The book contains an excellent and extended list of references, but the author has not made an effort to help the reader in finding his way through the list. Comments consisting of one sentence with a reference to fifteen or more papers are not very useful. The sections about applications are too modest, both in presentation and in quantity. These are minor criticisms on an otherwise excellent book, which thanks to the initiative of the AMS is now available to the international mathematical community.

REFERENCES

1. M. V. Keldysh, *On the eigenvalues and eigenfunctions of certain classes of nonselfadjoint equations*, Dokl. Akad. Nauk SSSR **77** (1951), 11–14.

2. ——, *On completeness of the eigenfunctions of certain classes of nonselfadjoint linear operators*, Uspehi Mat. Nauk **27** (1971), no. 4 (**160**), 15–41; English tranls. in Russian Math Surveys **27** (1971).

3. I. C. Gohberg and M. G. Kreĭn, *Introduction to the theory of linear nonselfadjoint operators*, Transl. Math. Monographs, vol. 18, Amer. Math. Soc., Providence, R.I., 1969.

4. M. G. Kreĭn and H. Langer, *On the theory of quadratic pencils of selfadjoint operators*, Dokl. Akad. Nauk SSSR **154** (1964), 1258–1261; English transl. in Soviet Math. Dokl. **5** (1964).

5. ——, *On some mathematical principles in the linear theory of damped oscillations of continua*, Applications of Function Theory in Continuum Mechanics (Proc. Internat. Sympos., Tbilisi, 1963), vol. 2, "Nauka", Moscow, 1965, pp. 283–322; English transl., Parts I, II, *Integral equations operator theory* **1** (1978), 364–399; 539–566.

6. L. Rodman, *An introduction to operator polynomials*, Operator Theory: Advances and Applications vol. 38, Birkhäuser Verlag, Basel, 1989.

I. GOHBERG
TEL AVIV UNIVERSITY

M. A. KAASHOEK
VRIJE UNIVERSITEIT

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 21, Number 2, October 1989

0273-0979/89 $1.00 + $.25 per page

Conformal geometry and quasiregular mappings, by Matti Vuorinen. Lecture Notes in Mathematics, vol. 1319, Springer-Verlag, Berlin, Heidelberg, New York, 1988, xix + 209 pp., $21.20. ISBN 3-540-19342-1

The theory of *quasiregular mappings* (q.r. mappings) is an extension to the Euclidean space $\mathbf{R}^n$ of the methods of geometric function theory in the complex plane $\mathbf{C}$. Very often properties of holomorphic functions in $\mathbf{C}$, which do not depend on power series developments, can be studied for mappings in $\mathbf{R}^n$. The main theme of this review is to provide examples of these properties and some of its applications. Let us remark that this extension is quite different from the theory of holomorphic functions in $\mathbf{C}^n$, $n \geq 2$. Indeed, a holomorphic function in $\mathbf{C}^n$ which is also quasiregular as a mapping of $\mathbf{R}^{2n}$ must be affine if $n \geq 2$ [**MaR**].

Let Ω be a domain in $\mathbf{R}^n$, $n \geq 2$, and $f: \Omega \to \mathbf{R}^n$ be a mapping in the space $W^{1,n}_{\text{loc}}(\Omega)$; that is, if $f = (f^1, \ldots, f^n)$, each f^i has first partial derivatives in the distributional sense which are functions in $L^n_{\text{loc}}(\Omega)$. In particular, the formal differential $Df(x)$ and the formal Jacobian $J_f(x)$ exist for a.e. $x \in \Omega$. The mapping f is said to be *quasiregular* if there exists $K \geq 1$ such that

$$|Df(x)|^n \leq K J_f(x) \tag{1}$$

for a.e. $x \in \Omega$, where $|Df(x)| = \sup\{|Df(x)h|: h \in \mathbf{R}^n, |h| = 1\}$. The smallest K satisfying (1) will be called the *dilatation* of f and we will say that f is K-q.r. If, in addition, f is injective then f is called K-*quasiconformal* (K-q.c.).

Take a point $x \in D$ such that $J_f(x) > 0$ and f is differentiable at x. Infinitesimal spheres centered at x will be mapped by f to infinitesimal ellipsoids centered at $f(x)$. Condition (1) means precisely that the ratio between the longest and shortest semiaxis of these ellipsoids is bounded independently of x.

An analytic function in the plane maps infinitesimal spheres to infinitesimal spheres, so they are examples of 1-q.r. mappings in $\mathbf{R}^2$. In this case (1) holds with equality. It is just another way of writing the Cauchy-Riemann equations. Moreover, *every* 1-q.r. *mapping in the plane is an analytic function and* 1-*q.c. mappings are conformal mappings.* The standard references for the two-dimensional theory are the books by Ahlfors **[A1]** and Lehto-Virtanen **[LV]**. A modern treatment is in Lehto's book **[L1]** and a complete historic account can be found in **[L2]**.

In this review we will emphasize the case $n \geq 3$. However, we cannot resist the temptation to mention a few instances in which plane q.c. mappings have provided the essential tool to solve problems from other branches of analysis. Let us cite Drasin's solution of the inverse problem of Nevanlinna theory **[D]**, Bers' theorem on simultaneous uniformization **[B1]**, applications to Kleinian groups and surface topology **[A2, B2, EE, K]**, Sullivan's recent solution of the Fatou-Julia problem **[S]** and the intimate connection with the theory of nonlinear elliptic equations in two variables **[GT**, Chapter 12**]**. Some excellent expository articles on plane q.c. mappings are **[A3, B3, B4** and **B5]**.

While there are plenty of 1-q.r. mappings in the plane, the situation changes drastically in $\mathbf{R}^n$, $n \geq 3$. Gehring **[G1]** and Rešetnjak **[R1]** extended a theorem of Liouville (1850) to prove that restrictions to Ω of 1-q.r. mappings are the only Möbius transformations, that is, finite composition of rotations, translations and reflections on spheres and planes (see **[BI1]** for a recent simpler proof). Therefore in higher dimensions it is necessary to consider the case $K > 1$.

Condition (1) implies that q.r. mappings have some degree of regularity. Rešetnjak **[R2, R3]** showed that a nonconstant q.r. mapping can be modified in a set of measure zero so that it becomes continuous, open and discrete ($f^{-1}(y)$ is a discrete set). Moreover, f is differentiable a.e., it preserves sets of measure zero, $m(f^{-1}(E)) = 0$ whenever $m(E) = 0$ and

$J_f(x) > 0$ a.e. Proofs of these properties in the q.c. case are in the fundamental papers of Gehring **[G1]** and Väisälä **[V1]** and (including the q.r. case) in the series of papers **[MRV1, MRV2, MRV3]**.

Let $B_f = \{x \in \Omega\colon f \text{ is not a local homeomorphism at } x\}$ be the *branch set* of f. Zoric proved in **[Z]** that if f is unbranched ($B_f = \emptyset$) and is defined in all of $\mathbf{R}^n$, $n \geq 3$, then f is *injective*. Compare this result with the exponential e^z in the two-dimensional case! In contrast to the two-dimensional case, in higher dimensions if a q.r. mapping is smooth enough it will be *unbranched*; for example, when $n \geq 3$ and $f \in C^2$ **[V2]**. These facts show that $\mathbf{R}^n$, $n \geq 3$, is much more "*rigid*" than $\mathbf{R}^2$. Another important result showing the difference between $n \geq 3$ and $n = 2$ was proved by Gehring in **[G1]**. It turns out that if $f\colon \mathbf{B}^n \to \mathbf{B}^n$ is q.c. and surjective, then f extends to a homeomorphism of $\overline{\mathbf{B}^n}$, **[V3]**. How regular is $g = f/\partial\overline{\mathbf{B}^n}$? If $n = 2$ Ahlfors and Beurling **[BA]** gave a complete characterization of these boundary mappings and constructed an example showing that g could be completely singular with respect to the Lebesgue measure in the circle ($|g(S^1)| = 0$). By contrast, Gehring showed that g is not only absolutely continuous when $n \geq 3$, but $g\colon \partial\overline{\mathbf{B}^n} \to \partial\overline{\mathbf{B}^n}$ is in fact q.c. (when properly defined). Gehring's theorem is a key step in the original proof of Mostow's rigidity theorem **[M]**: *If $n > 2$ and if M and M' are diffeomorphic compact Riemannian n-manifolds with constant negative curvature, then M and M' are conformally equivalent.*

We have seen how the higher-dimensional theory in some ways is substantially different from the case $n = 2$. Let us now look at some *similarities.* Many properties of analytic functions in the plane extend to higher dimensions. For example, the *uniform limit* (on compact subsets) *of K-q.r. mappings is K-q.r.* **[R2]**. The q.c. case of this fact was used by Lelong-Ferrand to settle a conjecture of Lichnerowicz **[LF1]**: *The group of conformal self-mappings of a compact Riemannian n-manifold. $n \geq 2$ is compact in the topology of uniform convergence.*

Nevanlinna's defect relation is a pillar in the theory of value distribution of analytic functions. S. Rickman in **[Ri1, Ri2]** has extended this theory to higher dimensions. Let us state his counterpart of Picard's theorem in space: *For $n \geq 3$ and any $K > 1$ there exists an integer $q = q(n, K)$ such that every K-q.r. map $f\colon \mathbf{R}^n \to \mathbf{R}^n \backslash \{a_1, \dots, a_q\}$ is constant whenever $a_1, \dots, a_q$ are distinct points in $\mathbf{R}^n$.* Rickam also constructed examples to show that, at least in $\mathbf{R}^3$, $q \to \infty$ as $n \to \infty$.

The relationship with elliptic partial differential equations goes over to higher dimensions. To prove that a q.r. mapping is open **[R3]** one depends on the nonlinear potential theory associated with equations of the type

$$\operatorname{div}((\sigma(x)\nabla u|\nabla u)^{((n/2)-1)}\sigma(x)\nabla u) = 0 \tag{2}$$

where $\sigma(x)$ is a symmetric positive definite $n \times n$ matrix satisfying

$$\alpha|h|^2 \leq \langle\sigma(x)h, h\rangle \leq \beta|h|^2$$

for all $h \in \mathbf{R}^n$, where $0 < \alpha < \beta < \infty$. In a sense made precise in **[GLM1]** equation (2) is "*quasi-invariant*" under q.r. maps, much as the Laplace operator is invariant under analytic functions. This kind of nonlinear potential theory has been developed by Martio and his school [**GLM1**, **GLM2**, **GLM3**, **GLM4**]. Among other things they have used ideas of q.r. mappings theory to obtain a *necessary and sufficient* Wiener-type criterion for continuity at the boundary of solutions of (2) [**LM**].

Suppose $f: \Omega \to \mathbf{R}^n$ is K-q.c. By definition the partial derivatives of f are in $L^n_{\text{loc}}(\Omega)$. When $n = 2$ Bojarski [**Bo**] showed, via singular integral theory, that in fact $f \in W^{1,2+\varepsilon}_{\text{loc}}(\Omega)$, for every $0 < \varepsilon < \varepsilon_0(K)$. In a seminal paper [**G2**] Gehring extended this higher integrability result to $n \geq 2$ using only distortion theory and the Hardy-Littlewood maximal function. *Let Q be a cube in Ω with* $\operatorname{dist}(Q, \partial\Omega) \geq \delta \operatorname{diam}(Q)$. *Then there exists* $\varepsilon_0(K, n)$ *and* $C = C(K, n, \delta)$ *such that, for* $0 < \varepsilon < \varepsilon_0(K, n)$,

$$(3) \qquad \left(\frac{1}{|Q|}\int_Q J_f^{1+\varepsilon}\,dx\right)^{1/(1+\varepsilon)} \leq \frac{C}{|Q|}\int_Q J_f\,dx.$$

Thus $f \in W^{1,n(1+\varepsilon)}_{\text{loc}}(\Omega)$.

A well-known conjecture states that $\varepsilon(K, n) = 1/(K-1)$, but this has not been proven even for $n = 2$.

Inequality (3) is a *reverse Hölder inequality* for J_f. In the language of Muckenhoupt's theory of weights, J_f is a weight of class A_∞ (see [**CF**]). Riemann [**Re1**] observed that $\log J \in \mathrm{BMO}(\Omega)$ and proved (under some additional conditions later removed by Astala [**As**]) that quasi-isometries between $\mathrm{BMO}(\Omega)$ and $\mathrm{BMO}(\Omega')$, Ω' being another domain in $\mathbf{R}^n$, are given by composition with q.c. mappings. In some ways q.c. mappings generalize bi-Lipschitz mappings like BMO functions generalize L^∞-functions. An account of the relationship between BMO and Q. C. is in the book [**RR**].

The ideas leading to (3) have proven to be very useful in the regularity theory of nonlinear elliptic equations and systems. See [**GG** and **G**].

For many other instances of the interplay between q.c. and q.r. theory and other branches of mathematics, and a much more comprehensive review of the state of the q-art, see Gehring's article in the Proceedings of the ICM-86, Iwaniec's article in the Proceedings of ICM-83, Väisälä's article in the Proceedings of the ICM-78, the book under review and [**BM**].

There are two principal methods for studying q.r. mappings. The *analytic* method was initiated by Rešetnjak. It is based on quasilinear elliptic PDE's and the theory of Sobolev spaces. A modern account of this approach, largely independent of the work of Rešetnjak, is given by Bojarski and Iwaniec in [**BI**]. The *geometric* method is based on the quasi-invariance of moduli of curve families. It was used by Gehring [**G1**] and Väisälä [**V1**, **V3**] to develop in a systematic way the properties of q.c. mapping, by Martio, Rickman and Väisälä [**MRV1**, **MRV2**, **MRV3**] for q.r. mappings.

The approach taken in the book under review is a variation on the geometric method, in which conformal invariants play a central role. The author considers two conformal invariants $\mu_\Omega(x, y)$ and $\lambda_\Omega(x, y)$ associated to a pair of points x and y in a domain Ω, first introduced by J. Ferrand [**LF2**] and S. Gál [**Ga**]. Both invariants are given as moduli of certain curve families. It turns out that μ_Ω is a metric in Ω, called the *modulus metric* or *conformal metric.* By relating μ_Ω and λ_Ω to more explicit geometric quantities, the author has proved a number of distortion theorems for quasiregular mappings which are sharp when K approaches 1 or are dimension independent.

Previous developments of this subject are in the monograph in Russian by Rešetnjak **[R4]** and the Lecture Notes by J. Väisälä **[V3]**. This last book contains a systematic development of the method of curve families applied to q.c. mappings.

Matti Vuorinen's book fills a gap in the literature, since it is the first monograph in English covering higher-dimensional quasiregular mappings. It begins with an excellent introduction and a survey of the aspects of the theory not covered in it. This is very useful since the theory of q.r. mappings is dispersed in research articles.

The book is written carefully with all necessary details spelled out and a generous use of pictures. It should serve as a good modern account of the young and rich subject of q.r. mappings. The bibliography is complete, and there is a good collection of open problems at the end.

References

[A1] L. Ahlfors, *Lectures on quasiconformal mappings*, Wadsworth & Brooks/Cole Mathematical Series, 1987.

[A2] ——, *Finitely generated Kleinian groups*, Amer. J. Math. **86** (1964), 413–429.

[A3] ——, *Quasiconformal mappings, Teichmüller spaces, and Kleinian groups*, Proceedings of the International Congress of Mathematicians, Helsinki, 1978, pp. 71–84.

[As] K. Astala, *A remark on quasi-conformal mappings and BMO-functions*, Michigan Math. J. **30** (1983), 209–212.

[BM] A. Baernstein and J. Manfredi, *Topics in quasiconformal mappings*, Topics in modern analysis, (Leo de Michelle, ed.) Istituto Nazionale di Alta Matematica, Roma, 1983, pp. 819–862.

[B1] L. Bers, *Uniformization by Beltrami equations*, Comm. Pure Appl. Math. **14** (1961), 215–228.

[B2] ——, *An extremal problem for quasiconformal mappings and a problem of Thurston*, Acta Math. **141** (1978), 73–98.

[B3] ——, *Uniformization, moduli, and Kleinian groups*, Bull. London Math. Soc. **4** (1972), 257–300.

[B4] ——, *Quasiconformal mappings, with applications to differential equations, function theory and topology*, Bull. Amer. Math. Soc. **83** (1977), 1083–1100.

[B5] ——, *Finite dimensional Teichmüller spaces and generalizations*, Bull. Amer. Math. Soc. (N.S.) **5** (1981), 131–172.

[BA] A. Beurling and L. Ahlfors, *The boundary correspondence under quasiconformal mappings*, Acta Math. **96** (1956), 125–142.

[Bo] B. Bojarski, *Generalized solutions of a system of first order differential equations of elliptic type with discontinuous coefficients*, Mat. Sb. **43** (1957), 451–503 (Russian).

[BI1] B. Bojarski and T. Iwaniec, *Another approach to Liouville theorem*, Math. Nachr. **107** (1982), 253–262.

[BI2] ——, *Analytical foundations of the theory of quasiconformal mappings in* $\mathbf{R}^n$, Ann. Acad. Sci. Fenn. Ser A I Math. **8** (1983), 257–324.

[D] D. Drasin, *The inverse problem of the Nevanlinna theory*, Acta Math. **138** (1977), 83–151.

[EE] C. Earle and J. Eells, *A fibre bundle description of Teichmüller theory*, J. Differential Geom. **3** (1969), 19–43.

[Ga] I. Gál, *Conformally invariant metrics and uniform structures*, Indag. Math. **22** (1960), 218–244.

[G1] F. Gehring, *Rings and quasiconformal mappings in space*, Trans. Amer. Math. Soc. **101** (1961), 499–519.

[G2] ——, *The L^p-integrability of the partial derivatives of a quasiconformal mapping*, Acta Math. **130** (1973), 265–277.

[G] M. Giaquinta, *Multiple integrals in the calculus of variations and nonlinear elliptic systems*, Ann. of Math. Studies 105, Princeton Univ. Press, Princeton, N. J., 1983.

[GG] M. Giaquinta and E. Giusti, *On the regularity of the minima of variational integrals*, Acta Math. **148** (1982), 31–46.

[GT] D. Gilbarg and N. Trudinger, *Elliptic partial differential equations of second order*, 2nd ed., Springer-Verlag, Berlin and New York, 1983.

[GLM1] S. Granlund, P. Lindqvist and O. Martio, *Conformally invariant variational integrals*, Trans. Amer. Math. Soc. **277** (1983), 43–47.

[GLM2] ——, *F-harmonic measure in space*, Ann. Acad. Sci. Fenn. A I Math. **7** (1982), 233–247.

[GLM3] ——, *Note on the PWB-method in the non-linear case*, Pacific J. Math. **125** (1986), 381–395.

[GLM4] ——, *Phragmén-Lindeöf's and Lindelöf's theorems*, Ark. Math. **23** (1985), 103–128.

[K] I. Kra, *On the Nielsen-Thurston-Bers type of some self-maps of Riemann surfaces*, Acta Math. **146** (1981), 231–270.

[LF1] J. Lelong-Ferrand, *Transformations conformes et quasi-conformes des variétés riemanniennes compactes* (Démonstration de la conjecture de A. Lichnerowicz), Acad. Roy. Belg. Cl. Sci. Mém. Coll. **39** (1971), 1–44.

[LF2] ——, *Invariants conformes globaux sur les variétés riemanniennes*, J. Differential Geom. **8** (1973), 487–510.

[L1] O. Lehto, *Univalent functions and Teichmüller spaces*, Graduate Texts in Math., vol. 109, Springer-Verlag, Berlin and New York, 1987.

[L2] ——, *A historical survey of quasiconformal mappings*, Lectures from the Euler Colloquium held at the Technische Universität, Berlin, May 12–14, 1983; (E. Knoblich, I. S. Louhivaara and J. Winkler, eds.) Birkhäuser, Basel-Boston, 1984, pp. 205–217.

[LM] P. Lindqvist and O. Martio, *Two theorems of N. Wiener for solutions of quasilinear elliptic equations*, Acta Math. **155** (1985), 153–171.

[LV] O. Lehto and K. Virtanen, *Quasi-conformal mappings in the plane*, 2nd ed., Springer-Verlag, Berlin and New York, 1973.

[MaR] A. Marden and S. Rickman, *Holomorphic mappings of a bounded distortion*, Proc. Amer. Math. Soc. **46** (1974), 226–228.

[MR] ——, *Measure properties of the branch set and its image of quasiregular mappings*, Ann. Acad. Sci. Fenn. Ser. A I **541** (1973), 1–16.

[MRV1] O. Martio, S. Rickman and J. Väisälä, *Definitions for quasi-regular mappings*, Ann. Acad. Sci. Fenn. Ser A I **465** (1970), 1–13.

[MRV2] ——, *Distorsion and singularities of quasiregular mappings*, Ann. Acad. Sci. Fenn. Ser A I **465** (1970), 1–13.

[MRV3] ——, *Topological and metric properties of quasiregular mappings*, Ann. Acad. Sci. Fenn Ser A I **488** (1971), 1–31.

[M] G. Mostow, *Quasi-conformal mappings in n-space and the rigidity of hyperbolic space forms*, Publ. Math. Inst. Hautes Études Sci. **34** (1968), 53–104.

[R1] J. Rešetnjak, *The Liouville theorem with minimal regularity conditions*, Sibirsk. Mat. Z. **8** (1967), 835–840.

[R2] ——, *Space mappings with bounded distortion*, Sibirsk. Mat. Z. **8** (1967), 629–658. (Russian)

[R3] ——, *Mappings with bounded distortion as extremals of Dirichlet type integrals*, Sibirsk. Mat. Z. **9** (1968), 652–666. (Russian)

[R4] ——, *Spatial mappings with bounded distortion*, Izdat. "Nauka", Sibirsk. Otdeleine, Novosibirsk, 1982. (Russian)

[Ri1] S. Rickman, *A defect relation for quasimeromorphic mappings*, Ann. of Math. (2) **114** (1981), 165–191.

[Ri2] ——, *The analogue of Picard's theorem for quasiregular mappings in dimension three*, Acta Math. **154** (1985), 195–242.

[RR] H. Reiman and T. Rychener, *Funktionen mit beschränkter mittlerer Oszillation*, Lecture Notes in Math., vol. 487, Springer-Verlag, Berlin and New York, 1975.

[S] D. Sullivan, *Quasiconformal homeomorphisms and dynamics*. I, *Solution of the Fatou-Julia problem on wandering domains*, Ann. of Math. (2) **122** (1985), 401–418.

[V1] J. Väisälä, *On quasiconformal mappings in space*, Ann. Acad. Sci. Fenn. **298** (1961), 1–36.

[V2] ——, *Quasiregular mappings in n-space*, XVI, Scandinavian Congress of Mathematicians, Göteburg, Sweden, 1972.

[V3] ——, *Lectures on n-dimensional quasiconformal mappings*, Lecture Notes in Math., vol. 229, Springer-Verlag, Berlin and New York, 1971.

[Z] V. A. Zŏric, *The theorem of M. A. Laurentiev on quasiconformal mappings in space*, Mat. Sb. **74** (1967), 417–433. (Russian)

JUAN J. MANFREDI
NORTHWESTERN UNIVERSITY

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 21, Number 2, October 1989

0273-0979/89 $1.00 + $.25 per page

Convex bodies and algebraic geometry: An introduction to toric varieties, by Tadao Oda. Ergebnisse der Mathematik und ihrer Grenzgebiete 3, Folge, Band 15, Springer-Verlag, Berlin, Heidelberg, New York, 1988, $78.00. ISBN 3-540-17600-4

The reader has heard the cliché that algebraic varieties are the locuses of solutions of polynomial equations; true as far it goes, but even in simple cases, one can know a lot about the equations and almost nothing about the solution set. Conversely, and more to the present point, even for a variety having an extremely natural and simple description, writing out the defining equations might be enormously expensive and unrewarding.

First of all, I want to give the flavour of toric geometry with two simple examples illustrating the main point, before discussing the background and the content of Professor Oda's very substantial book. Consider the quotient $\mathbf{C}^n/G$ of $\mathbf{C}^n$ by a diagonalised group action

$$(x_1, \dots, x_n) \mapsto (\varepsilon_1(g)x_1, \dots, \varepsilon_n(g)x_n),$$

where G is a finite Abelian group and $\varepsilon_i \colon G \to \mathbf{C}^*$ characters of G. This quotient can be seen as an explicit affine variety: make a list of G- invariant monomials, that is,

$$\left\{ x^{\mathbf{m}} = \prod x_i^{m_i} \,\middle|\, m_i \geq 0 \text{ and } \prod \varepsilon_i(g)^{m_i} = 1 \forall g \in G \right\},$$

then write out all the multiplicative relations between the generators, and finally, take these as the defining equations of a variety. Try it with

$$n = 2, \qquad G = \mathbf{Z}/22 \quad \text{and} \quad (x, y) \mapsto (\zeta x, \zeta^9 y)$$