines, and is determined by a section σ in the associated bundle E with fibre G/H [13, I, p. 57]. Let $\mathfrak{h}$ be the Lie algebra of H. If ω is a connection on P, we shall choose as connection on Q the projected connection $\hat{\omega}$, that is, the $\mathfrak{h}$-component of the restriction of ω to Q [13, I, p. 83]. In the cases which interest us this will always be a connection.

We are interested in the Lagrangian describing an octet of self-interacting scalar fields transforming according to the adjoint representation of SU_3, minimally coupled to a set of Yang-Mills fields.

In general, let P be the trivial bundle, and let ω be a connection on P. The Yang-Mills field is given by a section of P. A set of scalar fields—the Higgs fields—is given by a section of an associated G-bundle F with fibre $\mathfrak{g}$ (the Lie algebra of G), and G operating by the adjoint representation. The covariant derivative of ϕ,

$$D_\mu\phi=\partial_\mu\phi+[\Gamma_\mu,\phi],$$

is determined by ω. Let $R_{\mu\nu}$ be the curvature tensor and suppose, for simplicity, that G is semi-simple and simply connected.

The most general invariant Lagrangian which leads to second order field equations is of the form

$$\mathscr{L}=\frac{1}{2f^2}\,\mathrm{Tr}(R_{\mu\nu}R^{\mu\nu})-\mathrm{Tr}(D_\mu\phi D^\mu\phi)-V(\phi). \tag{II.1}$$

f is the Yang-Mills coupling constant and the coefficients in $\mathscr{L}$ are determined by the choice of normalization of the basis of the Lie algebra. For example, if G is SU_2 we choose $\sigma_j=\dfrac{-i}{2}\tau_j$ where τ_j are the Pauli matrices and if G is SU_3 we choose $\kappa_\alpha=\dfrac{-i}{2}\lambda_\alpha$ where λ_α are the Gell-Mann matrices.

The energy momentum tensor one derives from $\mathscr{L}$ is given by

$$T_{\mu\nu}=\frac{2}{f^2}\operatorname{Tr}(R_{\mu\sigma}R_\nu^\sigma)-2\operatorname{Tr}(D_\mu\phi D_\nu\phi)-\mathscr{L}g_{\mu\nu}\,. \tag{II.2}$$

Consider now the manifold M. Define $\mathbb{D}^3_\infty$, M, and M_0 by

$$\begin{aligned}&\mathbb{D}^3_\infty=\{x\in\mathbb{R}^3\,|\,|x|\leqq r\to\infty\}\,,\\ &M=\mathbb{R}\times\mathbb{D}^3_\infty, M_0=\mathbb{R}\times(\mathbb{D}^3_\infty-\{0\})\,.\end{aligned} \tag{II.3}$$

The boundaries of these manifolds are

$$\partial\mathbb{D}^3_\infty=S^2_\infty, \partial M=\partial M_0=\mathbb{R}\times S^2_\infty\,. \tag{II.4}$$

We are interested in everywhere regular, static, finite-energy (soliton) solutions to the field equations derived from (II.1). Each such solution defines by restriction a regular function ϕ on S^2_∞. From the expression (II.2) for $T_{\mu\nu}$ we see that if the energy is to be finite we must have

$$R_{\mu\nu}=o(r^{-2}), \qquad D_\mu\phi=o(r^{-2})\,. \tag{II.5}$$

From the field equations for ϕ and from (II.5) we deduce that the potential V must be extremal on ∂M:

$$\frac{\delta V}{\delta\phi_i}=o(r^{-2})\,. \tag{II.6}$$

If $G=SU_3$, and V is generic, then this implies [1, 2] if ϕ does not vanish identically on ∂M that it belongs to the orbit that contains the hypercharge generator and that the little group is U_2. In general we suppose that the extrema of V on ∂M all belong to one orbit of G. If we call a point in ϕ-space where V is extremal a classical vacuum, then this means that we assume that the degeneracy of the vacuum is due entirely to the action of the group [3]. In general let H be the little group. We see then that the boundary conditions at infinity for the soliton solutions to the field equations are in one-to-one correspondence with the applications of the sphere S^2_∞ into G/H. Any two boundary conditions on ∂M may be locally joined by a gauge transformation. If this gauge transformation can be extended globally over ∂M then the boundary conditions are equivalent—that is, the solutions corresponding to one are gauge transforms of the solutions corresponding to the other. It can be shown that this is the case if and only if the corresponding applications of S^2_∞ into G/H are homotopic. The inequivalent boundary conditions for soliton solutions are therefore in one-to-one correspondence with the elements of $\pi_2(G/H)$ [3]. This is the first step in the classification of magnetic monopoles. The second step consists in identifying the electromagnetic gauge group U_1, as a subgroup of H.

A soliton solution defines a section σ over ∂M of the associated G-bundle E with fibre G/H. σ can be always extended over M_0 but it can be extended over M if and only if the corresponding element in $\pi_2(G/H)$ is zero. Each soliton solution determines therefore a reduction of P over M_0 to an H subbundle Q—and therefore a broken symmetry. In general the extension of σ to M_0 is not unique.

Let $\hat{\omega}$ be the projected connection on Q; $\hat{\omega}$ determines an element c_1, the first Chern class, in $H^2(M_0;\mathbb{Z})$. For an intuitive introduction to characteristic classes, see

[14]. Since we have supposed that G is semi-simple and simply connected, $\pi_2(G/H)$ can be identified with $\pi_1(H)$, which we shall suppose to be isomorphic to $\mathbb{Z}$. The inequivalent boundary conditions for soliton solutions are therefore in one-to-one correspondence with the integers [3]. Let s be the integer corresponding to the given solution. Let τ be the fundamental cycle of $H_2(M_0)$ (any sphere surrounding the origin in $\mathbb{R}^3$). c_1 is an obstruction to extending a section of Q over τ; that is, $c_1(\tau)$ is an element of $\pi_1(H)$ and with the above identification it may be considered an element of $\pi_2(G/H)$. Therefore we have

$$c_1(\tau)=s. \tag{II.7}$$

This equality was first noticed by Arafune et al. [11] in the case $G=SU_2$.

III. SU_2 Magnetic Monopoles

Let P and Q be as in Section II, with $G=SU_2$ and $H=U_1$, and let ϕ be a given soliton solution. The orbit G/H of ϕ is the 2-sphere. We shall define on ∂M

$$n=\phi/\sqrt{\boldsymbol{\phi}^2}, \quad \phi=\boldsymbol{\phi}\cdot\boldsymbol{\sigma}, \quad \boldsymbol{\sigma}=\frac{-i}{2}\boldsymbol{\tau},$$

and designate by the same symbol an extension of n to M_0. If we identify G/H with the unit 2-sphere in $\mathbb{R}^3$:

$$G/H=S^2=\{\boldsymbol{n}\in\mathbb{R}^3 \,|\, \boldsymbol{n}^2=1\}, \tag{III.1}$$

the projection μ of P onto P/H, [13, I, p. 57] is given by

$$\mu(a)=\boldsymbol{n}, \quad a\sigma_3 a^{-1}=n=\boldsymbol{n}\cdot\boldsymbol{\sigma}, \quad a\in SU_2, \tag{III.2}$$

and the kernel of μ is

$$\operatorname{Ker}\mu=U_1=\{e^{-2\alpha\sigma_3} \,|\, 0\leqq\alpha\leqq 2\pi\}. \tag{III.3}$$

The electromagnetic field $eF_{\mu\nu}$ is the curvature of the projected connection $\hat{\omega}$ and the magnetic charge, in units of $1/2e$, is given by the element of $\pi_2(S^2)$ defined by the boundary values at space-like infinity, (II.7) [9, 10, 11].

In order to give an explicit expression for $F_{\mu\nu}$, and to identify the charge e in terms of the Yang-Mills coupling constant f, it is necessary to examine more closely how $\omega_{|Q}$ projects onto $\hat{\omega}$. P is the trivial bundle and it has a global section, for example the unit element of SU_2. Let Γ_μ be the Yang-Mills field given by this section; then ω may be written

$$\omega=a^{-1}\Gamma_\mu a dx^\mu+a^{-1}da.$$

Any local section of Q is given by $a=b(x^\alpha)$ where on ∂M, b is a solution of $n=b\sigma_3 b^{-1}$. Therefore, locally

$$\omega_{|Q}=\Gamma'_\mu dx^\mu-2\sigma_3 d\alpha, \quad \Gamma'_\mu=b^{-1}\Gamma_\mu b+b^{-1}\partial_\mu b. \tag{III.4}$$

The projected connection is locally of the form

$$\hat{\omega}=i\hat{\Gamma}_\mu dx^\mu+id\alpha, \quad \hat{\Gamma}_\mu=eA_\mu. \tag{III.5}$$

This is to be identified with the σ_3-component of $\omega_{|Q}$. This identification is most easily made by considering the corresponding covariant derivatives in an associated vector bundle F—for example, with fibre $\mathbb{C}^2$. Let

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

be a local section of such a bundle. Then

$$D'_\mu \psi = D'_\mu \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \partial_\mu \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} + \Gamma'^i_\mu \sigma_i \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \tag{III.6}$$

$$\hat{D}_\mu \psi_1 = \partial_\mu \psi_1 + i\hat{\Gamma}_\mu \psi_1 ,$$

and we have

$$\hat{\Gamma}_\mu = \frac{-1}{2} \Gamma'^3_\mu, \quad e = -f/2. \tag{III.7}$$

The sign of e is not significant. If we had used ψ_2 instead of ψ_1, in (III.6), we would have obtained $e = +f/2$. The possibility of a reduction of SU_2 to U_1 means that F splits into the sum of two U_1-bundles F_1 and F_2, with $F_2 = F_1^*$ (the action of U_1 on F_2 is the inverse of the action of U_1 on F_1). The Whitney sum formula [13, II, p. 306] implies therefore that the reduction was only possible because P was a trivial bundle. We shall encounter this obstruction to reducing an SU_2-bundle again in Section IV (case iii).

From (III.4, III.5, III.7) we find the following expression for A_μ:

$$eA_\mu = \mathrm{Tr}(\sigma_3 \Gamma'_\mu) = \mathrm{Tr}(n\Gamma_\mu) + \mathrm{Tr}(n\partial_\mu b b^{-1}). \tag{III.8}$$

A straightforward calculation yields the curvature tensor $eF_{\mu\nu}$ of $\hat{\omega}$:

$$eF_{\mu\nu} = \mathrm{Tr}(nR_{\mu\nu}) - \mathrm{Tr}(n[D_\mu n, D_\nu n]). \tag{III.9}$$

This calculation is most conveniently carried out either in a gauge where $n = \sigma_3$ or in a gauge where $\Gamma_\mu = 0$ (at a point). $F_{\mu\nu}$ is the electromagnetic field tensor introduced by 't Hooft [9].

It is of interest to consider under what condition the projected connection $\hat{\omega}$ coïncides with $\omega_{|Q}$; that is, under what condition ω is reducible to a connection on Q. One sees immediately that this is the case if and only if $R_{\mu\nu}$ commutes with n. The necessary and sufficient condition for this to be true is that the covariant derivative of some multiple λ of n vanish:

$$D_\mu(\lambda n) = 0, \quad \lambda \neq 0. \tag{III.10}$$

If this condition is satisfied everywhere in M_0, then the only field present is the electromagnetic field. What one has is a U_1 magnetic monopole imbedded in a trivial SU_2-bundle and no extra physical fields. We saw in Section II that (III.10) must be satisfied on ∂M.

IV. SU_3 Magnetic Monopoles

Let P and Q be as in Section II with $G=SU_3$ and $H=U_2$ and let ϕ be a given soliton solution. On ∂M the orbit of ϕ is G/H. We define

$$n=\phi/\sqrt{\boldsymbol{\phi}^2},\quad \phi=\boldsymbol{\phi}\cdot\boldsymbol{\kappa},\quad \boldsymbol{\kappa}=\frac{-i}{2}\boldsymbol{\lambda},$$

and we designate by the same symbol an extension of n to M_0. If we identify G/H as a 4-dimensional submanifold of the unit 7-sphere in $\mathbb{R}^8$, the projection μ of P onto P/H is given by

$$\mu(a)=\boldsymbol{n},\quad a\kappa_8 a^{-1}=n=\boldsymbol{n}\cdot\boldsymbol{\kappa},\quad a\in SU_3, \tag{IV.1}$$

and the kernel of μ is U_2. The second homotopy group of G/H is given by

$$\pi_2(G/H)=\pi_1(U_2)=\mathbb{Z}. \tag{IV.2}$$

Let $\hat{\omega}$ be the projected connection on Q. We can proceed in exactly the same way as in Section III to obtain the Yang-Mills fields for $\hat{\omega}$ in terms of the Yang-Mills fields for ω, with respect to any given local section of Q. Any such section of Q is given by $a=b(x^\alpha)$ where on ∂M, b is a solution of $n=b\kappa_8 b^{-1}$. Therefore, locally

$$\omega_{|Q}=a^{-1}\Gamma'_\mu a dx^\mu+a^{-1}da,\quad \Gamma'_\mu=b^{-1}\Gamma_\mu b+b^{-1}\partial_\mu b. \tag{IV.3}$$

where a is in U_2 and Γ_μ is given by the unit-element section of P. The projected connection is locally of the form

$$\hat{\omega}=a^{-1}\hat{\Gamma}_\mu a dx^\mu+a^{-1}da. \tag{IV.4}$$

Considering the corresponding covariant derivatives in an associated vector bundle with fibre $\mathbb{C}^3$, we obtain $\hat{\Gamma}_\mu$ in terms of Γ'_μ:

$$\hat{\Gamma}_\mu=\Gamma'^j_\mu\sigma_j-\frac{i}{2\sqrt{3}}\Gamma'^8_\mu\begin{pmatrix}1&0\\0&1\end{pmatrix}. \tag{IV.5}$$

If $\hat{R}_{\mu\nu}$ is the curvature tensor of $\hat{\omega}$, the first Chern class of any vector bundle associated to Q is given locally by

$$c_1=\frac{1}{4\pi i}\mathrm{Tr}(\hat{R}_{\mu\nu})\,dx^\mu\wedge dx^\nu. \tag{IV.6}$$

$c_1(\tau)$ is an integer s (II.7). As in Section III, if (III.10) is satisfied ω reduces to a connection on Q.

To identify the electromagnetic field we must identify the electromagnetic gauge group U_1 as a subgroup of U_2 and apply the reduction procedure once more, passing from a U_2-bundle Q to a U_1-subbundle R. There are essentially 3 ways of doing this depending on how U_1 intersects SU_2:

(i) $U_1\cap SU_2=\{1\}$,

(ii) $U_1\cap SU_2=\mathbb{Z}_2$,

(iii) $U_1\subset SU_2$.

We shall consider each of these three cases separately.

(i) U_1 may be considered a subgroup of U_2 by the embedding

$$e^{i\alpha} \mapsto \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}, \quad 0 \leqslant \alpha \leqslant 2\pi \tag{IV.7}$$

The quotient space U_2/U_1 is the 3-sphere S^3 and its second homotopy group vanishes. A reduction is defined by a section in the associated bundle E with fibre S^3. We shall choose as section the point (0, 0, 1) in S^3 corresponding to the unit element in U_2. Any other reduction yields an electromagnetic field with the same magnetic charge. Let R be the reduced U_1-subbundle and let $\hat{\hat{\omega}}$ be the projected connection on R. $\hat{\hat{\omega}}$ is locally of the form

$$\hat{\hat{\omega}} = i\hat{\hat{\Gamma}}_\mu dx^\mu + id\alpha, \quad \hat{\hat{\Gamma}}_\mu = eA_\mu. \tag{IV.8}$$

As previously, $\hat{\hat{\Gamma}}_\mu$ may be given in terms of $\hat{\Gamma}_\mu$ by considering the covariant derivative in an associated vector bundle F with fibre $\mathbb{C}^2$. For simplicity we suppose that the section of Q which we choose to obtain a local expression for $\hat{\omega}$ is also a section of R. Therefore we have

$$\hat{\hat{\Gamma}}_\mu = \hat{\Gamma}^0_\mu = \frac{-1}{2\sqrt{3}} \Gamma'^8_\mu, \quad e = -f/2\sqrt{3}. \tag{IV.9}$$

The possibility of this type of reduction of U_2 to U_1, means that F splits into the sum of two U_1-bundles F_1 and F_2 with F_1 equal to R and F_2 the trivial bundle. From the Whitney sum formula we see then that

$$c_1(Q) = c_1(R).$$

But $c_1(R)(\tau) = 2eg$ where g is the magnetic charge and $c_1(Q)(\tau) = s$; therefore, we have

$$2eg = s. \tag{IV.10}$$

The electromagnetic field tensor $F_{\mu\nu}$ is given by

$$\sqrt{3}\, eF_{\mu\nu} = \mathrm{Tr}(nR_{\mu\nu}) - \frac{4}{3}\mathrm{Tr}(n[D_\mu n, D_\nu n]). \tag{IV.11}$$

(ii) U_1 may be considered as subgroup of U_2 by the embedding

$$e^{i\alpha} \mapsto \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}, \quad 0 \leqslant \alpha \leqslant 2\pi. \tag{IV.12}$$

The quotient space U_2/U_1 is the group SO_3 and its second homotopy group vanishes. A reduction is defined by a section in the associated bundle E with fibre SO_3. We shall choose as section the unit element in SO_3. Any other reduction yields an electromagnetic field with the same magnetic charge. Let R be the reduced U_1-subbundle and let $\hat{\hat{\omega}}$ be the projected connection on R. As in case (i) we find (IV.8, 9). The possibility of this type of reduction of U_2 to U_1 however means that F splits into the sum of two isomorphic U_1-bundles F_1 and F_2. From the Whitney sum formula we see then that

$$c_1(Q)=2c_1(R);$$

therefore, we have

$$4eg=s. \tag{IV.13}$$

The electromagnetic field tensor $F_{\mu\nu}$ is given by (IV.11).

(iii) U_1 may be considered as subgroup of U_2 by the embedding

$$e^{i\alpha}\mapsto\begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha}\end{pmatrix}, \quad 0\leqslant\alpha\leqslant 2\pi. \tag{IV.14}$$

The quotient space $U_2\times U_1$ is $S^1\times S^2$ and the second homotopy group is isomorphic to Z. A reduction is defined by a section in the associated bundle E with fibre $S^1\times S^2$. There are therefore an infinite number of inequivalent ways of reducing to a U_1-subbundle R, but two reductions defined by homotopic sections of E yield electromagnetic fields with the same magnetic charge. Let R be a reduced U_1-subbundle and let $\hat{\hat{\omega}}$ be the projected connection on R. As in cases (i, ii) we have (IV.8) and we may suppose for simplicity that the section of Q which we choose to define a local expression for $\hat{\omega}$ is also a local section of R. But now $\hat{\hat{\Gamma}}_\mu$ is given locally by

$$-\hat{\hat{\Gamma}}_\mu=\frac{1}{2}\hat{\Gamma}^3_\mu=\frac{1}{2}\Gamma'^3_\mu, \quad e=-f/2. \tag{IV.15}$$

The possibility of this type of reduction of U_2 to U_1, means that F splits into the sum of two U_1-bundles F_1 and F_2 with $F_2=F_1^*$. From the Whitney sum formula we see that

$$c_1(Q)=0;$$

that is, Q must be isomorphic to the trivial bundle. The reduction from Q to R in this case is the same as the reduction P to Q in Section III. $c_1(R)(\tau)$ is an integer t and we have

$$2eg=t. \tag{IV.16}$$

There is a natural section of E given by

$$n_3=b\kappa_3 b^{-1}. \tag{IV.17}$$

We recall that $b(x^\alpha)$ is the element of SU_3 which defines the local section of Q (and R). The corresponding value of t is $t=0$ and the electromagnetic field tensor $F_{\mu\nu}$ is given by

$$\begin{aligned} eF_{\mu\nu}=&\operatorname{Tr}(n_3R_{\mu\nu})-\operatorname{Tr}(n_3[D_\mu n_3, D_\nu n_3]) \\ &-\sqrt{3}\operatorname{Tr}(n[D_\mu n_3, D_\nu n_3]). \end{aligned} \tag{IV.18}$$

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