This is probably a difficulty with any book presenting a young and active field, however. The author in his introduction acknowledges that some proofs are "arid and demanding" and encourages the reader to concentrate on their complement in the text. This is good advice, and I found it quite feasible to do so and still learn a great deal.

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Lectures on counterexamples in several complex variables, by John Erik Fornaess and Berit Stensønes. Mathematical Notes 33, Princeton University Press, Princeton, N. J., 1987, 247 pp., $22.50. ISBN 0-691-08456-4

This book is about examples in several complex variables, called for some strange reason counterexamples. This is a very nice and useful book, which gathers examples to be collected in many different places. It starts (the first 65 pp.) with an elegant survey, with proofs, of the most basic results about holomorphy, subharmonicity, and pseudoconvexity (including some material not to be found in the presently available textbooks). However, one sees once more the tremendous and amazing resistance to defining subharmonic functions by the fact that their Laplacian, in the sense of distributions, is a positive measure.

Although strict pseudoconvexity of a domain in C^n is a simple notion (a domain is strictly pseudoconvex if and only if, locally, it can be made strictly convex, under a holomorphic change of variables), the notion of (weak) pseudoconvexity is more subtle. The first basic example in this area is the Kohn-Nirenberg example which shows, in particular, that pseudoconvexity is not "locally equivalent" to convexity (after holomorphic change of variables). In fact, one can even start thinking about the crucial relation between convexity and subharmonicity in one complex variable. It is easy to see that a twice continuously differentiable strictly subharmonic function (Laplacian strictly positive) can be made strictly convex, in the neighborhood of any noncritical point, by a local holomorphic change of variables. The same fails to be true for subharmonic functions.

The world of weak pseudoconvexity had to be explored: exhaustion functions, neighborhoods .... Several examples by Diederich and Fornaess (including the famous worm domain) constituted a major achievement in this area. It is one of the main topics in the book.

It is impossible here to mention all the topics in the book, covered by about 40 examples! They bear on such basic questions as smoothing of pluri-subharmonic functions, embeddability of CR manifolds (a bad misprint is to be corrected in the statement of Kuranishi (-Akahori)'s theorem, p. 96, where one should read $n > 3$), Runge exhaustion, $\bar{\partial}$ theory, Stein spaces. They bear too on some questions which seem to me to be as important but somewhat more specialized: peak sets (not surprisingly, particularly well treated), boundary behavior of bounded holomorphic functions (inner functions), finite type, etc..

As a very minor remark, I wish to point out that the authors do not make a very precise use of the notion of Runge domain (lecture 46). At least for an inexperienced reader, they leave some possible confusion between Runge property (which for example every star shaped domain in C^n has), and polynomial convexity.

It would be just ridiculous to complain about "the" missing examples. However, I wish to list some few examples. Of course this is a very arbitrary list, and I certainly do not claim to give any kind of exhaustive list.

One should also mention, for less recent examples, the book **[St]** by E. L. Stout, which remains an excellent reference (see especially Chapter 6). There are just two things unfortunate about Stout's book: its title, and the fact that it seems to be no longer available.

I. Some examples are not in the book, because they are too recent. It has to be noticed that the book is based on a course given in '83, but some examples could at least have been given in references "for further reading." Let us mention the following ones:

(1) Barrett's examples, including the example of a smooth (nonpseudoconvex) domain, such that the Bergman projection does not map smooth functions to smooth functions **[B]**.

(2) Sibony's example of a smooth pseudoconvex domain for which the Corona theorem fails. This is an especially nice example, since it completes a series of two previous examples (lectures 19 and 39). This new example is again based on a construction by Catlin to put any polynomially convex set in C^n in the boundary of a smooth pseudo-convex set in C^{n+1}. Using a pathological nonsmooth polynomially convex set K leads to a smooth pathological domain. This new construction uses the example in lecture 19. Remark that in lecture 39 (non L^∞ estimate for $\bar{\partial}$), one can use, as compact set K, instead of the one constructed by Sibony, an example of E. Kallin, well known to be pathological for $\bar{\partial}$, **[K, Si2]**. Another nice example by Sibony is an example of a domain with a very "thin" Silov boundary, **[Si1]**.

(3) Trepreau's example. This is one of the most basic example in several complex variables (unpublished by the author!!!!), a CR manifold of codimension 2 in C^3, so that there are some CR functions which cannot be decomposed into the sum of boundary values of holomorphic functions on wedges.

II. It would have been nice to have had some topics covered in more detail: invariant distances, totally real embeddings **[A.R, R]**, Monge-Ampere equation.

A surprising omission is the examples of Diederich and Fornaess of complete pluripolar, and nonpluripolar smooth curves **[D.F]**. In the first example, one constructs a curve which is exactly the set on which a plurisubharmonic function is $-\infty$. Such a curve is interesting because, in some sense, it is a smooth curve as far as possible from being real analytic (e.g it cannot share a set of positive length with any real analytic curve). Complex analytic subsets of C^n are pluripolar, and a kind of partial converse is given by Siu's theorem on the Lelong number. The Diederich-Fornaess example is interesting too in this perspective, and is a very basic example.

III. However, if there is one omission that I find particularly shocking, it is the omission of Skoda's example.

A famous question (problem, conjecture?) raised by Serre was whether every holomorphic fiber bundle with base and fiber both Stein manifolds is necessarily Stein. This topic would have fitted perfectly well in the book. Serre's question has been the origin of some remarkable work. Positive results were obtained, in case the fiber is an arbitrary set in C, or a smooth bounded pseudoconvex set in C^n, or any arbitrary (nonsmooth) bounded pseudoconvex set in C^n with first Betti number equal to 0 **[Siu]**.

Then came Skoda's example, amazingly simple, totally transparent. In Skoda's example the fiber is C^2, and the basis is a finitely connected open set in C **[Sk]**. It has been significantly improved by Demailly who gave the example of a fiber bundle with fiber C^2 and base the unit disk in C, which is not Stein (and therefore not trivial!)—then of course the transition functions need not to be constant, **[D]**. Finally Coeure-Loeb gave an example with base $C - \{O\}$, and with fiber a bounded pseudoconvex domain in C^2. This example is especially striking, considering Siu's theorem mentioned above. Even if one is not interested in Serre's problem and fiber bundles, the Coeure-Loeb example is an interesting example of a domain in C^2 with a very "distorting" automorphism (which allows the construction of a fiber bundle for which the convexity property fails), **[C.L]**.

May the end of this review encourage some author to write a book as useful (and hopefully as pleasant to read) as the book under review, to gather some more examples, not in the book either because they are too recent, or for some other reason.

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Ring theory (2 volumes), by Louis H. Rowen. Pure and Applied Mathematics, volumes 127 and 128, Academic Press, San Diego, 1988, XXIV + 538 pp., $89.50 ISBN 0-12-599841-4 (volume I). xiv + 462 pp., $84.00 ISBN 0-12-599842-2 (volume II).

As in many areas of mathematics, most of the research in noncommutative ring theory—including some basic results—is less than thirty years old. As recently as 1956, Jacobson was able to include most of the subject in his classic *Structure of rings* [J], a book of moderate size. But by the time he revised *Structure of rings* in 1964, Goldie's Theorem and other new developments had appeared, and he merely sketched them in appendices.

It is not surprising that after *Structure of rings* no one has tried to write a book covering all of ring theory. Rowen has not gone quite so far in the two volumes under review, but his *Ring theory* is the most ambitious and encyclopedic book yet written in the area. It is both a textbook and an up-to-date encyclopedia of research. Roughly speaking, the greater part of volume I covers basic structure theory, and is meant to be the basis of a graduate level ring theory course; volume II and parts of volume I treat specialized areas of ring theory in which research is active today. These include noncommutative Noetherian rings, homological algebra, polynomial identities, central simple algebras, and enveloping algebras. The one major topic omitted is the representation theory of Artin algebras.

It is useful to consider separately three potential audiences for *Ring theory*, all beyond the undergraduate level: Graduate students, for whom it will be a textbook; experts (= ring theorists), for whom it will be a compendium of recent research; and nonexperts (= established mathematicians who are not ring theorists), for whom it will be a guide to the subject. For all of them, it also serves as a reference.

As a textbook for graduate students, *Ring theory* joins the best. The only other books which I think make excellent textbooks are *Noncommutative*