3. ——, *Elementary introduction to new generalized functions*, North-Holland, Amsterdam, 1985.

4. J. F. Colombeau and A. Y. Le Roux, *Multiplications of distributions in elasticity and hydrodynamics*, J. Math. Phys. **29** (1988), 315–319.

5. Y. C. Fung, *A first course in continuum mechanics*, Prentice-Hall, Englewood Cliffs, N.J., 1969.

6. J. Jelinek, *Characterization of the Colombeau product of distributions*, Comment. Math. Univ. Carolinae **27** (1986), 377–394.

7. L. Hörmander, *The analysis of linear partial differential operators*. I, II, Springer-Verlag, Berlin and New York, 1983.

8. H. Lewy, *An example of a smooth linear partial differential equation without solution*, Ann. of Math. (2) **66** (1957), 155–158.

9. J. L. Lions, *Quelques méthodes de résolution des problèmes aux limites non linéaires*, Dunod, Paris, 1969.

10. M. Oberguggenberger, *Products of distributions*, J. Reine Angew. Math. **365** (1986), 1–11.

11. ——, *Multiplications of distributions in the Colombeau algebra* $\mathfrak{G}(\Omega)$, Boll. Un. Mat. Ital. **5A** (1986), 423–429.

12. E. E. Rosinger, *Distributions and nonlinear partial differential equations*, Springer-Verlag, Berlin and New York, 1978.

13. ——, *Nonlinear partial differential equations: sequential and weak solutions*, North-Holland, Amsterdam, 1980.

14. L. Schwartz, *Sur l'impossibilité de la multiplication des distributions*, C. R. Acad. Sci. Paris **239** (1954), 847–848.

15. F. Treves, *Basic linear partial differential equations*, Academic Press, New York, 1978.

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Equivariant K-theory and freeness of group actions on C^*-algebras, by N. Christopher Phillips. Lecture Notes in Math., vol. 1274, Springer-Verlag, Berlin and New York, viii + 371 pp., $32.90. ISBN 3-540-18277-2

K-theory for C^*-algebras is also known under the name of "noncommutative" topology. A C^*-algebra is a Banach algebra that has the same abstract properties as the algebra $\mathscr{C}(X)$ of continuous complex-valued functions on a compact space X except for the fact that the multiplication is not necessarily commutative.

Noncommutative C^*-algebras arise naturally from group actions on topological spaces, foliated manifolds, pseudodifferential operators, etc., and they also formalize the noncommuting variables of quantum mechanics.

Even if one is only interested in spaces, one often has to extend the frame to the noncommutative category as certain natural constructions in K-theory automatically lead to noncommutative algebras. One might go as far as to compare this to the passage from real to complex numbers in analysis.

Nearly from the beginning of the development of topological K-theory it was known that the basic invariants can be defined also in the noncommutative setting, i.e., for C^*-algebras [7]. This also was the case for the dual Ext-theory introduced by Brown-Douglas-Fillmore [2].

K-theory for C^*-algebras however really got started around 1980 when:

1. Elliott used K-theory to show that the study of an important class of C^*-algebras (the AF-algebras) can largely be reduced to the study of certain ordered abelian groups [**6**].

2. Kasparov introduced his KK-theory which is a very far reaching generalization of K and Ext-theory and which fundamentally depends on the use of C^*-algebras even when restricted to spaces [**8**].

3. The first nontrivial computations of K-groups for noncommutative (simple) C^*-algebras appeared [**5**, **10**].

4. Connes applied the theory to index problems on foliated manifolds [**3**].

Since then the theory has undergone a very rapid development and is now also closely tied to "noncommutative" differential geometry [**4**]. Equivariant K-theory fits extremely naturally into the general setting. Kasparov defined equivariant K- and KK-theory even for noncompact locally compact groups [**9**].

It is natural, in K-theory for C^*-algebras, to take concepts from topological K-theory for topological spaces and to see what they give in the noncommutative context. The author does this here with the concept of a K-free action of a compact group G on a compact space X, which was introduced by Atiyah and Segal and shown to be equivalent to freeness of the action of G. Let $I(G)$ be the ideal of elements of dimension 0 in the representation ring $R(G)$ of G. Then an action of G on X is K-free if there exists n such that $I(G)^n K_G^*(X) = 0$, where $K_G^*(X)$ is the equivariant K-theory of X. Accordingly, an action of G on a C^*-algebra is defined to be K-free if there is n such that $I(G)^n K_*^G(A) = 0$.

Freeness of a G-action on a C^*-algebra is not a well-defined concept. Thus the author investigates if K-freeness could be used to replace it and also compares K-freeness to other substitutes of freeness for G-actions. Possible substitutes are e.g. fullness of the strong Connes spectrum, KK-freeness or also K-saturation: The action is K-saturated if the natural map $K_*(A^G) \to K_*^G(A)$, where A^G is the fixed point algebra, is an isomorphism. The result of the investigation is that each of the possible substitutes has its drawbacks and that the connections between them are rather messy except for the case of a finite cyclic group or S^1 acting on a separable type I algebra where at least part of them coincide and are equivalent to free action on $\mathrm{Prim}(A)$.

The book contains a good and detailed introduction to equivariant K-theory and KK-theory (for compact groups), many examples of actions on C^*-algebras and their K-theory and a wealth of further related material. These points constitute its real value. It is a good complement to [**1**] which is at the moment the only book on noncommutative topology available.

REFERENCES

1. B. Blackadar, *K-theory for operator algebras*, MSRI Publications, no. 5, Springer-Verlag, Berlin and New York, 1986.

2. L. Brown, R. Douglas and P. Fillmore, *Extensions of C^*-algebras and K-homology*, Ann. of Math. (2) **105** (1977), 265–324.

3. A. Connes, *A survey of foliations and operator algebras*, Proc. Sympos. Pure Math., vol. 38, Amer. Math. Soc., Providence, R.I., 1982, pp. 521–628.

4. ——, *Noncommutative differential geometry*, Inst Hautes Études Sci. Publ. Math. **62** (1986), 257–360.

5. J. Cuntz, *K-theory for certain C^*-algebras*, Ann. of Math. (2) **113** (1981), 181–197.

6. G. Elliott, *On the classification of inductive limits of sequences of semi-simple finite dimensional algebras*, J. Algebra **38** (1976), 29–44.

7. M. Karoubi, *K-theory: An introduction*, Springer-Verlag, Berlin and New York, 1978.

8. G. Kasparov, *The operator K-functor and extensions of C^*-algebras*, Izv. Akad. Nauk SSSR Ser. Mat. **44** (1980), 571–636.

9. ——, *Equivariant KK-theory and the Novikov conjecture*, Invent. Math. **91** (1988), 147–201 (preliminary version was circulated already in 1981).

10. M. Pimsner and D. Voiculescu, *Exact sequences for K-groups and* Ext-*groups of certain cross-product C^*-algebras*, J. Operator Theory **4** (1980), 93–118.

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Multiple forcing, by Thomas Jech. Cambridge Tracts in Mathematics, vol. 88, Cambridge University Press, Cambridge, 1986, vii + 136 pp., $34.50. ISBN 0-521-26659-9

In set theory truth is approached from four directions, not just the usual two. A given proposition may be true or false, but it may also be consistent with or independent of "the usual axioms for set theory", that is to say the Zermelo-Fraenkel system of axioms together with the Axiom of Choice, the whole denoted by ZFC. Moreover, these independent propositions take up part of the life of every set theorist. They are not the sort of propositions that only a logician could love; frequently they are powerful, fundamental assertions that occur naturally, and they require study. To follow this Fourfold Way of Truth one must master, in addition to proof and refutation, the method of forcing.

Forcing, of course, was invented 25 years ago by Paul Cohen as the key element in his proof that the Continuum Hypothesis is independent of ZFC. It can best be regarded as a way of adjoining to the universe of set theory new sets with special properties. For example, to make the Continuum Hypothesis false one might adjoin $\aleph_2$ new real numbers. Now from the point of view of the universe V of set theory any new sets have got to be fictitious, since V is nothing else than the collection of all (well-founded) sets, so there is a flavor of sand-castle-building to the whole enterprise of forcing. One way of dealing with this is to treat the extension of the universe as a collection of artificial constructs, "fuzzy" sets if you