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Mathematical problems from combustion theory, by Jerrold Bebernes and David Eberly. Applied Mathematical Sciences, vol. 83, Springer-Verlag, New York, 1989, 177 pp., $34.00. ISBN 0-387-97104-1

Combustion involves the liberation of energy by chemical reaction. Typically, the rate of energy release is a strongly sensitive function of temperature. Doubling the temperature, for example, may well increase the rate by a factor of ten thousand. Combustible materials, essentially inert at room temperature, can therefore ignite rapidly and explode when sufficiently heated.

Although combustion commences in chemistry, it also involves, in general, transport of matter and energy. A typical sequence of events in the combustion of a well-mixed mass of reactants may involve ignition followed by the establishment of a combustion front, which propagates across the vessel until the reactants are exhausted. A reasonable mathematical model of the underlying physicochemical processes, at least for gaseous combustibles, consists of the compressible Navier–Stokes equations augmented by balance equations for the individual chemical species taking part in the chemical reaction. From the mathematical point of view these equations form a formidable set; the complexities of fluid mechanics are only compounded by chemical kinetics, which can be quite daunting even for the simplest of combustibles.

Mathematical tractability demands drastic assumptions. It is not uncommon, for example, to replace the entire chemical scheme by an overall, one-step exothermic chemical reaction. Another frequently-held postulate takes the combustible to be a dilute mixture with an abundant inert. These and other similar approximations, aimed primarily at achieving mathematical simplicity, have nevertheless led to increased qualitative understanding of many aspects of combustion.

The sensitive dependence of the global reaction on temperature, at least for gaseous reactants, is governed by the Arrhenius law. The rate term then involves the factor $e^{1/\varepsilon - 1/\varepsilon T}$, where T is the (suitably nondimensional) temperature and ε, the reciprocal activation temperature, frequently a small parameter. The exponential nonlinearity has seduced many an applied mathematician in recent years. As a result, practitioners of the art of formal asymptotics have lustily exploited the limit $\varepsilon \to 0$ to examine a variety of combustion phenomena. This is especially true of ignition theory, the topic of this book.

Ignition is often characterized by a period of gradual chemical heating, when the reaction rate is low (the induction stage), followed by an extremely rapid, localized temperature rise (the explosion stage) as the reaction accelerates. In the small-ε analysis of ignition, it is recognized that during induction, changes in the state of the medium are measured properly on the ε scale. This leads to a set of small-disturbance equations which, although simpler than the full set, still retains a vestige of the chemical nonlinearity; with $T \sim 1 + \varepsilon\tau$, the Arrhenius factor $e^{1/\varepsilon - 1/\varepsilon T}$ does simplify, but only to e^{τ} at leading order. Transport terms in the governing equations are also linearized and some may even be negligible, the precise reduction depending upon the relative importance of diffusive and acoustic transport.

In the induction equations, ignition manifests itself as thermal runaway or blowup, i.e. the temperature (perturbation) grows without bound within a finite time. For prescribed initial and boundary data, it is important to determine whether the solution exhibits blowup, and if it does so, when, where and how the blowup occurs. Attempts to answer these questions form the subject of this book.

There are six chapters. The first contains a brief derivation of the governing equations for a reactive mixture. A series of simplifying assumptions leads to several sets of reduced equations, to be analyzed in subsequent chapters. With two exceptions (the solid-fuel and the full gaseous cases), the reduced sets all pertain to the induction stage under the small ε limit, differing from one another in the relative importance of the various kinds of transport mechanisms. Chapter 2 examines the steady-state reactive–diffusive problem as to questions of existence, multiplicity, and the qualitative shape of solutions. The unsteady reactive–diffusive problem is considered in Chapter 3. Both subcritical (without blowup) and supercritical (with blowup) solutions are examined, and for the latter, bounds are computed for the blowup time. A final-time description of the blowup profile remains elusive, even though formal asymptotic results for the same are available. Existence results for the solid-fuel model (two coupled reaction–diffusion equations for the temperature and the reactant mass fraction, and the full Arrhenius term) are given in Chapter 4, and for the acoustically filtered gaseous system in Chapter 5. The full gaseous model is considered in Chapter 6, along with the acoustic–reactive induction model. Existence results and solution bounds are obtained for the former, and one possible spatial structure of the blowup singularity justified for the latter. The mathematical tools employed include maximum principles, the notion of upper and lower solutions, invariance, comparison arguments, semigroup theory and energy estimates.

The book is written from the point of view of an analyst. To borrow a phrase from Titchmarsh, the authors appear to have "retained, as having a certain picturesqueness, some references to 'ignition', 'combustion', and so forth; but the interest is purely analytical, and the reader need not know whether such things exist." Once a problem is defined by a set of equations, theorems are stated. Proofs that follow are cogent and precise, to be sure, but also terse and dry. On occasions, certain choices and constructions seem to have been pulled out of thin air (e.g., the assumption $A + 2B = 1$ on page 135, and a few lines later, the selection of the similarity variable η). A livelier treatment would have employed physical considerations to motivate analytical steps (e.g., the switch to characteristic coordinates on page 134).

The comments at the end of each chapter are useful in relating the material to other mathematical literature, and in identifying open problems. There is no attempt, however, either to interpret the results physically or to identify their place within the general context of the science of combustion. This lack of attention to physics also seeps into the first chapter, presumably the most "physical" of all. To cite a few instances, equation (1.11) is said to have been derived by "neglecting certain higher-order terms," but there is no mention of any small quantities. The small fuel loss steady-state model is introduced in equations (1.32–33), without identifying it as a logical consequence of the full equations under the assumption of large heat release.

To summarize, the monograph presents rigorous results for a class of partial differential equations appearing in the theory of thermal ignition. Many of the results are due to the authors themselves, and they have done an excellent job of collecting and unifying the substantial mathematical literature on the subject. The material, certainly of interest to the analyst, should also be of value to those engaged in a more formal study of the subject, either through asymptotics or numerics. The applied mathematician in me would simply have preferred a greater interplay with the physical field from which the mathematical problems are drawn.

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