An important feature of ordered fields is the order-valuation, defined in terms of its valuation ring by the least convex subring, and the author devotes a separate chapter to its development, including a sketch of pseudo-completeness and a proof that $\xi\mathbf{No}$ may be regarded as a field of formal power series. These results are then put to use in studying 'hyperconvergent' power series; by this the author means the following: Let f be a power series in variables $X_1, \ldots, X_n$ over a surreal field $\xi\mathbf{No}$. Then there exists a convex prime ideal $\mathfrak{p} \neq 0$ in the valuation ring of $\xi\mathbf{No}$ such that f can be evaluated at each point of $\mathfrak{p}^n$. Hyperconvergence is just another way of looking at the theorem of H. Hahn and its extension by B. H. Neumann and A. I. Mal′cev on formal power series over an ordered group, and the author brings Neumann's proof, though strangely no reference to Mal′cev (except a negative one on p. 268). With this notion in hand it is possible to develop analysis and the author gives a few samples such as the implicit function theorem and a form of analytic continuation, but there are no real applications (as yet).

It is good to have an account drawing together several threads in the development of surreal field theory; many of the ideas are reminiscent of nonstandard analysis, which surprisingly is not mentioned. The author has taken some pains to make the volume self-contained, although the style is that of lecture notes rather than a text-book, and at a price of nearly 18 cents per page one would hope to get a typeset book rather than camera-copy using a fount which lacks $\cup$ and $\cap$. But persevering readers will find the prospect of an intriguing development of analysis, where much remains to be done.

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Theory of topological structures. An approach to categorical topology, by Gerhard Preuss. Translated by Andreas Behling, Mathematics and its Applications, D. Reidel Publishing Company, Dordrecht, Boston, Lancaster, Tokyo, 1988, xii + 304 pp., $69.00. ISBN 90-277-2627-2

Soon after A. Weil **[13]** introduced uniform structures in terms of entourages of the diagonal, J. W. Tukey **[12]** gave an equivalent description using uniform covers. The notion of uniform cover is particularly useful in topological dimension theory and proved to provide the more productive axiomatization of uniform spaces (cf. J. R. Isbell **[7]**). Thinking of spaces as sets X equipped with a system μ of covers of X stable under intersection (where the intersection of two covers is given by forming intersections of their members) and under making a cover coarser, various people were led to formulate more general notions: A. Frolík **[3]** introduced "P-spaces", J. R. Isbell **[7]** "quasi-uniform spaces", and M. Katětov **[11]** "merotopic spaces". For all such spaces (X, μ) (which are called "seminearness spaces" in the book under review), one may define the "interior" $\mathrm{int}_\mu A$ of a subset A of X as the set of points x for which $\{A, X\backslash\{x\}\}$ is a cover in μ. In this way one indeed obtains a topology on X, provided μ satisfies the axiom

$$(*) \qquad \mathscr{U} \in \mu \Rightarrow \{\mathrm{int}_\mu A \colon A \in \mathscr{U}\} \in \mu,$$

which is needed to ensure idempotency of the operator int_μ. The topology satisfies even the (very weak) R_0-separation axiom,

$$x \in \overline{\{y\}} \Rightarrow y \in \overline{\{x\}}.$$

Seminearness spaces with $(*)$ were studied first by H. Herrlich **[4]**, who called them *nearness spaces.* All R_0-topological spaces and all uniform spaces can be faithfully interpreted as nearness spaces. More precisely, in categorical terms, this means that Near contains R_0-Top as a full bicoreflective and Unif as a full bi-reflective subcategory.

As indicated in the author's Introduction, Preuss' book aims at presenting a categorical unification of topological structures, such as topologies, uniformities, and convergence and proximity structures in terms of nearness spaces. Several important reasons are stated for preferring nearness spaces to topological spaces: normality and paracompactness behave better with respect to the formation of subspaces and direct products in Near than in Top; completeness and compactness are subsumed under one umbrella in Near since there is a completion construction for nearness spaces which, as a particular instance, gives important compactifications or extensions for topological spaces (Stone-Čech, Alexandroff, Hewitt, Wallman), Čech cohomology groups, as well as Isbell's uniform dimension and the large dimension, may be defined for nearness spaces, and both dimensions allow for cohomological characterizations. In addition, the author also mentions the need for natural function space structures as a reason for studying nearness spaces. This is indeed far less convincing than the reasons given before, since Near, like Top, or like the celebrated category Loc of locales (cf. Johnstone **[9]**) (in which we study "spaces without points" by lattice-theoretic methods), is not cartesian closed: the fact that the larger category of seminearness spaces contains good cartesian closed subcategories can hardly attract topologists to nearness structures when already Top contains such a subcategory. Still, there remain a sufficient number of very good reasons to justify writing a modern monograph on nearness spaces which might open the subject to a wider community of mathematicians. (There exists a nice introductory text-book on nearness spaces by H. Herrlich **[6]** which can be recommended to anybody who reads German; however, it does not deal with normality, paracompactness, or cohomology.)

However, Preuss tries to do much more in his book than present the theory of nearness spaces, but combines the presentation of topological structures with an introduction to Category Theory. Chapter 0 ("Preliminaries") gives the definition of a category with the notion of (extremal) monomorphism and direct product, and a short section on Weil's uniform spaces. Chapter I deals with Set-based "Topological categories", the most important property of which is the existence of initial (= weak) structures. It is here that, as an example, the category Near is introduced, with the formal definition of a nearness space added in parentheses. Completeness as well as factorization properties are established for such topological categories. The chapter also contains a much more specialized and quite isolated section on so-called relative (dis-) connectednesses, a subject considered first (for Top) by Arhangelskii and Wiegandt [2]. More could have been said here if the topic of reflective and coreflective subcategories, which is discussed in Chapter II, had been already introduced. Chapter II also introduces the notion of adjointness and establishes the existence of epi-reflective and mono-coreflective hulls, first for topological and then for abstract categories. Chapter III finally informs the reader in which sense nearness spaces generalize R_0-topological spaces and uniform spaces. It also defines various generalizations of nearness structures (prenearness, seminearness) as well as subconcepts of such generalizations (grill-determined prenearness, subtopological nearness). More such concepts follow in Chapter IV which deals with "Cartesian closed topological categories".

Having completed reading the first half of the book, any conceivable reader will be longing either for some guidance through the considerable number of notions introduced, or for some deeper facts concerning these notions, be they topological or categorical. But more patience is needed: Chapter V on "Topological functors" indeed recapitulates almost all categorical themes treated before, in slightly greater generality or with a slightly different point of view, introducing quite a few new names and re-proving various of the earlier facts in the new context, so that many readers will wonder why they did not study this chapter first. Finally, Chapters VI ("Completions") and VII ("Cohomology and dimension of nearness spaces") provide reward for patience. Although Chapter VI deals with two totally different subjects, so called initial completions of concrete categories and completions of nearness spaces (the only common aspect of which seems to be that the reals can be obtained from the rationals both ways), here one learns about two constructions of prominent importance to the study of faithful functors (cf. H. Herrlich [5] and J. Adámek [1]) and of nearness spaces, respectively. Most of the facts referred to in the Introduction as a motivation for studying nearness spaces are in fact given in a very comprehensive manner in the second part of Chapter VI and in Chapter VII, which account for 40 pages of the 300-page book. There is an Appendix on "Representable functors" and a collection of exercises at the end of the book.

As much as I believe that nearness structures provide a modern and very useful tool in "uniform topology", and that topologists will benefit a great deal from acquiring a categorical background, I also fear that Preuss' book, despite the great potential of the subject, will promote neither nearness spaces nor categories to the degree its author wishes. To make the case for nearness spaces as the "right category" for topologists, more areas within topology, including homotopy theory, need to be presented. Even without discussing what "Categorical Topology" ought to be (for different viewpoints or aspects cf. P. T. Johnstone **[10]** and J. R. Isbell **[8]**), I believe that topologists will remain in doubt whether it is worth the effort to study all categorical notions provided in this book, and that categorists will say that indeed too small a portion of their subject has been presented, even if repetitiously. As a researcher I welcome Preuss' book as a reference manual on the great many notions on generalized topological structures used in the literature, but I am disappointed that the opportunity to provide genuine guidance through a new field, in which the important material still needs to be selected from the many concepts offered, has not been used to its fullest.

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Multidimensional Brownian excursions and potential theory, by K. Burdzy. Longman Scientific & Technical, Essex, New York, (Pitman Research Notes in Mathematics Series, vol. 164), 1987, 172 pp., $44.95. ISBN 0-582-00573-6

Burdzy's monograph deals with a recent addition to the probabilistic arsenal, Brownian excursion laws, and their application to boundary problems in classical potential theory and complex analysis. Excursion laws first arose in the work of K. Itô, and later in that of B. Maisonneuve.