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Orthogonality and spacetime geometry, by Robert Goldblatt. Universitext, Springer-Verlag, New York, Berlin, Heidelberg, 1987, viii + 189 pp., $26.00. ISBN 0-387-96519-x

This book is devoted to the axiomatic construction of Special Relativity. It is known that the geometry of spacetime of Special Relativity is the geometry of Minkowskian space, i.e. the four-dimensional pseudo-Euclidean space of signature $(+ - --)$. The author investigates the problem of the determination of Minkowskian space in the class of all affine spaces that carry binary relation of orthogonality of lines. In other words, beginning with an abstract set of points and lines and by considering properties of the orthogonality relation (presence of self-orthogonal or singular lines, etc.) the author obtains categorical axiomatic descriptions for all discussed geometries and the geometry of Minkowskian space in particular.

The book begins with the chapter, *A trip on Einstein's train.* By making clear the basic statements of Relativity theory the author attracts the reader's attention to the pairs of worldlines which are called orthogonal: one of them is a worldline of an observer moving at constant speed and the other is a line of locations that are simultaneous for him, the choice of the term "orthogonal" being explained at once thanks to the introduction of the Minkowskian inner product. Thus the orthogonality relation turns into a notion that deserves serious attention. And what is more, the author clarifies that there exist geometries with different properties of orthogonal lines. Apart from the Minkowskian plane these are the Robb plane, the optical plane and their many-dimensional analogs.

The question arises: how many different geometries based on the orthogonality notion exist, and how is the Minkowskian space to be selected among them. The answer is contained in the following chapters. The presentation of material of the first chapter is undoubtedly a success. It certainly attracts attention of the reader, and stirs his imagination; he is looking forward to the following chapters.

Chapter 2, *Planes* begins with the notion of the affine, Desarguesian and Pappian planes. The definition of affine plane α_F over the field F is given, and the solution of the Coordinatisation Problem is stated. In §§2.4–2.6 the reader learns about existence of many different metric geometries, such as null, singular and Artian planes, the degenerate metric Fano plane and the Artian Fano plane, and so on. The author proves that every nonsingular Pappian metric plane, except for the degenerate Fano case, can be characterised by the associated inner product $-k \cdot x_1 \cdot x_2 + y_1 \cdot y_2$. These planes are denoted as $\alpha_F(-k)$. Among the planes of the form $\alpha_F(-k)$ the Artian planes are selected. The end of Chapter 2 contains the theorem which states that the three metric planes over real numbers (Euclidean plane, the planes of Robb and Minkowski) can be specified among the Desarguesian metric planes that carry the ternary relation "between" B. The relation B satisfies five axioms, one of which is Pasch's Law, and the other is a version of Dedekind continuity.

Chapter 3, *Projective transformations* "develops an alternative method of coordinatising a metric affine plane by embedding it into a projective plane, and using the orthogonality relation to define a matrix-representable transformation on the line at infinity. The construction will be central to the subsequent treatment of metric affine spaces of higher dimension."

The investigations conducted by the author in Chapters 3 and 4 show that there exist only two nonsingular metric affine threefolds (Euclidean and Minkowskian spaces) and only three nonsingular metric affine fourfolds (Euclidean, Artian and Minkowskian spaces), if these spaces carry the ternary relation "between." Such spaces are called continuously ordered.

The word "order" occurs in Appendix B "After and the Alexandrov-Zeeman Theorem" for the second time, where the author shows that the notions "between" and "orthogonal" can be defined in terms of the notion of "after", i.e. an ordering which is given on affine space. Hence the method of axiomatization of spacetime based on the orthogonality relation must be considered as part of the program of the construction of causal theory of spacetime which was proposed by A. D. Alexandrov [**1**, **2**].

This book is read with pleasure, and will be useful for the students who are wishing to become geometers.

REFERENCES

1. A. D. Alexandrov, *Mappings of space with families of cones and space-time transformations*, Ann. Mat. Pura Appl. **53** (1975), 229–257.

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Partially ordered abelian groups with interpolation, by K. R. Goodearl. Mathematical Surveys and Monographs, number 20, American Mathematical Society, Providence, R.I., 1986, xxii + 336 pp., ISBN 0-8218-1520-2

In [**19**], F. Riesz introduced what has come to be called the Riesz decomposition property. An ordered (abelian) group is said to have the Riesz decomposition property if the sum of two order intervals is again an order interval. Riesz showed, among other things, that the cone of positive additive functionals on an ordered group with this property is a lattice. (In the case that the group has an order unit, this says that the compact