A large set of tools having been developed for the above work, the author proceeds to apply much of it to questions about commutative and skew field subalgebras of the skew field coproduct and the simple Artinian coproduct. In particular, he shows that even if two skew fields contain no elements algebraic over the center, the skew field coproduct of the two over the center may contain a (commutative) algebraic subfield. However, the transcendence degree of subfields of a skew field coproduct cannot be much bigger than those of the factors. He also shows that free skew fields on different numbers of generators cannot be isomorphic.

Finally, by introducing a generalization of the skew field coproduct, Schofield gives a complete solution to Artin's problem for skew fields: for any integers $m, n > 1$ he constructs skew fields $D \subseteq E$ such that E has right D-dimension m and left D-dimension n. This allows him to construct a hereditary Artinian ring of (finite) representation type $I_2(5)$ as suggested by Dowbor, Ringel, and Simson.

The book is definitely a work of power and of breadth, with much to interest a wide class of ring theorists. The reader is given the strong impression that mathematics is in the process of being done here. Perhaps inevitably in such a case, there are many misprints, some of which are serious. The most important are described in Bergman's review (MR 87c:16001). The reader should also be warned that the index is of limited usefulness, since it has only a selection of the important subjects.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 19, Number 2, October 1988

0273-0979/88 $1.00 + $.25 per page

Partial differential equations: An introduction to a general theory of linear boundary value problems, by Aleksei A. Dezin. Translated by Ralph P. Boas. Springer-Verlag, Berlin, Heidelberg, New York, xii+163 pp., $82.00. ISBN 3-540-16699-8

Partial differential equations, by J. Wloka. Translated by C. B. and M. J. Thomas. Cambridge University Press, Cambridge, London, New York, New Rochelle, Melbourne, Sydney, 1987, xi+518 pp., $79.50. ISBN 0-521-25914-2

For centuries scientists have known that many phenomena in nature are described by partial differential equations. Every student of physics comes across the potential, heat, and wave equations as well as equations bearing such names as Poisson, Helmholtz, Maxwell, Schrödinger, Klein-Gordon, Navier-Stokes, Korteweg–de Vries, etc. All of these are partial differential equations or systems of partial differential equations that describe various aspects of the physical world. It was generally accepted from the very beginning that a more accurate description of the universe is given by nonlinear equations, but the difficulty in dealing with such equations led most researchers to study linearized versions. It is no surprise that much more is known about linear equations than nonlinear ones. The two books under review consider only linear partial differential equations (with one exception as noted below).

Three partial differential equations are found very frequently in classical physics: the potential equation, the heat equation, and the wave equation. From both the physical and mathematical viewpoints it was clear that these equations could not and should not be solved alone but only under additional posed conditions. If such conditions are imposed on the boundary of a region, they are called "boundary conditions." If they are imposed at a given time t_0, they may be called "initial conditions." Moreover, it became quite clear that the three equations required different additional conditions in order to lead to a "well posed problem." Conditions that were suitable for one of these equations were not usually suitable for the others. This led people to wonder what there was in the nature of each equation that required a particular kind of boundary condition.

Other partial differential equations began to arise in applications together with the problem of finding suitable boundary conditions for each of them. It was noticed that there were fairly large classes of equations that behaved like one of the three basic equations. Those that were similar to the potential equation were called "elliptic," those similar to the heat equation were called "parabolic," and those similar to the wave equation were called "hyperbolic." The same boundary conditions are suitable for all equations of the same "type." No matter how complicated the equation or system of equations, the type determines the proper boundary conditions for it. Many authors have studied very general equations of these three types.

The Wloka book considers the three basic types of equations from the L^2 Hilbert space approach. The first chapter prepares all of the background material including Sobolev spaces and domain mappings. All details are carefully explained.

The second chapter studies general boundary value problems for properly elliptic equations. Included are the index theorem, normal boundary value problems, Green's formulas, and adjoint problems. The third chapter presents the Garding-Agmon theory for strongly elliptic equations. Included are discussions of the existence and regularity of weak solutions and boundary value problems in unbounded domains. An exception to the consideration of only linear problems, this chapter discusses a nonlinear elliptic boundary value problem. Included is an elementary proof of the Schauder fixed point theorem.

Chapters 4 and 5 give existence, uniqueness, and regularity theorems for solutions of parabolic and hyperbolic equations. The final chapter shows how finite-difference methods can be used to approximate solutions of boundary value problems for second-order partial differential equations.

There are many illustrations and examples presented in the text, and some of the sections have problems. There is an extensive bibliography. The book is a translation of *Partielle Differentialgleichungen* published by B. G. Teubner, Stuttgart, 1982. The translation is good, preserving the clear presentation of the original. The reviewer would have preferred the term "boundary" to "frontier."

If a partial differential operator does not fall into one of the well-known classes and there is no known application, it is extremely difficult to determine which boundary conditions will be suitable for such equation. Clearly, it is the structure of the equation that determines this, but to date no one has developed a general method of finding such conditions. It is to this and related problems that Dezin addresses his book. He asks if the well-known classes of partial differential equations are special in any way. Given an arbitrarily chosen equation, can one find reasonable boundary conditions that will lead to a well posed problem? What happens when a boundary value problem is incorrectly posed?

Dezin has undertaken a study of these problems for special classes of equations. He does not give general theorems, but he studies many different examples. Cases that yield to his methods include ordinary differential equations, equations on tensor products, model operators (operators that have a complete system of eigenfunctions that form a Riesz basis), and equations with constant coefficients. The approach is to work within L^2 theory and make use of the spectral properties of operators.

The book is a translation of *Obshchie veprosy teorii granichnykh zadach*, Nauka, Moscow, 1980. The first two chapters present the elements of spectral theory, special classes of operators, function spaces, and operators generated by differentiation. The next five chapters apply the methods developed to ordinary differential operators, model operators, and proper operators. The final chapter presents a special operational calculus. Further developments are added at the end of the English edition in the form of an appendix.

The problems considered are extremely difficult, and this book only attempts to make a significant beginning. There is much that remains to be done before there is the possibility of formulating a general theory.

Both authors are to be commended for contributing worthwhile additions to the mathematical literature.

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