Finally, we remark that a number of new graduate texts on measure theoretic probability are now being written, soon to appear. The prior appearance of Dudley's book is certain to define a new standard of rigor and completeness for the decade of the 1990s.

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*Noncommutative Noetherian rings*, by J. C. McConnell and J. C. Robson. Pure and Applied Mathematics. Wiley Interscience, John Wiley and Sons, New York, 1987, xiv+596 pp., $138.00. ISBN 0-471-91550-5

A ring $R$ is said to be *right Noetherian* if every right ideal of $R$ is finitely generated. There are two equivalent conditions: The *ascending chain condition* (every ascending chain of right ideals becomes stationary) and the *maximal condition* (every nonempty set of right ideals contains maximal elements). It has been 100 years since Hilbert [H] proved his basis theorem, which nowadays is stated in the following form: If $R$ is a Noetherian ring, so is $R[x]$, the polynomial ring over $R$ in one variable.

Hilbert used his result to conclude that certain rings of invariants were finitely generated. The name "Noetherian" honors

Emmy Noether whose work showed the power of this abstract condition. Her proof of the primary decomposition of ideals (the Lasker-Noether theorems) is often called the first result of modern (or abstract, or noncomputational) algebra. Lasker [L] had proved primary decomposition for ideals in polynomial rings over fields by lengthy computational means; Noether [N] achieved the same result for the much wider class of commutative Noetherian rings by elegant indirect arguments which have become standard tools of today's algebraists, but which remain magical.

It follows from the Hilbert basis theorem that commutative rings which are finitely generated as algebras over a field (or even a commutative Noetherian ring) are themselves Noetherian. Thus, the most commonly studied commutative rings are Noetherian, and it is not surprising that the Noetherian condition is a standing hypothesis in most of commutative ring theory.

For noncommutative rings the Noetherian condition is not usual, although there are important classes of noncommutative Noetherian rings; among them are group rings of polycyclic groups, Weyl algebras, and enveloping algebras of finite-dimensional Lie algebras. The importance of the Noetherian hypothesis in noncommutative ring theory comes from its utility—it can be used to prove theorems. However, it often happens that easy results for commutative rings become difficult for noncommutative rings, and are true only with extra hypotheses, which most often include the Noetherian hypothesis.

This phenomenon is illustrated by Goldie's theorem, which is the fundamental theorem of noncommutative Noetherian rings. It says that a semiprime ring $R$ has a right quotient ring which is semisimple Artinian if and only if $R$ has the following two properties: (1) $R$ has no infinite direct sums of right ideals; (2) $R$ has the ascending chain condition on right annihilators. Rings satisfying (1) and (2) are called *right Goldie rings.* Right Noetherian rings form a large class of right Goldie rings. Goldie's theorem is a deep and difficult result, but if we assume that $R$ is commutative and also assume that $R$ is prime (for the sake of illustration—it does not essentially simplify the proof in the noncommutative case), then $R$ is a commutative integral domain, and Goldie's theorem reduces to the elementary fact that a commutative integral domain has a field of quotients.

After a brief review of basic ring theory, *Noncommutative Noetherian rings* is divided into four parts. Part I, *Basics*, includes Goldie's theorem, noncommutative localization, and many other topics. Part II, *Dimensions*, is devoted to Krull, global, and Gelfand–Kirillov dimensions. Part III, *Extensions*, consists mostly of noncommutative versions of commutative theorems. These include nullstellensatz and going-up theorems for certain rings, and some $K$-theory, such as stable range theorems and the functor $K_0$. Part IV, *Examples*, consists of three chapters, each of which is a brief introduction to an area of ring theory: polynomial identity rings, enveloping algebras of Lie algebras, and rings of differential operators on algebraic varieties.

*Noncommutative Noetherian rings* is a splendid book because of the excellent presentation of the material, not because of the novelty of the material presented, most of which can already be found in well-written monographs. Part I has considerable overlap with *Localization in Noetherian rings* by Jategaonkar [J], a much more personal book which reflects its author's methods. Among the sources for the material in the remainder of *Noncommutative Noetherian rings* are the following: *Krull dimension* by Gordon and Robson [GR], *Growth of algebras and Gelfand–Kirillov dimension* by Krause and Lenagan [KL], *Algebraic K-theory* by Bass [B], *Fixed rings of finite automorphism groups of associative rings* by Montgomery [M], *Polynomial identities in right theory* by Rowen [R], *Enveloping algebras* by Dixmier [D], and *Rings of differential operators* by Bjork [Bj]. Some material does appear for the first time in a monograph. Most notable is Stafford's generalization of the stable range theorems of Serre (large projectives have free direct summands) and Bass (free summands can be cancelled) to noncommutative Noetherian rings.

*Noncommutative Noetherian rings* is a model of mathematical writing, as perfectly written a mathematics book as I have seen. This is a consequence of several fortuitous circumstances. First, except for Jategaonkar, nearly all the major researchers in noncommutative Noetherian rings are part of a loosely connected and benevolent Mafia centered at Leeds, the home university of both authors. This gave the authors early access to nearly all work in the area, above all to the better and better presentations of basic results which arose over time. Then the fact that the completion of the book was delayed for many years allowed these improvements to be incorporated. It also meant that the book was not written until the most important trends in the subject had been recognized. We can thank the authors not only for their ability to take advantage of these circumstances, but also for the accessibility of their highly readable presentation. It can be profitably read by nonexperts, even nonalgebraists. Especially noteworthy are the copious use of examples to illustrate the theory, and the brief but precise and useful historical remarks at the end of each chapter.

*Noncommutative Noetherian rings* does not attempt to be at the frontier of research nor to be more than an introduction to any particular class of Noetherian rings. Rather, it is an almost perfectly conceived account of major developments and general methods. It will remain a basic reference for many years. I am grateful for this book to which I will often return.

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