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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 18, Number 1, January 1988

0273-0979/88 $1.00 + $.25 per page

Modern geometry—methods and applications. Part II, *The geometry and topology of manifolds*, by B. A. Dubrovin, A. T. Fomenko, and S. P. Novikov, Graduate Texts in Mathematics, vol. 104, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1985, xv + 430 pp., $54.00. ISBN 0-387-96162-3

This is the second volume of an excellent series of books on modern aspects of geometry. A review of the English translation of Part I of the series, also published by Springer-Verlag, appeared in this Bulletin, vol. 13 (1985), 62–65. In this volume the modern conceptions and ramifications of manifolds are treated. Perhaps no other concept in modern mathematics has been so central to the development of mathematics in this century. This volume tries successfully to take a straightforward, wide-angle view of this immense and intricate topic.

The volume differs from Part I in that prime billing is given to the uses of the relevant geometry and topology described, in other areas of mathematics, rather than science.

Thus, topics in Chapter 3 tied in with other areas in mathematics include the Poincaré-Bendixson theorem in differential equations, the notions of Lie groups and Lie algebras, Riemann surfaces and the maximum modulus principle in complex analysis.

In Chapter 7 we are treated to an excursion and to the applications of manifolds in dynamical systems, Hamiltonian mechanics, and the calculus of variations.

To begin with the table of contents, we note that the book is divided into eight chapters, with Chapter 1 describing in a very authoritative way down-to-earth examples of manifolds: Riemann surfaces, Lie groups, symmetric spaces, and vector bundles.

Chapter 2 passes on to the questions concerning functions on manifolds. Chapter 3 follows this idea further by outlining the notion of the degree of mappings between two manifolds and the various applications of this idea as well as the material outlined above in the first paragraph. Chapter 4 treats covering spaces and the fundamental group in an intuitive and clear-cut manner with numerous examples taken from analysis and geometry. This chapter also discusses the calculation of the fundamental group, discrete groups of motions on constant curvature surfaces, and an introduction to homology groups.

Chapter 5 discusses the higher homotopy groups, the suspension map, and the calculation of the homotopy groups of spheres. Chapter 6 passes on to the discussion of higher homotopy groups and the differential geometry of fiber bundles. Again clear intuitive ideas are stressed, and the connection with various topics in mathematical physics is mentioned. For example, this book does not hesitate to mention simple, special cases from mathematical physics such as the relationships of curvature, Maxwell's equations, and the electromagnetic field tensors. Thus the discussion of characteristic classes given in this book is much more transparent than in more axiomatic versions. The chapter ends with a discussion on knots, links, and braids.

Chapter 7 treats various problems in nonlinear dynamics of both finite and infinite dimensional systems. Hamiltonian dynamics is stressed, with Liouville's theorem on complete integrability given prominence, as well as the famous Korteweg–de Vries equation from an infinite-dimensional Hamiltonian viewpoint.

Chapter 8 is a tour de force of good writing in advanced mathematical physics. It is entitled *The global structure of solutions of higher-dimensional variational problems.* Consistent with the book's geometric approach, this chapter starts out with the general theory of relativity in a geometrical formulation based on Einstein's equations and their geometric meaning. First, spherically symmetric solutions are discussed, including those of Schwarzchild and Kruskal. This chapter then passes on to axially symmetric solutions including the famous Kerr metric for rotating black holes. The chapter then passes on to a discussion of various cosmological models from a geometric point of view. The calculus of variations is stressed in the next section, involving gauge theories and global geometric versions of the Yang-Mills equations. Self-duality symmetry in four dimensions is described and linked with the geometry of previous chapters. The chapter then passes on to a discussion of chiral fields and the Dirichlet integral. This section has a number of surprises, ending with the nonlinear sine Gordon equation.

Thus, the book surveys many both important and interesting topics, but has a number of glaring omissions. Perhaps the most important is that no geometrical discussion of hydrodynamics is attempted. This is a nonlinear geometrical field par excellence. However, it is normally omitted from the conventional mathematicians' and physicists' education. A pity!

This book is a remarkably up-to-date view of the geometry and topology of manifolds. It omits most connections between analysis and the topics discussed. For example, nonlinear analytic aspects of geometry are given short shrift. Yet, the book is a remarkable modern document covering major geometrical ideas and their applications in modern mathematics, taken in the broadest sense.

I tried using this text in a graduate course on nonlinear dynamics. It proved to be too difficult for students, so it seems best for researchers and possibly advanced graduate courses in geometry.

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