[M1] S. Montgomery, *Outer automorphisms of semiprime rings*, J. London Math. Soc. **18** (1978), 209–220.

[M2] ——, *Fixed rings of finite automorphism groups of associative rings*, Lecture Notes in Math., vol. 818, Springer-Verlag, Berlin and New York, 1980.

[M3] ——, *Prime ideals and group actions in noncommutative algebras*, Invariant Theory, Contemp. Math., vol. 88, Amer. Math. Soc. Providence, RI, (1989), pp. 103–124.

[Mo] J. A. Moody, *Induction theorems for infinite groups*, Bull. Amer. Math. Soc. (N.S.) **17** (1987), 113-116.

[NT] Y. Nakagami and M. Takasaki, *Duality for crossed products of von Neumann algebras*, Lecture Notes in Math., vol. 731, Springer-Verlag, Berlin and New York, 1979.

[NA] T. Nakayama and G. Azumaya, *On irreducible rings*, Ann. of Math. (2)**48** (1947), 949–965.

[N] E. Noether, *Nichtkommutative algebra*, Math. Z. **37** (1933), 514–541.

[P1] D. S. Passman, *The algebraic structure of group rings*, Wiley-Interscience, New York, 1977.

[P2] ——, *Semiprime crossed products*, Houston J. Math. **11**(1985), 257–268.

[P3] ——, *Prime ideals in polycyclic crossed products*, Trans. Amer. Math. Soc. **301** (1987), 737–759.

[Q] D. Quinn, *Integrality over fixed rings*, J. London Math. Soc. **40** (1989), 206–214.

[Ri] M. A. Rieffel, *Actions of finite groups on $C^*$-algebras*, Math. Scand. **47** (1980), 157–176.

[R] J. E. Roseblade, *Prime ideals in group rings of polycyclic groups*, Proc. London Math. Soc. **36** (1978), 385–447.

[vdW] B. L. van der Waerden, *A history of algebra*, Springer-Verlag, Berlin and New York, 1985.

[W] J. H. M. Wedderburn, *A type of primitive algebra*, Trans. Amer. Math. Soc. **15** (1914), 162–166.

[Z] A. E. Zaleskiĭ, *Irreducible representations of finitely-generated nilpotent torsion-free groups*, Math. Notes **9** (1971), 117–123.

[ZN] A. E. Zaleskiĭand O. M. Neroslavskii, *On simple Noetherian rings*, Proc. Akad. Bylorussia SSR (1975), 38–42. (Russian)

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*Perfect groups*, by Derek F. Holt and W. Plesken. Oxford University Press, Oxford, New York, 1989, xii + 364 pp., $70.00. ISBN 0-19-853559-7

Around 1980 the completion of the classification of the finite simple groups was announced (see [G]). Group theorists of a classifying bent were left to ponder what to classify next. All finite groups is an obvious naive guess, but the asymptotic estimates of Higman [Hi] and Sims [S] on the number of nonisomorphic $p$-groups of order $p\hat{}n$ and the tables of small 2-groups compiled by Hall and Senior [H-S] and others rapidly dampen one's enthusiasm for this project. Retrenchment to a more tractable class of groups is definitely in order.

Holt and Plesken appear at first to be offering perfect groups as a suggestion. A group $G$ is *perfect* if $G$ is equal to its commutator subgroup. In particular all nonabelian simple groups are perfect, while no finite $p$-groups are perfect. However bad news emerges rapidly. Firstly, an almost complete description is needed of the nonisomorphic indecomposable $GF(p)$ [G]-modules for nonabelian simple groups $G$, a hopelessly difficult task for most blocks of noncyclic defect. Even when the necessary modules are in hand, as for $\mathrm{Alt}(5)$, the problem is almost as difficult as the classification of $p$-groups, as the authors note on page 4.

So in what sense is a "theory of perfect groups" attainable? This is not at all clear, but the study of these groups certainly brings into play a fascinating assortment of group-theoretic techniques and provides a context for students to get their hands on a panoply of nontrivial examples and see many interesting methods at work in concrete examples, a situation which is not often the case in books on representation theory or cohomology of groups.

Certainly many groups of critical interest to mathematicians, physicists and chemists are not simple but are perfect or close to perfect. For example, the groups of motions of affine or Euclidean spaces and the "space groups" of motions preserving a lattice are all far from simple but close to perfect. Typically they are splitting extensions of large abelian groups by simple or almost simple groups. These groups are of such fundamental importance that their absence from most graduate algebra texts is shocking. Space groups, whose study goes back to Bieberbach [B], receive considerable attention in this work and in particular a system of representatives of the genus classes of all ordinary irreducible perfect space groups up to dimension 10 is tabulated.

For a finite group $G$, the authors define the complete reducible residue of $G$, $\mathrm{CR}(G)$, to be the intersection of the kernels of all epimorphisms of $G$ onto simple groups. It is easy to see that the smallest perfect group $G$ with nonsolvable $\mathrm{CR}(G)$ is $\mathrm{Alt}(5)\,\mathrm{wr}\,\mathrm{Alt}(5)$. As the authors restrict their attention to perfect groups of order less than one million and to space groups in dimension at most 10, it is always the case for them that $\mathrm{CR}(G)$ is solvable and the problem becomes that of studying extensions of solvable groups (usually $p$-groups) by direct products of non-abelian simple groups. To this problem, the authors bring to bear, in addition to modular representation theory, methods involving pro-finite groups, Frattini extensions and $p$-adic groups. This general methodology occupies the first 75 pages of the book.

Over 200 pages of the book are devoted to tables enumerating the perfect groups of order less than a million and the space groups in dimension at most 10. The former tabulation is not quite complete and the authors indicate exactly what is covered.

With only 20 pages devoted to modular representation theory, it is not surprising that, as the authors say, "the reader is assumed to have some knowledge of the elements of representation theory." Indeed on all topics, more than a little a priori sophistication would be helpful. This book operates then on two levels. For a highly trained graduate student, the book pulls together in a novel way many topics she may have seen from a different angle before and unifies them in the service of a well-defined, concrete but highly nontrivial problem. For a "customer" of advanced group theory, the tables may be used as a source of data, for example, on low-dimensional space groups. The book also comes with microfiches tabulating the character tables of factor groups for some perfect space groups.

Personally, I wish the general theory in Chapter 2 were presented at a more leisurely pace. There is a wealth of interesting mathematics there, but I fear it is too densely packed for all but the hardiest souls. Perhaps this is fair warning. Lasciate ogni speranza, voi ch'entrate. Otherwise, the authors have an engaging writing style and the book seems to be low on errors, both mathematical and typographical.

The moral of the book seems to be: Classifiers despair! But aficionados of fascinating groups and gnarly problems can find a wealth of both in the domain of perfect groups. And for such, this book provides several keys to the kingdom.

## REFERENCES


[B] L. Bieberbach, *Uber die Bewegungsgruppen der Euklidischen Raume* I, II, Math. Ann. **70** (1910), 297–336; Math. Ann. **72** (1912), 400-412.

[G] D. Gorenstein, *Finite simple groups: an introduction to their classification*, Plenum Press, New York, 1982.

[H-S] M. Hall, Jr. and J. K. Senior, *The groups of order* $2n (n = 6)$, Macmillan Press, New York, 1964.

[Hi] G. Higman, *Enumerating p-groups* I, Proc. London Math. Soc. **10** (1960), 24–30.

[S] C. C. Sims, *Enumerating p-groups*, Proc. London Math. Soc. **15** (1965), 151–166.



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*Transformation groups and algebraic K-theory*, by Wolfgang Lück. Lecture Notes in Math., vol. 1408, Springer-Verlag, Berlin and New York, 1989, 455 pp., $42.70. ISBN 0-387-51846-0

To understand a manifold, it is necessary to understand its symmetries. This is the basic theme of equivariant topology. Typically, one studies a group acting on a manifold by diffeomorphisms. A basic example of this is when the manifold is $R^n$, $n$-dimensional euclidean space. To understand this manifold, one associates the various matrix groups such as $Gl(n, R)$ or the orthogonal group $O(n)$. It is also important to study smaller groups such as the finite subgroups of $O(n)$. A representation is an action by a group of orthogonal matrices on $R^n$, inducing an action on the unit sphere, or on the unit disk. These are model examples of the kind of group action one considers in equivariant topology. The basic problem is to construct and classify actions with given properties.

In many interesting cases the action of the group is cellular. This means that the manifold has a cellular decomposition, so that the action of the group is given by permuting cells. This is, for instance, the case when the group is finite and the action is smooth. It is, thus, natural to start out studying cellular actions