that the book is a valuable addition to the literature both for mathematical logicians because of the systematic and exhaustive account of interpretation in weak systems of arithmetic, and for philosophers of mathematics for the way in which conclusions compatible with strict finitism are deduced from assumptions based purely on a formalist viewpoint.

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Commutative rings with zero divisors, by James A. Huckaba. Marcel Dekker, New York and Basel, 1989, x+216 pp., $79.75. ISBN 0-8247-7844-8

This excellent monograph on the titled subject covers a huge amount of research over the past thirty years. The author manages in just over 200 pages (not densely printed—more about this later) to include works from over 200 papers. The Index of Main Results lists 120 theorems, and the remarkably complete end-of-chapter notes tell where each and every one comes from! The work is estimably enriched by more than 20 mostly difficult examples (and counterexamples) worked out in the last chapter, which the motivated reader reads appropriately alongside the foregoing. (No it-can-be-showns for Professor Huckaba!)

References in the sequel, especially to the chapter notes may be found in the text.

Chapter I (Total Quotient Rings) introduces various properties of the commutative ring R, its total quotient ring denoted

$$T(R) = \{a/b | a \in R, b \in R^*\},$$

where R^* is the set $R \backslash Z(R)$, and $Z(R)$ is the set of zero divisors of R. Also frequently used is the so-called complete (or maximal) ring $Q(R)$ of quotients, for which the author refers to the classic book of Lambek *Lectures on Rings and Modules* (currently reprinted by Chelsea).

Some frequently used properties are introduced in Chapter I. One called property A is the following: If $I \subseteq Z(R)$ is a finitely generated ideal, then I has nonzero annihilator. Elsewhere I call a ring with this property a McCoy ring since the proof that a Noetherian ring has property A requires McCoy's ubiquitous theorem. *If an ideal* I *is contained in a product* $I_1 I_2 \cdots I_n$ *of ideals, then some power* I^t *of* I *is contained in one of the* I_j. *(If the* I_j *are prime ideals, then* $I \subseteq I_j$, *for some* j.) Kaplansky's *Commutative Rings* is referred to for the proof of the case where each I_j is prime.

Theorem 2.7 states that any graded ring, e.g., *any* polynomial ring $R[X]$ (in *any* number of variables X), has property A.

Graded rings A also have another annihilator condition, denoted (a.c.), namely, every finitely generated ideal I has annihilator $\operatorname{Ann} I = \operatorname{ann}(x)$ for some $x \in A$. Neither property (a.c.) and A imply the other but examples are given much later (on p. 177).

The major theorem of Chapter I is Mewborn's characterization of compactness of

$$\operatorname{Min} R = \operatorname{min\,spec} R$$

via flatness of $Q(R)$, with other characterizations given of Quentel and Henriksen and Jerison. Another theorem, due to Quentel, characterizes von Neumann regularity of $T(R)$ via the two properties: R has property A, and $\min R$ is compact. The Henriksen–Jerison refinement states that property A can be replaced by (a.c.) in this characterization.

Chapter II (Valuation Theory) introduces valuations of an arbitrary (commutative) ring R (after Samuel) with Manis' characterization via max pairs (V, M) containing a pair (R, P) consisting of a ring R and a prime ideal. Thus: If T is a ring $\supseteq R$, then (V, M) is maximal in the set of ordered pairs (S, L) where S is a between ring $T \supseteq S \supseteq R$ and L is an ideal of S such that $L \cap R = P$. In this case V is the valuation ring of a valuation of T, and M is the associated prime ideal. (Note: When R is a domain, and $T = T(R)$, then by Krull's theorem V is a chained ring, i.e., the ideals of V are linearly ordered by inclusion, but in general only the "closed" prime ideals of V are linearly ordered.)

Prufer rings R (= finitely generated regular ideals are invertible in $T(R)$) are included in this chapter, and various localizations are studied in connection with work of Davis, Marto, Griffin, Butts and Smith, Portelli and Spangher, and Huckaba.

Another section of Chapter II is devoted to Krull rings and the work of Marot, Gilmer, Kennedy, D. D. Anderson, D. F. Anderson, and Markanda.

Chapter III (Integral Closure), Section 9, presents Griffin's generalization of Krull's characterization of integral closed domains as intersections of the valuation overrings to rings integrally closed in their total quotient ring: They are intersections of "paravaluation" overrings, where a paravaluation $v: T \to \Gamma\{\infty\}$ of T to an ordered Abelian group Γ union $\{\infty\}$ need not be onto as is required in the definition of a (Manis) valuation.

Section 10 proves Huckaba's theorem: The integral closure of a Noetherian ring is a Krull ring. Nagata's theorem on integrally closed domains of dimension ≤ 2 appears in §11, and in generalizations to zero divisors in §12. Section 13 is devoted to integrally closed polynomial rings: Results of Quentel and Endo are applied to deduce for a reduced integrally closed ring with compact min spec that $R[X]$ is integrally closed if $T(R)$ is von Neuman regular.

Theorem 13.11, due to Akiba, states that $R[X]$ is integrally closed over an integrally closed reduced ring having property A.

Chapter IV (Overrings of Polynomial Rings) discusses many of the foregoing properties for polynomial rings and their various overrings. Define:

$R\langle X\rangle = R[X]_W$, where W is the set of monic polynomials.
$R(X) = R[X]_S$, where S is the set of polynomials of content 1.

Thus, $R(X)$ is a quotient ring of $R\langle X\rangle$.

Assuming $\dim R < \infty$, then in §17 Huckaba proves theorems of LeRiche, namely:

$$\dim R\langle X\rangle = \dim R[X] - 1 = \dim R(X),$$

and if, in addition, R is Noetherian, then

$$\dim R = \dim R\langle X\rangle = \dim R(X) = \dim R[X] - 1.$$

Gilmer's and Heinzer's theorems are of evident interest:

$$R(X) = R[X] \Leftrightarrow \dim R = 0;$$

and

$$R\langle X, Y\rangle = R\langle Y, X\rangle \Leftrightarrow \dim R = 0,$$

where $R\langle X, Y\rangle = R\langle X\rangle\langle Y\rangle$. (Contrast this with D. D. Andersen's theorem (Lemma 15.3) which asserts that $R(X, Y) = R(Y, X)$.)

Section 18 studies "divisibility" (which I would call factorization) properties of $R\langle X\rangle$ and $R(X)$. For domains $R\langle X\rangle$ and $R(X)$ behave responsibly: GCD, UFD, Krull, PID, and Dedekind are mirrored in R, $R\langle X\rangle$, and $R(X)$. In addition, for rings with zero divisors, ZPI (another Krull contribution, literally "*Zerlungen in primen idealen*") and PIR are mirroring properties, but Prufer rings are not: R must be "strongly" Prufer for $R\langle X\rangle$ to be Prufer, e.g., if R is Prufer with property A.

Chapter IV is rounded out with $*$-operations and Kronecker functions (in §20), Arnold's theorems generalized to Marot rings with property A, by Hinkel and Huckaba (§21), and more on Kronecker Function rings (§22).

Chapter V (Chained Rings) is concerned with aspects of a Kaplansky question: Is every chained ring a homomorphic image of a chained (i.e., valuation) domain? Fuchs and Salce have given a counterexample in their book (Marcel Dekker, 1985, *Modules over Valuation Domains.* Incidentally, this is one of 11 books in References.) An affirmative answer by Hungerford (1968) and MacLean (1973) holds when $T(R)$ is Noetherian, and by Ohm–Vicnair, when R is a chained monoid ring $A[S]$. (Also see Fuchs and Shelah, PAMS 105 (1989, pp. 25–30).)

I. S. Cohen's structure theorem for complete local rings is stated on p. 154 and applied in the proof of the Hungerford–MacLean theorem. It is worthy of note that up to this point in the book nothing has been said about completions, and neither Zariski–Samuel's nor Nagata's book contains the statement in full generality. (The author refers to Cohen.)

Also in Chapter V is the following curiosity: A theorem of Froeschl states for a chained ring $T = T(T)$ that a valuation ring V with $T(V) = T$ is chained iff $V \subseteq Z(T)$. Otherwise, V has exactly two maximal ideals, one of which is $Z(V)$. (So much for one's intuition.)

Chapter VI (Constructions and Examples) has already been alluded to, but it ought to be mentioned that many of the examples are "idealizations" and "$A+B$ rings" whose originals are discussed by Huckaba in the chapter notes. Actually, idealizations are called split-null or trivial extensions elsewhere.

In my copy I added these to the index: integrally closed (p. 54), integral closure R' (p. 53), Jacobson radical (p. 60), nil radical $N(R)$ (p. 1), Noetherian (p. 4), and primary ideal (pp. 12–13). The page numbers are where they appear in the text, but they are left undefined by the author. There is no list of symbols.

It may note be amiss here to remark on the price of this outstanding book. At approximately \$80 the price comes to roughly 40 cents per page! Ouch! But, don't leave; there are roughly 30 lines per page compared to the ≈ 40 lines per page of Van der Waerden's Algebra (Viertel Auflage, Springer-Verlag (1959)), which is of comparable size. Thus, set in the denser Springer-Verlag mode, this book would shrink to 3/4's the present number of 215 pages, to ≈ 155 pages, which comes to ≈ 55 cents per page while Springer books average ≤ 20 cents per page!

I sympathize with an author's plight: He does not set prices! Regardless, I recommend this excellent text for *those who can afford it.* I found nary a typo, and the treatment of the selected topics is not only lucid but impeccable. It brought the same delight that I experienced reading Kaplansky's non-pareil *Commutative Rings* and Lambek's *Lectures on Rings and Modules*, which is to say that the author's love and command of the subject shines on every page.

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Harmonic analysis in phase space, by Gerald Folland. Princeton University Press, Princeton, NJ, 1989, \$17.50 (paper), \$55.00 (cloth). ISBN 0-691-08528-5

"The phrase *harmonic analysis in phase space* is a concise if somewhat inadequate name for the area of analysis $\mathbf{R}^n$ that involves the Heisenberg group, quantization, the Weyl operational calculus, the metaplectic representation, wave packets, and related concepts: It is meant to suggest analysis *on* the configuration space $\mathbf{R}^n$ done by working *in* the phase space $\mathbf{R}^n \times \mathbf{R}^n$. The ideas that fall under this rubric have originated in several fields—Fourier analysis, partial differential equations, mathematical physics, representation theory, and number theory, among others. As a result,