Chapter 4 is devoted to the study of estimations of the parameters of stable distributions. In this chapter a new approach to the problem of estimating the parameters of stable distributions is presented based on the use of explicit expressions for the corresponding characteristic transforms and the method of sample moments well known in statistics.

Chapter 1 gives examples of the occurrence of stable laws in applied fields. §1.1 studies a model of point source of influence. Examples include the gravitational field of stars; temperature distribution in a nuclear reactor; distribution of stresses in crystalline lattices; distribution of magnetic field generated by a network of elementary magnets. §§1.2 and 1.3 deal with the stable laws occuring in radio engineering and electronics and economics and biology respectively.

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Nonlinear functional analysis, by Klaus Deimling. Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1985, xiv + 450 pp., $39.00. ISBN 3-540-13928-1

The author's purpose is to "present a survey of the main elementary ideas, concepts and methods which constitute nonlinear functional analysis." He has ably accomplished his goal. Moreover, by interlacing extensive commentary and foreshadowing subsequent developments within the formal scheme of statements and proofs, by the inclusion of many apt examples and by appending interesting and challenging exercises, Deimling has written a book which is eminently suitable as a text for a graduate course.

The central theme of nonlinear functional analysis is the study of systems of nonlinear equations—algebraic, functional, differential, integral—which are formulated as the solutions of an operator equation

$$F(x) = 0, \qquad x \in D, \tag{1}$$

where X and Y are linear spaces, $D \subset X$ and $F: D \to Y$ is some operator, which is not necessarily linear. There are, of course, many ways to cast the solutions of a system of equations as the zeros of a nonlinear operator, and the choice of set-up is often crucial. After the existence and multiplicity of solutions of (1) have been determined, more particular questions need to be considered. For instance, if a solution of equation (1) corresponds to a solution of a differential equation, one needs to study stability, regularity, dependence on initial data, etc.

In studying equation (1) there are analytical, topological and variational methods which can be invoked, as examples of which we mention iterative methods, degree theories and mini-max theorems, respectively. When $X = Y = \mathbf{R}^n$, the study of equation (1) for continuous F is already formidable. When the spaces are infinite-dimensional there are additional basic difficulties stemming from the noncompactness of closed, bounded sets: When X is infinite dimensional, the Brouwer fixed-point theorem does not extend to the unit ball, $GL(X)$ does not necessarily have two components, and a continuous functional on a closed, bounded set does not necessarily assume its infimum. In developing the theory when X is infinite-dimensional, one singles out, depending on the particular methodology, subclasses of nonlinear mappings which are narrow enough to be able to prove interesting results yet broad enough to include important examples. There are significant equations for which this general functional-analytic approach has not yet been useful.

The initial chapter of Deimling's book is devoted to a development of the Brouwer degree and an examination of some of its applications. The passage from the smooth, regular case to the continuous case is steered via the representation of the degree as an integral. The author uses the Amann-Weiss Axioms **[A-W]** as a natural device for mapping this journey.

In Chapter 2, Deimling turns to degree theory for mappings acting between infinite-dimensional spaces. The Leray-Schauder degree for compact perturbations of the identity is thoroughly discussed, as well as other degrees which have been defined over the last twenty years for more general perturbations of the identity, including perturbations consisting of the sum of a compact mapping and a Lipschitz mapping whose Lipschitz constant is less than one. Along the way the author provides precise details concerning the pertinent results from the linear theory. For instance, there is a clear, self-contained treatment of the requisite spectral theory.

Monotone and accretive mappings are the subject of the third chapter. There is an extensive discussion of the structure theory of these classes, of existence theorems, of applications to the study of solvability for differential equations and of the role of these mappings as generators of semigroups.

Perhaps the first part of the next chapter might well have been a very suitable opening for this book. In it one finds the Inverse Function Theorem, the Implicit Function Theorem and Newton's Method. Then there is a version of the Nash-Moser Theorem, accompanied by an application to a small divisor problem. In the last part of the chapter, existence theorems from the previous chapters are used in conjunction with the Implicit Function Theorem to form a Lyapunov-Schmidt method for attacking resonance problems.

Deimling's Chapter 5 is titled "Fixed Point Theory," and he begins it with some editorial commentary. He has selected a few notable offspring which have grown from the seeds of the Banach and Schauder Fixed Point Theorems, among which are results on nonexpansive mappings and inwardness conditions. The discussion of the fixed-point principle which he titles the Browder-Caristi Theorem would be greatly improved by inclusion of the equivalent principle of Ekeland and some of the remarkable examples of this simple, powerful idea (cf. **[E]**).

The maximum principle for second order elliptic partial differential equations gives rise to various order preserving properties for the corresponding functional operator. Such properties may usefully be framed within the setting of operators acting in cones, in which context one has the classic results of Krein and Rutman for linear operators and of Birkhoff and Kellogg for nonlinear operators. Chapter 6 is titled "Solutions in Cones." Included here are structure theorems for cones, fixed point theorems and extensions of degree, together with applications to differential equations. Curiously enough, the Birkhoff-Kellogg Theorem **[B-K]** is not to be found.

Chapter 7 covers some aspects of the Galerkin method for studying nonlinear equations. Given an operator equation formulated in the context of Banach spaces having Schauder bases, the Galerkin approximations are a sequence of finite-dimensional equations the solutions of which will, under favorable circumstances, approximate a solution of the original equation. Within this context, W. V. Petryshyn introduced the class of A-proper mappings **[P]**, a class which has been useful in ordering a number of existing ideas in the study of both nonlinear and linear equations and also in providing a context within which many classical results have been extended. The author has selected topics within this circle of ideas.

The penultimate chapter is titled "Extremal Problems." Convexity, Lagrange multipliers and mini-max theorems are covered. While Morse theory is not discussed, the author does have a clear discussion of the topological ideas involved in the Lyusternik-Shnirel'man category theory, where one obtains weaker, although still interesting, information about critical points under much weaker assumptions than needed for Morse Theory. It would have been natural to include the Mountain Pass Lemma here (cf. **[E]**).

The final tenth chapter comprises an introduction into bifurcation theory. The introduction is brief, but illuminating, in that when treating bifurcation problems one sees both the interdependence and the particular applicability of the broad range of tools that have been constructed in the first nine chapters. The local analytical tools are used in proving the simple-eigenvalue theorem of Crandall and Rabinowitz and its consequences regarding exchange of stability. Degree theory is the crucial tool in deriving the Rabinowitz Global Bifurcation Theorem. Lyusternik-Shnirel'man category is used to prove local bifurcation for variational problems, in the absence of parity assumptions on the dimension of the null space of the linearization. The chapter and book conclude with a proof of the local Hopf Bifurcation Theorem.

Deimling has chosen a rather personal, informal writing style, one which took me some time to get comfortable with. He also expresses opinions, through selection of material or reference and by choice of adjective, with which I sometimes disagreed. In spite of these minor reservations, this book will be the text I will choose for the next course I teach on the subject.

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Asymptotic methods in statistical decision theory, by Lucien Le Cam. Springer Series in Statistics, Springer-Verlag, New York, Berlin, Heidelberg, 1986, xxvi + 742 pp., $49.95. ISBN 0-387-96307-3

It is rather odd that in statistics no "general" theory has yet been formulated that would investigate from a single point of view the properties of estimates, tests and other inference procedures and would be accepted by the majority of specialists. A fundamental concept such as optimality is usually defined *ad hoc* in each special case, depending on the nature of the problem. Therefore, the number of concepts of optimality in statistics grows along with the number of problems treated and models studied. The work of A. Wald, D. Blackwell and others in abstract statistical decision theory has shown how successfully the ideas and methods of the theory of optimization and convex analysis work in statistics. This has led to some systematization of the various concepts of optimality. The development of the asymptotic theory of statistical inference has shown that under mild regularity assumptions various concepts of optimality often lead to the same or almost the same solutions. These ideas have been especially well studied in the more traditional schemes of statistical inference, in which for n independent identically distributed (i.i.d.) observations $X^n = (X_1, \dots, X_n)$ representing a sample of size n from the distribution F_θ, one wishes to estimate the value of the parameter θ or derive a test for verifying some hypothesis about this parameter. The similarity, in such a situation, of these different approaches to the concept of asymptotic optimality is explained by the fact that they all depend on using an appropriate form of the scaled likelihood ratio

$$\frac{dP^n_{\theta+h/\sqrt{n}}}{dP^n_\theta}(X^n) \equiv Z^n_h \equiv Z^n_h(\theta; X^n),$$

where P^n_θ is the distribution of X^n derived from F_θ. Let

$$Z_h = \exp(L_\theta(h) - \tfrac{1}{2}(J_\theta h, h))$$