Ramanujan has left a legacy that will keep mathematicians busy for many more decades.

ADDED IN PROOF (MAY 23, 1988). The above conjecture has been proven by Dean Hickerson.

REFERENCES

1. P. A. MacMahon, *Combinatory analysis*, vol. II, Cambridge Univ. Press, 1916, Reprinted Chelsea, New York, 1960.

2. S. Ramanujan, *Notebooks*, 2 vols., Tata Inst., Bombay, 1957, Reprinted Narosa, New Delhi, Madras, Bombay, 1984.

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Topological graph theory, by Jonathan L. Gross and Thomas W. Tucker, John Wiley and Sons, New York, Chichester, Brisbane, Toronto, Singapore, xv + 351 pp., $59.95. ISBN 0-471-04926-3

Topological graph theory began with the 1890 paper of Heawood [**1**] in which it was pointed out that Kempe's proof of the 4-colour theorem was incorrect. Heawood proved instead that every map on S_h, the sphere with h handles attached, can be coloured in $\chi(S_h) = [\frac{1}{2}(7 + \sqrt{1 + 48h})]$ colours for each $h \geq 1$. He claimed that this is best possible since a map with $\chi(S_h)$ pairwise adjacent countries (or, equivalently, a complete graph with $\chi(S_h)$ vertices) can be drawn on S_h for each $h \geq 1$. While this claim, which became known as the Heawood conjecture, is correct, it took almost 80 years until a proof was completed. The main ideas and the major part of the proof were provided by G. Ringel who wrote a book on the proof [**2**]. The final cases of the proof were done by Ringel and Youngs.

The Heawood conjecture led to the following general question: Given a graph G and a natural number h, can G be embedded into S_h? While this problem is NP-complete, (and thus probably hopeless), as shown recently by the reviewer, there are many results for special classes of graphs. Most of the investigations motivated by the Heawood conjecture are concerned with the existence and properties of certain embeddings. However, the recent Robertson-Seymour theory on minors has shown that topological graph theory is also important as a tool and has a natural place in general discrete mathematics. One of the highlights in the Robertson-Seymour theory is the following: Let p be a graph property which is preserved under minors, that is, if G has property p and H is obtained from G by deleting or contracting edges, then also H has property p. Then there exist only finitely many minor-minimal graphs that do not have property p, and there exists a polynomially bounded algorithm for deciding if a graph has property p. In order to understand the proof of this general result it is necessary to be familiar with some topological graph theory.

The first comprehensive treatment on topological graph theory is the book of White [**3**]. The present book by Gross and Tucker is another valuable source on the subject. For obvious reasons it does not include the Robertson-Seymour theory (which has not yet been published in full) but emphasizes, like White's book, the methods for describing embeddings in 2-dimensional compact orientable or nonorientable surfaces and the interaction between embeddings and groups.

Embeddings of graphs on orientable surfaces can be described completely by the local clockwise orientations around the vertices. This observation, which is attributed to Heffter, Ringel and Edmonds, reduces many embedding problems, including the Heawood conjecture, to purely combinatorial problems. Thus the problem of finding an embedding of minimum genus of a graph amounts to describing local orientations such that the number of facial walks (that is, walks in the graph such that one turns sharp left at every vertex) is maximum. A clever device, the so-called current graphs, for describing such orientations was introduced by W. Gustin. These ideas and their applications (illustrated with several cases of the proof of the Heawood conjecture) are carefully described in this book. As a special feature, the book treats in detail the dual concept of current graphs, called voltage graphs, introduced by one of the authors (Gross). While current graphs are convenient for actually describing embeddings, it seems that voltage graphs are more natural for explaining the underlying ideas. In addition, voltage graphs are of interest in studying symmetry properties of abstract graphs as well as branched coverings and group actions on surfaces, as is clearly demonstrated in the present book.

White introduced the genus of a graph as the smallest genus of one of its Cayley graphs. This useful concept has lead to the study of highly symmetric embeddings of graphs on surfaces. The final chapter of the present book (which is more about groups than graphs) describes the results on the classification of the groups of a given genus and the interesting relationship between the genus of a group and the minimum genus surface on which the group acts.

Rather than concentrating on deep results and technical proofs the book contains many exercises and examples as well as intuitive ideas and historical remarks which makes it easy for a nonexpert to get a good feeling for the subject. The book emphasizes the interaction between graphs, groups and surfaces and can be recommended for anyone who wants to become familiar with the methods and results in topological graph theory.

REFERENCES

1. P. J. Heawood, *Map-color theorem*, Quart. J. Math. Oxford Ser. **24** (1890), 332–338.

2. G. Ringel, *Map Color Theorem*, Springer-Verlag, Berlin, 1974.

3. A. T. White, *Graphs, groups and surfaces*, North-Holland, Amsterdam, 1973 (rev. ed., 1984).

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