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Mathematical problems in viscoelasticity, by Michael Renardy, William J. Hrusa, and John A. Nohel. Pitman Monographs and Surveys in Pure and Applied Mathematics, vol. 35, Longman Scientific and Technical, Essex, and John Wiley and Sons, New York, 1987, 273 pp., $87.95. ISBN 0-852-00320-2

Initial value problems for elastic materials involve nonlinear hyperbolic equations, and smooth solutions usually exist for only a finite time interval, after which a discontinuous solution representing a shock wave must be used. In viscous materials, on the other hand, the equations are parabolic, and signals travel by diffusion rather than as well-defined waves. The viscosity has a smoothing effect that prevents the finite time breakup seen in elastic problems.

Viscoelastic materials exhibit effects of both elasticity and viscosity. The stress is a functional of the past history of the strain, instead of being a function of the present strain value (elastic) or of the present value of the time derivative of strain (viscous). When an integral model is used for the history dependence, the dynamical equations become partial integrodifferential equations with a character somewhere between hyperbolic and parabolic. The book at hand deals with existence theorems for problems involving such equations.

The principal type of stress-strain relation that is used has either an explicit elastic term or an explicit viscous term, plus an integral over the past history of some nonlinear function of the strain. The dynamical equations can then be classified as either hyperbolic or parabolic, with the history integral as a perturbation term. Existence theorems can be obtained by an extension of methods used for pure partial differential equations, principally energy estimates for hyperbolic equations and the implicit function theorem for parabolic cases.

In hyperbolic cases, nonlinearity usually causes smooth solutions to break up in finite time, but for sufficiently small initial data this is prevented by the smoothing effect that the history integral provides. In parabolic cases, apparently solutions are smooth for all time, but for large initial data this has not been proved in the same generality as for small data.

Much less is known about solutions with shocks. When discontinuous solutions are admitted, solutions are highly nonunique, and some admissibility criterion for shocks is needed. Criteria for elastic materials can be formulated intuitively, but for viscoelastic materials the question is still under study. Although the existence of constant-velocity shocks can be proved under very broad assumptions about the dependence of stress on the strain history, the formation and subsequent propagation of shocks in initial value problems is an open area.

Stress-strain relations with no explicit elastic or viscous term are not considered in the present volume, so the truly in-between cases in which the equations are neither hyperbolic nor parabolic are not discussed. A start in this direction is made by considering materials whose stress-relaxation moduli have initially infinite slopes. Even for initially elastic materials this produces a smoothing effect like that of viscosity, although weaker. When the stress relaxation modulus itself is initially infinite, the governing integrodifferential equation is squarely not a perturbation on a partial differential equation, and no results of any great generality are available for such cases.

The book is complete in itself so far as the fundamentals of nonlinear viscoelasticity are concerned. The methods that are used for proving existence theorems are always introduced and explained in terms of simpler model equations, and the difficulties are taken one at a time. This style of exposition makes the proofs comprehensible even to one who is not an analyst, and for that I am most grateful.

ALLEN C. PIPKIN
BROWN UNIVERSITY

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The Malliavin calculus, By Dennis R. Bell. Pitman Monographs and Surveys in Pure and Applied Mathematics, vol. 34, Longman Scientific and Technical, Essex, and John Wiley, New York, 1987, x + 105 pp., $62.95. ISBN 0-582-99486-1

The Malliavin calculus refers to a part of Probability theory which can loosely be described as a type of calculus of variations for Brownian motion. It is intimately concerned with the interplay between Markov processes with continuous paths (i.e., *diffusions*) and partial differential equations.

A time homogeneous diffusion X with values in $\mathbf{R}^n$ can be represented as a solution of a stochastic integral equation of the form

$$X_t = x + \int_0^t a(X_s)\,dB_s + \int_0^t b(X_s)\,ds, \tag{1}$$

where B is a Brownian motion on $\mathbf{R}^m$ (also known as a Wiener process), provided X satisfies mild regularity conditions. From a statistical standpoint, the diffusion X is determined by its *transition probabilities*, since it is a Markov process $P_t(x, A) = P(X_{u+t} \in A | X_u = x)$, all $u \geq 0$, all $t > 0$. The measures $P_t(x, dy)$ induce operators on bounded Borel functions $P_t f(x) = \int f(y) P_t(x, dy)$, and since they are a semigroup of operators there is an *infinitesimal generator* ($P_0 = I$),

$$Lf(x) = \lim_{t \downarrow 0} \frac{P_t f(x) - f(x)}{t},$$