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Correlation theory of stationary and random functions. vol. I; *Basic results*, vol. II, *Supplementary notes and references*, by A. M. Yaglom. Springer Series in Statistics, Springer-Verlag, New York, Berlin and Heidelberg, 1987, vol. I, xiv + 526 pp., $58.00. ISBN 0-387-96268-9, vol. II, vii + 258 pp., $56.50. ISBN 0-387-96331-6

This work has been set forth in two volumes. The first volume is described as *Basic results* and the second *Supplementary notes and references*. The title *Correlation theory of stationary and related random functions* indicates that the exposition does not attempt to discuss general aspects of the study of stationary processes but rather confines itself to the important but more limited aspect dealing with first and second order moment properties.

The object apparently is to give a direct development of results on a heuristic basis supplemented by illustrations in terms of applications and graphical representations in the first volume. The second volume consists of notes on the material developed in volume one together with an extensive set of references. Generally the first volume has no proofs. Occasionally sketches of proofs are given in the notes of volume two. However, for the most part these are not given. The reader is referred to material in the references. As remarked by the author, the book is intended as an elementary introduction to the second order theory of stationary processes for readers without a special mathematical background. It is assumed that most readers will be interested in applications of the theory.

The book starts with an introduction in which basic properties of distribution functions, probability densities and moments of random variables are mentioned. Random processes are discussed heuristically in the context of Brownian motion, shot noise, turbulence, electroencephalography as well as other applications.

First the basic properties of stationary processes are considered. The concept of a random process is first dealt with in terms of compatability of the family of joint distribution functions. Stationarity is mentioned in terms of invariance of joint probability structure under time shift. Weak stationarity is discussed as the restriction of stationarity just to conditions on first and second order moment structure. The characterization of the correlation function of a stationary process as a positive definite function is given. Convergence in mean square and the associated concepts of derivatives and integrals of processes in mean square are presented.

The exposition continues with an enumeration of a number of examples. These include uncorrelated sequences, moving averages, autoregressive sequences, and autoregressive—moving average (ARMA) sequences. A number of point processes like the Poisson process and Poisson pulse process are also analyzed. The spectral representation of the covariance function of a weakly stationary process in terms of the spectral distribution function of the process is detailed. The corresponding spectral representation of the weakly stationary process (assuming continuity in mean square) in terms of the random spectral function of orthogonal increments is also laid out. Both of these are given in terms of Fourier integral representations, the first as a Stieltjes integral and the second as a random Stieltjes integral in mean square. Then there is a discussion of examples of correlation functions and of linear transformations.

There is an extended discussion concerned with the estimation of aspects of the structure of the process on the basis of observations of the process. The first question dealt with is estimation of the mean or first order moment. Conditions for convergence of the sample mean to the true mean in mean square are given in terms of an ergodic result that the volume refers to as Slutsky's ergodic theorem. The condition amounts to continuity of the spectral distribution function at zero. The question of estimation of the correlation function is then considered. In the Gaussian case a necessary and sufficient condition for consistency of estimates of the correlation function, given observations $x(t)$, $0 \leq t \leq T$, on the process as $T \to \infty$, is continuity of the spectral distribution function. As already remarked, the focus of the book is on second order moment properties, or their equivalent (for example spectra) for weakly stationary processes.

However, every so often remarks are made about conjectured asymptotic normality of certain estimates. References are given in volume 2 to a number of works on the central limit theorem for dependent processes. Such results depend on some formulation of weak dependence. Since such a concept does depend on more than second order moment information in the non-Gaussian case, a more detailed discussion has been avoided. Usually the notion of weak dependence is formulated in terms of a version of a "strong" mixing condition employing a big block, small block argument in terms of partial sums (see Rosenblatt [**8**]) or else in terms of a criterion that can eventually be used to reduce the argument to the application of a martingale difference central limit theorem (see Hall and Heyde [**4**]). Yaglom continues with a discussion of statistical spectral analysis. The spectral density is the derivative of the spectral distribution function. Assuming that it exists and is smooth the object is to estimate it. There are several methods that have been proposed. One of these amounts to estimating the convariance function $r(t)$ by $r_T(t)$, $0 \leq t \leq T$ (assuming mean zero)

$$r_T(t) = \frac{1}{T}\int_0^{T-t} x(u)x(u+t)du,$$

and using a weight function $a_T(t)$ close to one near zero and tapering down to zero for large t, e.g.

$$a_T(t) = \begin{cases} \left(1 - \frac{|t|}{T^\alpha}\right), & 0 \leq |t| \leq T^\alpha, \\ 0, & \text{otherwise,} \end{cases}$$

with $0 < \alpha < 1$. One then just Fourier transforms the product $r_T(t)a_T(t)$. Another amounts to computation of the periodogram (originally due to Schuster) and smoothing it appropriately. Historically, it was remarked at a later point, that if the data is discretely sampled, a computationally efficient method of proceeding would be to compute the periodogram by using the fast Fourier transform. This observation is especially important because of the existence now of high speed modern computers. Initially it was thought that the proposal for such an efficient algorithm was new. However, Yaglom states that the idea for such an algorithm has been traced back to Gauss (for a historical discussion see Cooley, Lewis and Welch, [**2**] as well as Heideman, Johnson and Burns, [**6**]). Initial insights into spectral estimation are due to Daniell, Bartlett and slightly later Tukey in the 1940s. Yaglom remarks in a startling note that on a heuristic basis Einstein already in an incredibly early paper (1914) had suggested smoothing the periodogram to obtain an estimate of the spectral density. The asymptotic properties of the spectral estimates are described by Yaglom as $T \to \infty$. There is also given a description of what are termed parametric methods of spectral density estimation. One could describe this as an attempt to approximate the spectral density by an autoregressive spectral density by estimating autoregressive coefficients. The number of coefficients estimated depends on the sample size. Much of the initial insight into finite parameter estimates is due to Whittle. A discussion of finite parameter estimation on its own can be found in Hannan and Deistler, [**5**] and Brockwell and Davis,

[1]. An extensive treatment of finite parameter models and their estimates is not given in Yaglom's books. One should note that most of this finite parameter theory is centered on Gaussian models. More recently there has been some investigation of finite parameter non-Gaussian models (see Kreiss, [7]). Due to the restriction to second order moments and their estimation, phase information (which is available in the non-Gaussian case) cannot be resolved. This is possible in the non-Gaussian case using higher order moment or spectral information. Also a larger class of models (so-called non-minimum phase models) can be dealt with. There are recent researches in this direction which are naturally not covered in Yaglom's opus (see Rosenblatt, [8]). The maximum entropy spectral estimate (as an example of a parametric spectral estimate) is discussed at some length.

Finally generalizations of the concepts of a stationary process and a spectral representation are presented. At first vector-valued stationary processes are considered with the corresponding spectral representations of the covariance (matrix-valued) function and the vector-valued process. The nondecreasing Hermitian character of the now matrix-valued spectral distribution function is noted. Homogeneous random fields are now introduced. They are processes indexed by a multidimensional parameter and weakly stationary with respect to a shift in that parameter. Again spectral representations are introduced. Statistical inference for homogeneous random fields is then addressed. Isotropic random fields, that is homogeneous random fields whose covariance function depends only on the length of τ,

$$E[X(t+\tau)\overline{X(t+\tau)}] = B(\tau)$$

are considered. The corresponding spectral representations are now what might be termed Fourier-Bessel representations. Two different generalizations of isotropy to the vector-valued case are considered. The one of greatest interest is that in which mean and matrix-valued covariance function do not change when one replaces the pair of points t_1, t_2 by a new pair of points t_1', t_2' obtained from t_1, t_2 by making a rotation or reflection in R^n and the linear transformation of X is carried out corresponding to the rotation or reflection. The concepts of homogeneity and isotropy are partially motivated by the study of the model of homogeneous turbulence. A brief discussion of the motivation is given. It would have been interesting if Yaglom, so knowledgeable relative to turbulence, had given a more extended discussion of the field as well as the usefulness of these statistical models there. Homogeneous fields are described on spheres and other homogeneous spaces (spaces with a transitive group of motions on them). Random processes with stationary increments and generalized random processes (or random distributions as introduced by K. Ito and I. M. Gelfand, independently) are then treated. Locally isotropic fields (or equivalently random fields with isotropic increments) are characterized. A number of comparatively isolated topics are examined at the end of the book. The Karhunen-Loève expansion of a random process in terms of the eigenfunctions and eigenvalues of the integral equation having the covariance function as kernel (and compact operator) is developed. Remarks are made about oscillatory and evolutionary spectra. The object is to see whether a local version of the type of spectral representation one has for a stationary process might hold for some nonstationary processes. Finally there are some words about harmonizable processes (a class of processes introduced by Loève) where a Fourier representation for the process is possible but not generally in terms of a random spectral function with orthogonal increments.

The book is extensively illustrated by many examples and illustrations. The second volume has over 800 references to an extensive literature in theory and applications with brief comments on the text in volume one or on the references. The work provides a much more rapid introduction to the probabilistic background, the extensive applications and basic results on stationary processes and spectral analysis than is possible in a conventional exposition and is excellent in this way. A reader who wants a more formal background should supplement the book by referring to other texts or to original papers. The two volumes are incredibly free of all except trivial typographical errors. The author is to be hailed for his extended and richly rewarding exposition.

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Representations of algebraic groups, by Jens Carsten Jantzen. Pure and Applied Mathematics vol. 131, Academic Press, Orlando, 1987, xiii + 443 pp., $59.50. ISBN 0-12-380245-8

The appearance of this book marks an important point in the development of the theory of rational representations of algebraic groups. Many