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Lie groupoids and Lie algebroids in differential geometry, by Kirill Mackenzie. London Mathematical Society Lecture Note Series, vol. 124, Cambridge University Press, Cambridge, 1987, xv + 327 pp., $34.50. ISBN 0-521-34882-X

It is worthwhile to first examine what the author has to say about groupoids and about his book:

> The concept of groupoid is one of the means by which the twentieth century reclaims the original domain of application of the group concept. The modern, rigorous concept of group is far too restrictive for the range of geometrical applications envisaged in the work of Lie. There have thus arisen the concepts of Lie pseudogroup, of differentiable and of Lie groupoid, and of principal bundle—as well as various related infinitesimal concepts such as Lie equation, graded Lie algebra and Lie algebroid—by which mathematics seeks to acquire a precise and rigorous language in which to study the symmetry phenomena associated with geometrical transformations which are only locally defined.
>
> This book is both an exposition of the basic theory of differentiable and Lie groupoids and their Lie algebroids, with an emphasis on connection theory, and an account of the author's work, not previously published, on the abstract theory of transitive Lie algebroids, their cohomology theory, and the integrability problem and its relationship to connection theory. [p. vii]

> The primary aim of this book is to present certain new results in the theory of transitive Lie algebroids, and in their connection and cohomology theory; we intend that these results establish a significant theory of abstract Lie algebroids independent of groupoid theory. As a necessary preliminary, we give the first full account of the basic theory of differentiable groupoids and Lie algebroids, with emphasis on the case of Lie groupoids and transitive Lie algebroids. [p. ix]

> An important secondary aim of these notes is to establish that the theory of principal bundles and general connection theory is illuminated and clarified by its groupoid formulation; it will be shown in Chapter III that the Lie theory of Lie groupoids with a given base is coextensive with the standard theory of connections. [p. vii]

> Indeed it should perhaps be emphasized that this is a book about the general theory of connections, since this may not be fully evident from a glance at the table of contents. General connection theory has traditionally taken place on principal bundles, but we argue here that the proper setting for much of connection theory is on a Lie algebroid, and that the relationship between principal bundles and Lie algebroids is best understood by replacing principal bundles by Lie groupoids. [p. xi]

The above passages do portray rather accurately the book's aims as well as its content. The author's presentation is detailed, thorough and pleasant to read despite the fact that the subject matter requires lengthy technical elaborations, the latter being counterbalanced by many examples. Another pleasant feature is a kind of "personal touch" by which the author occasionally comments on his previous work and shows how certain ideas matured over the years. Moreover, he provides a wide spectrum of references linking permanently the matter under discussion with the work of other authors. One has in fact the impression that the author is aware of almost every piece of information ever written on groupoids, algebroids, and related topics. As for the criticisms, they mainly reflect the reviewer's personal taste. Given the initial aims of the author's work—in particular, proving some version of Lie's third theorem—and given the cohomological techniques to be developed and used, it is understandable that the book should center on transitive groupoids. Nevertheless, it seems a pity that so little attention is devoted, in a book, to the intransitive situation. In fact, general (intransitive) groupoids seem far more relevant in geometrical applications and some beautiful constructions devised by Pradines [**8**] and later by Almeida [**1**] are thus almost entirely disregarded. Furthermore, contrary to the author's claims, the book seems too much "bundle-like" and, contrary to the reviewer's taste, too much "connection-like". Bundles are of course extremely useful objects but, as Ehresmann would probably say, groupoids are somehow closer to the truth. As for connections, they are an extremely useful algorithm whereas groupoids (and algebroids) are an extremely useful concept.

Since this is a first publication, in book form, on the theory of differentiable and Lie groupoids, it seems appropriate to give a brief account of how this book came to existence.

Differentiable groupoids first appeared in 1951, in connection with Ehresmann's theory of jet spaces [**4**]. Vested as groupoids of invertible jets, their fundamental relevance in several domains (e.g., connection theory, foliations, partial differential equations, Lie pseudogroups, etc.) was of course demonstrated by Ehresmann, who subsequently defined the abstract notion of differentiable category and groupoid and developed the basic theory for these new objects [**5**].

Differentiable groupoids generalize Lie groups (the multiplication is only required to be partially defined); hence, the first natural step in trying to investigate the structure of such objects consists of introducing a notion, the Lie algebroid extending that of Lie algebra, and trying to develop a basic Lie theory that would establish the relationship between the category of differentiable groupoids and that of Lie algebroids. In other words, it consists in investigating the validity of Lie's second and third fundamental theorems in the extended context, the first theorem reducing to a sound definition of the Lie algebroid associated to a differentiable groupoid.

Needless to say, many authors contributed towards the maturing of these ideas, though being more concerned with Lie pseudogroups. The basic contribution, specifically for differentiable groupoids, is due to Pradines who, during the years 1966–1968, published four notes [**8**] laying solid foundations for such a Lie theory and claiming the validity of Lie's third theorem. It is worthwhile to mention that this Lie theory was far from being routine work and gave signs of considerable resistance to study right from the beginning. For instance, it was pointed out at that time, by the reviewer, that Lie's second theorem did not hold—even for groupoids of jets—in its standard formulation except when certain transitivity conditions were fulfilled. Pradine's first note [**8**(a)] resolved the difficulty by introducing the holonomy groupoid associated to a given microdifferentiable groupoid. The other three notes then paved the way, via beautifully designed constructions, towards the highly desired third theorem.

These four notes being published, the much expected full account never came to print and, in the subsequent years, the subject somehow passed into oblivion. It thus remained until about 1976 when, coincidentally, two young mathematicians—Almeida in São Paulo and Mackenzie in Monash—working towards their doctoral dissertations, decided to give it a new try [**1**, **6**]. Both faithfully believed in the validity of Lie's third theorem and both, fortunately, failed in their objectives. However, their work substantially contributed towards bringing new insight and clarity to the subject, as well as developing important new techniques. Almeida's work was more directly inspired by Pradines' notes [**8**] while Mackenzie's was inspired by van Est's cohomological approach to the integrability problem of infinite-dimensional Lie algebras [**9**]. Whereas Almeida stumbled on the globalization, via a universal groupoid construction, of a *piece* of differentiable groupoid, Mackenzie stumbled on the nonvanishing of certain cocycles.

As for the ultimate outcome, let us see again the author's words:

> For many years the major outstanding problem in the theory of differentiable groupoids and Lie algebroids was to provide a full proof of a result announced by Pradines (1968b) [[**8**(d)]], that every Lie algebroid is (isomorphic to) the Lie algebroid of a differential groupoid. This problem was resolved recently in the most unexpected manner by Almeida

> and Molino (1985) [[**2**]] who announced the existence of transitive Lie algebroids which are not the Lie algebroid of any Lie groupoid... The examples of Almeida and Molino (1985) arise as infinitesimal invariants attached to transversally complete foliations, and represent an entirely new insight into the subject. [p. 259]

The next step consisted, of course, in reexamining the cohomological data and pinpointing the obstruction. This work was carried out independently by Almeida and Molino [**3**] and by Mackenzie (Chapter V of the book under review), as discussed in the following passages:

> We now construct a single cohomological invariant, attached to a transitive Lie algebroid on a simply-connected base, which gives a necessary and sufficient condition for integrability. The method is from Mackenzie (1980) [[**7**]], which gave the construction of the elements here denoted e_{ijk} and the fact that if the e_{ijk} lie in a discrete subgroup of the center of the Lie group involved, then the Lie algebroid is integrable. (In particular, a semisimple Lie algebroid on a simply-connected base is always integrable.) However in Mackenzie (1980) the author believed that sufficient work would show that the e_{ijk} could always be quotiented out. [p. 259]

> Ce résultat a été obtenu de façon indépendante par nous et [dans le cas où W est simplement connexe] par K. Mackenzie. En réalité, une bonne part de la construction de $\chi(\underline{L})$ se trouvait dans les travaux de K. Mackenzie [**8**], [**9**], [[**6**, **7**]], et ce qui l'avait empêché d'avancer jusqu'à la conclusion était sans doute essentiellement la croyance à la validité du "3^e théorème de Lie". Dès que l'on "recollait" ses travaux avec nos remarques, le résultat s'imposait. Nous en donnons ici une version détaillée et, nous semble-t-il, particulièrement élémentaire. [**3**, p. 41]

> With the discovery of counterexamples to the general result by Almeida and Molino (1985), it is easy to see that the e_{ijk} form a cocyle; it should be noted that Almeida and Molino independently made this observation for the corresponding elements in Mackenzie (1980). The method now yields a cohomological obstruction to the problem of realizing a transitive Lie algebroid on a simply-connected base as the Lie algebroid of a Lie groupoid. [p. 259]

It is of course disconcerting to find that reference [**3**] is not recorded in the book.

The existence of an obstruction to the integrability of Lie algebroids might seem a serious drawback to the theory. However, mathematics has its own magic and what indeed is occurring is quite the contrary. The overall meaning and implications of the obstruction are rapidly giving rise to entirely new lines of research. This was already outlined by Almeida and Molino [**3**] and, quite recently, confirmed by Weinstein [**10**]. The publication of the book is therefore very opportune and timely.

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Inverse spectral theory, by Jürgen Pöschel and Eugene Trubowitz. Academic Press, Orlando, 1987, x + 192 pp., $29.95. ISBN 0-12-563040-9

Inverse spectral theory and its close relative inverse scattering theory form an active branch of mathematics. The basic question addressed is: given certain data of spectral or scattering type, can one deduce what the underlying operator which governs the process is? Most commonly this becomes the question of determining an unknown coefficient function in operators of a given form. The unknown might be a potential or a mass distribution or even the shape of a scattering obstacle. Such problems are notoriously hard. They are sometimes ill-posed and the solutions may not be unique. Typically the problem has two parts: (I) to characterize the set of data which might arise from a given class of operators and (II) to find which coefficients give rise to a particular admissible data point.