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The lost notebook and other unpublished papers, by Srinivasa Ramanujan, with an introduction by George E. Andrews, Narosa Publishing House, New Delhi, Madras, Bombay, 1988, xxv + 419 pp., 250 rupies. ISBN 81-85198-06-3, North American and European distribution: Springer-Verlag, ISBN 0-387-18726-X

In the spring of 1976, G. Andrews was looking through a box of Watson's material in the library of Trinity College when he came across about 90 sheets of paper, most of them in Ramanujan's handwriting. In 1957 the Tata Institute for Fundamental Research had published photostatic copies of Ramanujan's early notebooks [**2**], so Ramanujan's writing was well known to Andrews and quite a few others. However very few people would have been able to recognize exactly what was in this box in the Trinity library. Andrews had written a thesis on mock theta functions, so when he saw that some of these sheets contained claims of Ramanujan about mock theta functions, he knew this was a major find. These sheets consist primarily of work Ramanujan did in the last 15 months of his life, after he left England and returned to India. For the last ten years, Andrews has published a number of papers proving results in these sheets, and a few other people have published a little more, but the mathematical community at large has not had access to this fascinating collection. Thanks to Narosa Publishing House, anyone who wants to can now try his or her hand at proving some of Ramanujan's last results.

Many other fascinating things are contained in this book. There is Littlewood's letter to Hardy commenting on Ramanujan's second letter. Among other perceptive comments in this letter is the following: "I can believe that he's at least a Jacobi."

There are some manuscripts of Ramanujan that were not published before, either because of financial problems that the London Mathematical Society had, or because they were unfinished. There is a fascinating sheet (p. 358) which is undated, but was probably written in 1915. It contains four reasons why

$$\begin{aligned}(1)\qquad &1+\frac{x}{1-x}+\frac{x^4}{(1-x)(1-x^2)}+\frac{x^9}{(1-x)(1-x^2)(1-x^3)}+\cdots\\ &=\frac{1}{(1-x)(1-x^6)(1-x^{11})\cdots(1-x^4)(1-x^9)(1-x^{14})}\cdots\end{aligned}$$

should be true. The first is the statement that "Mr. MacMahon has verified up to x^{55} and found the result correct up to that term." By the time MacMahon published [1] in 1916, he had verified it up to the coefficient of x^{89}. The second reason is that the two sides each have the same asymptotic behavior as $x \to 1$, i.e., the log of both sides looks like $\frac{\pi^2}{15(1-x)}$ as $x \to 1^-$. The third reason deals with numerical results for the continued fraction

$$C(x) = \cfrac{1}{1+\cfrac{x}{1+\cfrac{x^2}{1+\cdots}}}$$

Recall that Ramanujan in his first letter to Hardy claimed that

$$C(e^{-2\pi}) = \left\{\sqrt{\frac{5+\sqrt{5}}{2}} - \left(\frac{\sqrt{5}+1}{2}\right)\right\} e^{2\pi/5}.$$

He could prove this if two series-product identities were true. One of these two identities is the one mentioned above. This is one of a number of reasons why Ramanujan was interested in (1). The fourth also deals with this contained fraction. If

$$v(x) = xC(x^5)$$

then Ramanujan claimed that $v^{-1} - v - 1$ vanishes when $x = e^{i\pi m/n}$ when m and n are relatively prime integers except when n is a multiple of 25. Schur later studied this continued fraction when x has this form.

In 1915 Ramanujan read a paper of L. J. Rogers that contained a proof of (1). After studying this proof, Ramanujan put in an extra parameter that was implicit in Rogers's derivation, and was then able to find another proof of (1).

Pages like this give a small indication how Ramanujan thought about certain problems, and show that he was much more than a calculating prodigy.

Andrews has written a short introduction, setting Ramanujan's work in a broader setting when this can be done. Many of Ramanujan's claims in this work have been proven, but there are still many that no one knows how to prove. Andrews has said he will write a survey paper, telling what he has done, and highlighting some of the claims that are still open. One that Andrews mentioned in the introduction is the following.

A partition of n is a set of positive integers $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r > 0$ with $n = \lambda_1 + \lambda_2 + \cdots + \lambda_r$. Define the rank of a partition to be $\lambda_1 - r$, the largest part minus the number of parts. If $R_b(n)$ denotes the number of partitions of n with rank congruent to $b \bmod 5$, then Andrews and Garvan have conjectured that $R_1(5n) - R_0(5n)$ equals the number of partitions of n with unique smallest part and all other parts less than or equal to double the smallest part. For example, $n = 2$ gives $R_1(10) - R_0(10) = 9 - 8 = 1$, and 2 is the only partition of 2 with unique smallest part and all other parts at most double the smallest part. This or the corresponding analytic identity is true if and only if any one of five of Ramanujan's claims about a class of fifth order mock theta functions is true.

Ramanujan has left a legacy that will keep mathematicians busy for many more decades.

ADDED IN PROOF (MAY 23, 1988). The above conjecture has been proven by Dean Hickerson.

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Topological graph theory, by Jonathan L. Gross and Thomas W. Tucker, John Wiley and Sons, New York, Chichester, Brisbane, Toronto, Singapore, xv + 351 pp., $59.95. ISBN 0-471-04926-3

Topological graph theory began with the 1890 paper of Heawood [**1**] in which it was pointed out that Kempe's proof of the 4-colour theorem was incorrect. Heawood proved instead that every map on S_h, the sphere with h handles attached, can be coloured in $\chi(S_h) = [\frac{1}{2}(7 + \sqrt{1+48h})]$ colours for each $h \geq 1$. He claimed that this is best possible since a map with $\chi(S_h)$ pairwise adjacent countries (or, equivalently, a complete graph with $\chi(S_h)$ vertices) can be drawn on S_h for each $h \geq 1$. While this claim, which became known as the Heawood conjecture, is correct, it took almost 80 years until a proof was completed. The main ideas and the major part of the proof were provided by G. Ringel who wrote a book on the proof [**2**]. The final cases of the proof were done by Ringel and Youngs.

The Heawood conjecture led to the following general question: Given a graph G and a natural number h, can G be embedded into S_h? While this problem is NP-complete, (and thus probably hopeless), as shown recently by the reviewer, there are many results for special classes of graphs. Most of the investigations motivated by the Heawood conjecture are concerned with the existence and properties of certain embeddings. However, the recent Robertson-Seymour theory on minors has shown that topological graph theory is also important as a tool and has a natural place in general discrete mathematics. One of the highlights in the Robertson-Seymour theory is the following: Let p be a graph property which is preserved under minors, that is, if G has property p and H is obtained from G by deleting or contracting edges, then also H has property p. Then there exist only finitely many minor-minimal graphs that do not have property p, and there exists a polynomially bounded algorithm for deciding if a graph has property p. In order to understand the proof of this general result it is necessary to be familiar with some topological graph theory.