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WILLIAM F. REYNOLDS

BULLETIN (New Series) OF THE
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Sobolev spaces, by Vladimir G. Maz'ja. Translated from Russian by T. O. Shaposhnikova. Springer-Verlag, Berlin, Heidelberg, New York, and Tokyo, 1985, xix + 486 pp., $59.00. ISBN 3-540-13589-8

Vladimir G. Maz'ya is one of the foremost authorities on the subject of Sobolev spaces. However, many of his papers are published in Soviet publications with very limited circulation. As a result he has sometimes suffered the misfortune of seeing his results rediscovered or attributed to others. With the appearance of the book under review, this state of affairs should belong to the past. (For other books by the same author, see the review by David R. Adams in this Bulletin, **15** (1986), 254–259.)

In the present book the author has collected and rewritten the results of many years of research by himself and his collaborators. He has added introductory material, and results due to others, but most of the contents of the book are due to the author himself. Naturally there is very little overlap with other existing books on Sobolev spaces.

An earlier version of the book appeared in German in three volumes [**1**, **2**], but the new book is considerably expanded. A Russian version [**3**] was published almost simultaneously, which speaks well of the alertness of Springer-Verlag. The English translation is due to T. O. Shaposhnikova, who is a mathematician in her own right. As far as this reviewer is able to judge a language that is not his own, the translation reads very well.

If $\Omega \subseteq \mathbf{R}^n$ is open, $p \geq 1$, and m is a positive integer, the Sobolev space $W^p_m(\Omega)$ consists of those functions in $L^p(\Omega)$ whose weak partial derivatives (i.e., derivatives taken in the sense of distributions) of order $\leq m$ also belong to $L^p(\Omega)$. The space can be equipped with the norm

$$\|f\|_{W^p_m(\Omega)} = \sum_{|\alpha|\leq m} \|D^\alpha f\|_{L^p(\Omega)},$$

and it is then a Banach space. One of the basic results in the theory is that for any Ω the space so obtained is the closure of $L^p(\Omega) \cap C^\infty(\Omega)$ with respect to the norm $\|\cdot\|_{W^p_m(\Omega)}$.

The motivation for the introduction and study of Sobolev spaces comes from the theory of partial differential equations and can be traced back to the justification of Dirichlet's principle by Hilbert and Lebesgue in the first years of the century. (See, e.g., the book by C. B. Morrey, Jr. [**4**] for interesting historical remarks.)

One of the fundamental theorems about Sobolev spaces is the Sobolev embedding theorem, proved by S. L. Sobolev in 1938. This theorem states that if $f \in W_m^p(\Omega)$, where $\Omega \subseteq \mathbf{R}^n$ is a domain that satisfies a so-called cone condition, and $1 < p < n/m$, then $f \in L^{p^*}(\Omega)$, where $1/p^* = 1/p - m/n$. If $p > n/m$, then $f \in C(\Omega)$, and if $p = n/m$, then $f \in L^q(\Omega)$ for all $q < \infty$. (The theorem was later extended to $p = 1$ by L. Nirenberg and E. Gagliardo.)

Generalizations and refinements of this theorem are one of the main themes of the book under review. The author is especially interested in finding necessary and sufficient conditions on the domain Ω for the validity of various embedding theorems. For $p = 1$ such conditions can be given in terms of isoperimetric inequalities, relating the volume and the surface area of portions of the domain. For $p > 1$ the area has to be replaced by "p-capacity".

Another type of extension theorem is obtained if the domain is allowed to be all of $\mathbf{R}^n$, but embeddings into spaces $L^q(\mu)$ are considered for positive measures μ. The measures allowing such embeddings are characterized in terms of capacities. These results are in part due to D. R. Adams.

An interesting and useful chapter, written jointly with Yu. D. Burago, treats spaces of functions of bounded variation, i.e., functions whose derivatives are measures.

Generally speaking, this book is not the right choice for someone who is just trying to learn a few simple facts about Sobolev spaces. The author's taste is for completeness. He treats every conceivable aspect of his problems, which makes the book rather overwhelming for the general reader.

On the other hand, this makes the book all the more valuable as a work of reference. It is a treasure house, for example, for someone who is looking for a weird domain as a counterexample to some theorem, and for many others. Every good mathematical library should have it.

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LARS INGE HEDBERG

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Complex cobordism and stable homotopy groups of spheres, by Douglas C. Ravenel, Academic Press, Orlando, 1986. xix + 413 pp., \$90.00 cloth, \$45.00 paperback. ISBN 0-12-583430-6

The author has previously written [**4**, p. 407]: "I am painfully aware of the esoteric nature of this subject and of the difficulties faced by anyone in the past who wanted to become familiar with it." The subject in question is the topic of the book under review, namely the study of stable homotopy theory