18. H. Tanaka, *Propagation of chaos for certain purely discontinuous processes with interaction*, J. Fac. Sci. Univ. Tokyo Sect. IA Math. **17** (1970), 259–272.

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Transformation groups, by Tammo tom Dieck. Studies in Mathematics, vol. 8, Walter de Gruyter, Berlin, New York, 1987, x + 311 pp., $71.00. ISBN 0-89925-029-7

In 1981 **[9]**, I reviewed a book with a similar title **[5]** by the same author. There has been considerable progress in this general area since then. Both books focus on topics in equivariant topology, the study of spaces with group actions, primarily actions by compact Lie groups.

In geometric topology, the study of high dimensional manifolds has more and more come to focus on problems concerning smooth, *PL*, and topological group actions. In algebraic topology, classical homotopy theory has moved more and more in the direction of equivariant theory, although there is still a little gap between those who approach problems from an equivariant point of view and those who approach problems from a more classical point of view.

Some of the most important work in algebraic topology since 1981 has concerned the Segal conjecture, the Sullivan conjecture, and various generalizations and applications of those results. Much of this work is intrinsically equivariant in nature. Perhaps a little discussion of these results will illuminate the difference in points of view one can take on these matters.

The Sullivan conjecture, in its generalized form, starts with a finite p-group G, a contractible space EG with a free action by G, and a G-space X. One defines the "homotopy fixed point space of X," denoted X^{hG}, to be the space of G-maps $f\colon EG \to X$. To say that f is a G-map just means that $f(gy) = gf(y)$ for $g \in G$ and $y \in EG$. If x is a fixed point of X, so that $gx = x$ for all $g \in G$, then we have the constant G-map f_x specified by $f_x(y) = x$ for all $y \in EG$. There results an inclusion $\imath\colon X^G \to X^{hG}$. A special case of the "homotopy limit problem" **[12]** asks how near this map is to being a homotopy equivalence. Roughly speaking, the generalized Sullivan conjecture asserts that this map becomes an equivalence after p-adic completion when X is finite dimensional. The conjecture has been proven independently by Haynes Miller, Jean Lannes, and Gunnar Carlsson **[4, 7, 10]**, and numerous authors have obtained interesting applications. While the statement may seem technical and unintuitive, the fact is that the result opens the way to a variety of concrete calculations in homotopy theory of a sort unimaginable just a few years ago.

There is a slightly different, more equivariant, way of thinking about the generalized Sullivan conjecture. One can consider the space $\mathrm{Map}(EG, X)$ of all maps $f: EG \to X$. This space admits the conjugation G-action specified by $(gf)(y) = gf(g^{-1}y)$. One has a constant map $f_x: EG \to X$ for any $x \in X$, not necessarily a fixed point. There results an inclusion of G-spaces $\varepsilon: X \to \mathrm{Map}(EG, X)$, and one can ask how near this map is to being a G-homotopy equivalence. Since $\iota = \varepsilon^G$, the original version of the question is obtained by passing to fixed point spaces.

In this case, the answer to the original "nonequivariant" version of the problem implies the answer to the "equivariant" version of the problem. Again, roughly speaking, the G-map ε becomes an equivalence after p-adic completion [**6**]. The connection between the two versions is given by the equivariant Whitehead theorem, which asserts, essentially, that a G-map $\alpha: X \to Y$ is a G-homotopy equivalence if its fixed point map α^H is an ordinary homotopy equivalence for all subgroups H of G. Technically, the methods of attack developed by Miller and Lannes are basically nonequivariant in nature while the methods developed by Carlsson are basically equivariant in nature.

The Segal conjecture can be viewed as the "stable" version of the equivariant form of the Sullivan conjecture. The relevant idea of stability is described intuitively in [**9**], and a full dress treatment of equivariant stable homotopy theory has since appeared [**8**]. There is a notion of a stable G-space, technically called a "G-spectrum". For a G-spectrum X, there is a function G-spectrum $\mathrm{Map}(EG, X)$ and a natural G-map $\varepsilon: X \to \mathrm{Map}(EG, X)$. For finite p-groups, the Segal conjecture asserts that ε becomes an equivalence upon p-adic completion. It was proven by Carlsson [**3**], with lesser contributions by several other mathematicians. There is an equivalent nonequivariant version of the result, but in this case genuinely equivariant methods seem to be essential to the proof.

Moreover, in contrast to the Sullivan conjecture, the Segal conjecture works perfectly well for general finite groups. One must replace completion at p by completion at the augmentation ideal of the Burnside ring $A(G)$; see e.g. [**1**], which gives a much more general result. Here $A(G)$ is the Grothendieck ring obtained from the semiring of isomorphism classes of finite G-sets. The problem of obtaining anything like the Sullivan conjecture for general finite groups seems to be quite intractable, although a little is known [**6**].

The above is all something of a digression. A more geometric topologist asked to review this book might have talked about equivariant surgery theory, equivariant simple homotopy theory, the role of algebraic K-theory, Weinberger's theory of homology propagation [**13**], or any of a number of other areas of recent activity. Some excellent survey articles may be found in [**11**].

It is tom Dieck's point of view that there really is a subject of "transformation groups", however broad and diffuse it may have become, and that any good algebraic topologist needs to know something about it. With that in mind, he has made a valiant effort to compile an introduction that is broad enough to be useful both to the algebraic and the geometric topologist and yet focused enough to allow a coherent presentation. The book concentrates on those topics that are essential preliminaries to both algebraic and geometric work, with more emphasis being given to the algebraic side of things. One justification for the emphasis is that topics which are covered in Bredon's excellent and more geometrically oriented introduction [2] are deliberately gone over lightly or omitted.

The first chapter is simply called "foundations" and gives capsule introductions to a wide variety of the standard notions in the business. The second chapter is called "algebraic topology" and introduces G-CW complexes, stable homotopy, Bredon homology, and obstruction theory. While the Segal and Sullivan conjectures are not discussed in the book, a fair amount of the relevant background material may be found here. For example, the equivariant Whitehead and equivariant suspension theorems are proven, and equivariant spectra are introduced. This chapter also includes some more specialized material on represented spheres. While one might quibble with a few of the choices and emphases, this is mainly very basic information that everybody working in algebraic topology should know, and little if any of this material appears in textbooks.

The third chapter gives a very nice treatment of localization in equivariant homology theory, including the classical theorems of Smith and Borel and an illuminating selection of computations and applications. This too is basic material that every algebraic topologist should know and that is not to be found in textbooks. The fourth and last chapter largely overlaps the earlier book [5], although it does have some new material. It treats the Burnside ring of a compact Lie group and its role in induction theory and in the study of equivariant homology theory. The novice might find this chapter hard going since there are few computations and examples. In fact, there are few calculations in the literature at present, but there is a great deal of work going on in this area. A later writer may well find that he has a wealth of results to choose from to illustrate the force of the seminal ideas presented here.

REFERENCES

1. J. F. Adams, J.-P. Haeberly, S. Jackowski, and J. P. May, *A generalization of the Segal conjecture*, Topology **27** (1988), 7–21.

2. G. E. Bredon, *Introduction to compact transformation groups*, Academic Press, New York,1972.

3. G. Carlsson, *Equivariant stable homotopy and Segal's Burnside ring conjecture*, Ann. of Math. (2) **120** (1984), 189–224.

4. ——, *Equivariant stable homotopy and Sullivan's conjecture*, Preprint.

5. T. tom Dieck, *Transformation groups and representation theory*, Lecture Notes in Math., vol. 766, Springer-Verlag, Berlin and New York, 1979.

6. J.-P. Haeberly, *Some remarks on the Segal and Sullivan conjectures*, Amer. J. Math. (to appear).

7. J. Lannes, *Sur la cohomologie modulo p des p-groupes abeliens elementaires*, Preprint.

8. L. G. Lewis, Jr., J. P. May, and M. Steinberger (with contributions by J. E. McClure), *Equivariant stable homotopy theory*, Lecture Notes in Math., vol. 1213, Springer-Verlag, Berlin and New York, 1986.

9. J. P. May, *Review of Transformation groups and representation theory*, by Tammo tom Dieck, Bull. Amer. Math. Soc. (N.S.) **4** (1981), 90–93.

10. H. Miller, *The Sullivan conjecture and homotopical representation theory*, Preprint.

11. R. Schultz (ed.), *Group actions on manifolds*, Contemp. Math. vol. 36, Amer. Math. Soc., Providence, R.I., 1985.

12. R. W. Thomason, *The homotopy limit problem*, Contemp. Math. vol. 19, Amer. Math. Soc., Providence, R.I., 1983, pp. 407–419.
13. S. Weinberger, *Constructions of group actions: a survey of some recent developments*, Contemp. Math. vol. 36, Amer. Math. Soc., Providence, R.I., 1985, pp. 269–298.

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One-dimensional stable distributions, by V. M. Zolotarev. Translations of Mathematical Monographs, vol. 65, American Mathematical Society, Providence, 1986, ix + 284 pp., $92.00. ISBN 0-8218-4519-5

The concept of weak convergence of distribution functions plays a very fundamental role in Probability Theory. This concept may be defined as follows:

Let F be a distribution function, that is, a nondecreasing right-continuous function defined on $\mathbf{R}$ such that $F(-\infty) = 0$ and $F(+\infty) = 1$, and let $\{F_n\colon n \geq 1\}$ be a sequence of distribution functions. The sequence $\{F_n\}$ is said to *converge weakly* to the distribution function F on $\mathbf{R}$ if

$$F_n(x) \to F(x), \quad \text{as } n \to \infty$$

at all continuity points x of F. In this case we write $F_n \xrightarrow{w} F$, as $n \to \infty$. We note that the weak limit of the sequence $\{F_n\}$, if it exists, is unique. Moreover, let $\mathscr{C}_0 = \mathscr{C}_0(\mathbf{R})$ be the space of all bounded, real-valued continuous functions on $\mathbf{R}$. Let $\{F_n\colon n \geq 1\}$ be a sequence of distribution functions, and let F be a distribution function. Then $F_n \xrightarrow{w} F$ if and only if $\int_{-\infty}^{\infty} g\,dF_n \to \int_{-\infty}^{\infty} g\,dF$, as $n \to \infty$ for every $g \in \mathscr{C}_0$.

A necessary and sufficient condition for weak convergence was obtained by Lévy which can be stated as follows:

Let $\{F_n\}$ be a sequence of distribution functions and let $\{\varphi_n\}$ be the corresponding sequence of characteristic functions, that is, φ_n is the Fourier-Stieltjes transform of F_n for $n \geq 1$,

$$\varphi_n(t) = \int_{-\infty}^{\infty} e^{itx}\,dF_n(x), \qquad t \in \mathbf{R}.$$

Then $F_n \xrightarrow{w} F$ if and only if the sequence $\{\varphi_n\}$ converges (pointwise) on $\mathbf{R}$ to some function φ, which is continuous at the point $t = 0$. Moreover in this case, the limit function φ is the characteristic function of the limit distribution function F. This result plays a fundamental role in the study of the limit theorems for sums of independent (real valued) random variables.

The first result in this direction was obtained by Lévy which is known as the Lévy Central Limit Theorem and can be stated as follows.

Let $\{X_n\colon n \geq 1\}$ be a sequence of independently and identically distributed (i.i.d.) random variables with a finite variance $\sigma^2 > 0$. Let $S_n =$