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Asymptotic theory of statistical inference, by B. L. S. Prakasa Rao. John Wiley and Sons, New York, Chichester, Brisbane, Toronto, Singapore, 1987, xiv + 438 pp., $49.95. ISBN 0-471-84335-0

Statistics, generally speaking, addresses the problem of how to determine from data knowledge of the underlying mechanism, presumed random, which produces that data. Usually the mechanism is idealized as a probability law which is assumed to belong to a collection of possible laws. If we have abundant data, we expect that we can determine fairly accurately the unknown law, or some aspect of it, say the mean, μ, in which we are interested. The asymptotic theory of statistical inference is the study of how well we may succeed in this pursuit, in quantitative terms. Any function of the data, when the amount of data is n, is called a "statistic" or estimator $\hat{\mu}(n)$ of, e.g., the mean μ. The sequence $\{\hat{\mu}(n)\}$ is said to be consistent for μ if $\hat{\mu}(n)$ converges to μ as n goes to infinity. The sequence is said to be asymptotically normal (regrettably, language is abused this way) if $\hat{\mu}(n) - \mu$ can be normalized so that the law of the resulting sequence converges to a normal distribution. Proofs that particular estimators have these and other nice properties in various versions and settings comprise much of the work of classical and modern asymptotic statistics.

In purely mathematical terms, the subject is about convergence of sequences of functions or measures in various senses; in particular its tools are drawn from that part of real analysis and measure theory called probability theory.

Until rather recently, some would say "classically", a large portion of probability theory dealt with operations on sequences of independent random variables, and statistical models assumed that data consisted of sequences of independent observations. As probability theory began to focus on other processes—Markov processes in the 50s and 60s, and stationary time series in the 60s and 70s, mathematical statistics began to deal with models where observations were assumed to follow these patterns.

The last ten or fifteen years have produced a strong thrust of activity in several areas associated with stochastic processes: stochastic integrals, stochastic analysis, stochastic differential equations, weak and strong convergence of stochastic processes, etc. A class of processes receiving a lot of attention is the very broad class called semimartingales. During the same time period there has been a burst of activity, partly in response to computing power and convenience, in techniques of data analysis, statistical software packages, and adaptive statistical procedures. The subject of asymptotic statistics, buoyed up, perhaps, by the prosperity of its neighbors, has taken off energetically in a number of fresh directions.

In such a situation it is a daring step to write a book whose stated aim is to bring up to date the interface between probability theory and asymptotic statistics. It is indeed to throw large stones into a torrent with the aim of providing some kind of bridge.

Prakasa Rao's first and longest chapter collects a variety of probability topics, each introduced with several lines of orientation. A number of points receive special emphasis. One is recent work of Sheu and Yao on a moment inequality for embedding times. Another is the extensive development in the related areas of absolute continuity and contiguity of measures. The exposition extends from Kakutani's well-known result to recent work of Liptser and Shiryayev and others on contiguity of stochastic process measures. The topics in this chapter are indeed primarily concerned with stochastic processes even though the statistical content of the book is, as stated in the preface, mostly about independent data as opposed to more general stochastic process data. It is implicit here that the potential for use of these topics in statistics is far ahead of their exploitation. Particularly on this account, it would have been convenient to have some forward indexing in the form of additional notes in the "Remarks" which appear at the end of each section, telling us where in this book or elsewhere statistical application or significance of the probability results may be found.

The remaining chapters are about asymptotic statistics, and here Prakasa Rao has done a large job in assembling and selecting pieces of work from an enormous literature spanning the fifteen-year period beginning about 1971. As a guideline for emphasis he has used insights gained through his own research on several topics. As a pattern of exposition for each topic, he has selected a particular author or authors whose writing lends itself to presentation in book form, whose methods are attractive and innovative and whose work is important for that topic, and presented that person's work as a kind of feature article. In this way he has managed to cover an impressive array of topics without getting bogged down with different approaches and interplays. The expense is that there is little integration or amalgamation of the work presented or of the literature. We have, for instance, Strasser on global and local asymptotic bounds for risk, Sweeting on maximum likelihood estimation for processes, Deshayes and Picard on a particular change-point problem, Khmaladze on goodness-of-fit and so on. Each of these works represents a large development. The collection is extensive and provides a useful introduction to many topics and access to their literature. Separate reference lists at the end of each chapter and large author and subject indices make this a source-book for orientation and reference.

There are occasional signs that the author's own review of the literature may not be very thoroughgoing. For example, following Moore [**2**] he points out a difficulty about chi-square tests where bins are based on an estimated parameter. But this problem has been resolved by Dzhaparidze and Nikulin [**1**], and recent literature in this direction is not explored.

Even a cursory study of the book will yield an impression of the magnitude of the task it attempts and also of areas where research effort is especially invited. It is evident, for instance, that the exploitation of results about rate of convergence and of "martingale" methods for proving weak convergence and for constructing statistics has only just begun.

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Invariant manifolds, entropy and billiards; smooth maps with singularities, by Anatole Katok and Jean-Marie Strelcyn, with the collaboration of F. Ledrappier and F. Przytycki. Lecture Notes in Mathematics, vol. 1222, Springer-Verlag, Berlin, Heidelberg, New York, London, Paris, Tokyo, 1986, viii + 283 pp., $23.60. ISBN 0-387-17190-8

Many dynamical systems arising in physics, meteorology, chemistry, biology, engineering and other fields exhibit chaotic behavior. There is no precise definition of "chaos"; however, in simple terms, chaotic behavior means that a typical orbit seems to wander aimlessly in the phase space with no identifiable pattern and its future is unpredictable although the system itself is deterministic in nature. The only known cause of chaos is hyperbolicity. Suppose we begin moving along a hyperbolic orbit with the speed prescribed by the system and observing the relative motion of nearby orbits that start on a codimension 1 transversal to our orbit. Then in an appropriate coordinate system the relative motion up to first-order terms will be the same as in a neighborhood of a saddle point $\dot{x} = \Lambda x$, $\dot{y} = My$. The eigenvalues of Λ have strictly negative real parts; the eigenvalues of M have strictly positive real parts. These real parts are called Lyapunov characteristic exponents (LCEs) and give us the exponential rates with which nearby trajectories move to or away from our orbit. If all orbits are hyperbolic, all LCEs are uniformly separated from 0 and all estimates are uniform, then we have an Anosov system; a good example is the geodesic flow on a compact surface of curvature -1. D. Anosov and Ya. Sinaĭ studied such systems about 20 years ago. They constructed invariant families (or foliations) of stable and unstable manifolds and used them to prove ergodicity (i.e., chaos) for Anosov systems preserving an absolutely continuous measure. In the mid-70s Ya. Pesin generalized the whole theory for smooth nonuniformly hyperbolic dynamical systems, i.e., systems for which LCEs are not bounded away from 0 and some may actually equal 0. He followed a similar path by constructing and using the stable and unstable manifolds and proved that the measure-theoretic entropy equals $\sum_j \int \mu_j \, dm$, where the μ_j's are positive exponents and m is an absolutely continuous invariant measure. Later D. Ruelle, R. Mane and others simplified and generalized some of Pesin's results.

In their book A. Katok and J. M. Strelcyn generalize the Pesin theory for the case of a dynamical system with singularities. An example of such a system and one of the main motivations for the book is a billiard system, i.e.,