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Foundations of analysis over surreal number fields, by N. L. Alling. North-Holland Mathematics Studies, vol. 141, North-Holland, Amsterdam, 1987, xvi + 374 pp., $66.75. ISBN 0-444-70226-1

The field **R** of real numbers may be characterized abstractly as a Dedekind-complete ordered field. Moreover, **R** is real-closed and by Tarski's theorem it shares its first-order properties with all other real-closed fields, so to distinguish it from such fields one has to invoke higher-order properties (like Dedekind-completeness). The author has set himself the task of studying these fields. The usual methods involve assumptions equivalent to a form of the generalized continuum hypothesis; here he follows the route taken by Conway in his account "On numbers and games" (London Math. Soc. Monographs No. 6, Academic Press, 1976) to construct such fields and to describe the beginnings of analysis in the new setting.

Conway's method of building up number systems may be regarded as Dedekind cuts taken to extremes. His basic principle states that "If L, R are two sets of numbers and no member of L is $\geq$ any member of R, then $\{L|R\}$ is a number. All numbers are constructed in this way." Starting from $0 = \{|\}$ one obtains in a countable number of steps all dyadic numbers $m2^{-n}$; by continuing transfinitely one reaches all real numbers and indeed can go beyond **R** to construct a Field **No**, i.e. a proper class whose elements admit the operations and satisfy the laws of fields. The members of **No** are the *surreal numbers*. The pairs $\{L|R\}$ used in the construction process are called *Conway cuts* in the ordered set X that is being considered. The special case where L, R form a partition of X into nonempty sets is the familiar Dedekind cut. An intermediate notion, where L or R may be empty (but their union is still X) was introduced in 1954 by N. Cuesta Dutari. Such Cuesta Dutari cuts are used here to construct η_ξ-fields, i.e. real-closed fields that are η_ξ-sets. For $\xi = 0$ such fields are of course well known, e.g. **R**, or the subfield of all real algebraic numbers, but η_ξ-fields for $\xi > 0$ are harder to construct. They are fields displaying a high degree of density and any ordered field whose cardinal is bounded by ω_ξ can be embedded in an η_ξ-field; thus every such field can be embedded in the 'universal' η_ξ-field ξ**No**.

The author's aim may be described as a study of analysis over ξ**No**. For a satisfactory development he has to work with a ξ-topology. This is a modification of the usual definition in that the family of all open sets is closed under finite intersections and unions of fewer than ω_ξ sets (ω_ξ being regular). Notions such as ξ-continuous, ξ-compact, ξ-connected etc. are introduced and it is reassuring to find that addition and multiplication are ξ-continuous and the ξ-connected subsets are just the intervals. In this terminology an η_ξ-set is just an ordered set which is ξ-complete. In fact much of this development is carried out generally for η_ξ-sets and later specialized to apply to surreal fields.

An important feature of ordered fields is the order-valuation, defined in terms of its valuation ring by the least convex subring, and the author devotes a separate chapter to its development, including a sketch of pseudo-completeness and a proof that $\xi\mathbf{No}$ may be regarded as a field of formal power series. These results are then put to use in studying 'hyperconvergent' power series; by this the author means the following: Let f be a power series in variables $X_1, \ldots, X_n$ over a surreal field $\xi\mathbf{No}$. Then there exists a convex prime ideal $\mathfrak{p} \neq 0$ in the valuation ring of $\xi\mathbf{No}$ such that f can be evaluated at each point of $\mathfrak{p}^n$. Hyperconvergence is just another way of looking at the theorem of H. Hahn and its extension by B. H. Neumann and A. I. Mal′cev on formal power series over an ordered group, and the author brings Neumann's proof, though strangely no reference to Mal′cev (except a negative one on p. 268). With this notion in hand it is possible to develop analysis and the author gives a few samples such as the implicit function theorem and a form of analytic continuation, but there are no real applications (as yet).

It is good to have an account drawing together several threads in the development of surreal field theory; many of the ideas are reminiscent of nonstandard analysis, which surprisingly is not mentioned. The author has taken some pains to make the volume self-contained, although the style is that of lecture notes rather than a text-book, and at a price of nearly 18 cents per page one would hope to get a typeset book rather than camera-copy using a fount which lacks $\cup$ and $\cap$. But persevering readers will find the prospect of an intriguing development of analysis, where much remains to be done.

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Theory of topological structures. An approach to categorical topology, by Gerhard Preuss. Translated by Andreas Behling, Mathematics and its Applications, D. Reidel Publishing Company, Dordrecht, Boston, Lancaster, Tokyo, 1988, xii + 304 pp., $69.00. ISBN 90-277-2627-2

Soon after A. Weil **[13]** introduced uniform structures in terms of entourages of the diagonal, J. W. Tukey **[12]** gave an equivalent description using uniform covers. The notion of uniform cover is particularly useful in topological dimension theory and proved to provide the more productive axiomatization of uniform spaces (cf. J. R. Isbell **[7]**). Thinking of spaces as sets X equipped with a system μ of covers of X stable under intersection (where the intersection of two covers is given by forming intersections of their members) and under making a cover coarser, various people were led to formulate more general notions: A. Frolík **[3]** introduced "P-spaces", J. R. Isbell **[7]** "quasi-uniform spaces", and M. Katětov **[11]** "merotopic