(2) The approach of Esnault-Viehweg and Kollår towards vanishing theorems, which is related to the above-mentioned connection of weak Lefschetz and vanishing theorems.

(3) The "non-vanishing theorem" of Shokurov and the application of Kawamata et al. to $\bigoplus_{m \geqslant 0} H^q(X, K_X^m)$.

Third, the book deals only with the compact theory, the noncompact theory being mentioned only in the references.

In spite of these remarks, this book should have many readers since various mathematical fields come together in an exciting way: real analysis, complex differential geometry, complex analysis and algebraic geometry.

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Singularities of differentiable maps, Vol. 1. *The classification of critical sets, caustics and wave fronts*, by V. I. Arnol'd, S. M. Gusein-Zade, and A. N. Varchenko, translated from Russian by Ian Porteous. Birkhäuser, Boston, Basel and Stuttgart, 1985, x + 382 pp., $44.95. ISBN 0-8176-3187-9.

Singularity theory is a subject of comparatively recent origin, and spans a wide range of disciplines: one may study singularities of differentiable or of holomorphic maps, or of algebraic varieties, under differentiable or topological equivalences: thus topology, complex analysis, and algebraic geometry all play a part.

Although the theory has important global aspects, it is dominated by local considerations, and we will focus on these. The central object of study is the germ at a point of a C^∞-map between smooth manifolds. One commonly takes local coordinates in source and target: then the Taylor expansion of the map defines a k-jet (truncating the expansion at terms of degree k), and the germ is said to be k-determined (for the equivalence relation E) if all germs with the same k-jet are E-equivalent to it. The most important equivalence relations are defined by local diffeomorphisms of source or target or both; one may also wish to consider homeomorphisms. This idea, of approximating the infinite-dimensional space of germs by the finite-dimensional spaces of jets, is central to the whole subject.

A map is *stable* if all nearby (in the C^∞-topology) maps are equivalent to it. The equivalence may be by diffeomorphism of source and target, by homeomorphism, or many other choices. There are corresponding local notions; their precise definitions are rather technical.

Singularity theory has its origins in papers of Whitney and Thom: the latter full of ideas and proposals but not easy to follow at the time (1955–1965) when they were written. Towards 1970 several major developments gave this area of mathematics a new impetus and cohesion.

Milnor's book on topology of hypersurface singularities, while drawing extensively on the literature, had a major popularizing effect. The central topic of the book is now known as the Milnor fibration; the curve selection lemma was thrown into a key position.

Mather's work on stability of C^∞-mappings in a profound and far-reaching series of papers opened up a whole new range of ideas: finite determinacy, versal unfoldings in a sense more general than that of Thom, multitransversality as a condition for stability, and large numbers of examples. The equivalence of "stability" and "infinitesimal stability" was merely one aspect of this work.

Arnol′d's survey article [**1**] was also influential: it was more accessible than Mather's work and overlapped substantially with it. Tougeron's work on ideals of differentiable functions, building on but substantially extending that of Malgrange, also overlapped significantly with Mather's papers.

At about the same time the work of Grothendieck, Griffiths, and Deligne on integrals and monodromy culminated in the monodromy theorem and the study of limit mixed Hodge structures, with the so-called nilpotent orbit theorem and SL_2-orbit theorem. Brieskorn gave an important account of monodromy in the context of isolated singularities. Brieskorn also gave an account of the relation between du Val singularities and algebraic groups.

The 1970s was a period of consolidation. The Moscow school of Arnol′d and collaborators developed the theory of isolated singularities of complex functions. Enumerations, when interpreted in terms of modality, revealed characterizations of interesting classes of examples (simple, simple elliptic, cusp, ...) which had alternative characterizations in terms of uniformization or of signatures; and also from the viewpoint of elliptic Gorenstein normal surface singularities. The relation between asymptotic expansions of oscillatory integrals and monodromy of the corresponding singularities was explored by a succession of authors.

Mather's work on deformation and unfolding theory for mappings and on determinacy theorems was systematized and popularized by several workers, and his estimates enormously improved—notably by Gaffney, who also gave geometric interpretations using sheaf theory. An account of this, with full references, was given in a survey [**7**] by the reviewer.

There were also major advances in the theory of topological equivalence. The density of topologically stable maps was proved by Mather, fleshing out a programme proposed by Thom; as well as the development of (Whitney) stratification theory and the proof of "isotopy lemmas," this demanded a close understanding of the relation between source and target singularities, the theory of C^∞-stable unfoldings, and a good deal more. On a more local level, an early criterion of Kuo for topological determinacy of functions was built (notably by Wilson) into an extensive theory of topological determinacy (again see [**7**]).

More recent work has expanded the scope of the theory in several quite different directions: as illustrative examples I cite the study of representation type of categories of reflexive sheaves over isolated singularities and the extension of Mather's theory to the equivariant bifurcation situation.

I now come to the book under review. This is a translation of the Russian original, which was written in 1979, so the absence of recent results is inevitable. The translation itself is good: the book reads as though written in English. It is divided into three rather different parts.

Part I, entitled "*Basic concepts*", is a general introduction to the subject. Its scope is revealed by the chapter headings: the simplest examples, the classes Σ^I (the Thom-Boardman symbol), the quadratic differential, local algebra and the Weierstrass preparation theorem, local multiplicity, stability and infinitesimal stability, proof of the stability theorem, versal deformations, classification of stable germs, review of further results. This is pretty much the same selection of material as in the other books on the subject [**2**, **3**, **4**, **5**]; while the others have their value, I feel the book under review has more style. The reader is introduced to the subject by simple examples; important concepts are introduced in successive chapters in a "user-friendly" manner—there are ample illustrative examples, clear discussion and well-laid-out proofs. (I note in parenthesis that Sard's theorem is proved here; the differentiable preparation theorem is not.) The discussion of local degree goes further than the other books, with useful treatments of the Eisenbud-Levine theorem and of the Grothendieck residue (here called the symbol).

Part II is devoted to the classification of critical points of smooth functions. The treatment, and also the choice of material, closely follows those papers of Arnol'd in the Russian Math. Surveys from 1973–1978. What is here is well presented, but one might have hoped for a broader viewpoint in such a book; for example, I regret the absence of any discussion of the geometry of the singularities of modality $\leqslant 2$, to which only brief reference is made here.

The inclusion of Part III, on caustics, wave fronts, etc., in such a book is characteristic of the Russian school. Indeed, Professor Arnol'd is celebrated for his earlier book on Hamiltonian mechanics, which was one of his main sources of interest in this whole area. The connections with differential geometry go back to Thom.

The cotangent bundle T^*M of any manifold M has a canonical symplectic structure ω; if $L \subset T^*M$ is a Lagrangian submanifold (i.e. $\omega \mid L = 0$ and $\dim L = \dim M$) one studies the projection of L on M. A normal form is obtained using a "generating function," so the classifications of Part II can be applied here to classify the singularities of Lagrangian maps. These, and a modified theory (Legendrian, using contact structures) provide the right context for a mathematical description of singularities of caustics and wave fronts. This is all explained in detail and illustrated by pictures and examples.

The present volume is the first of a two-volume treatise, and the translation of the second should appear shortly (it is in proof stage at the time of writing). The reviewer has not yet seen this, but looks forward to doing so: volume 2 is to include the complex-analytic and algebro-geometric aspects (monodromy, intersection theory, asymptotics of integrals and mixed Hodge structures).

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The higher calculus: A history of real and complex analysis from Euler to Weierstrass, by Umberto Bottazzini; translated by Warren Van Egmond. Springer-Verlag, New York, Berlin, Heidelberg, London, Paris, Tokyo, 332 pp., $39.00. ISBN 0-387-96302-2

This book is the second, much revised and augmented edition of one first published in Italian [**1**]. The first edition was good, and this one is better. The subject is not really analysis as a whole but the foundations of analysis, the origins of concepts and rigorous proofs, by no means devoid of examples to show how need for change arose and how new modes of thought developed. Of course Bottazzini makes good use of his few recent predecessors' works, for example [**2**], which covers a greater range, and [**3**, **4**], which suffer from their authors' lack of experience in mathematics itself and in mathematical ways of thinking, and [**5**], which treats only the concept of function. Bottazzini's book is much better than [**2**, **3**, **4**], for he speaks with authority, understands and treats fairly his sources, quotes neither too much nor too little, and writes compactly yet with precision. He lets his authors speak for themselves to a great part, aiding the reader to pass from one quotation or paraphrase to the next by brief yet informative transitions, and at the ends of many sections are excellent summaries in a few well chosen words, free of the pontifications in unsupported generalities that often deaden academic theses and writings by authors still close to them.

A standard defect in historical writings on mathematics comes from their authors' failure to see that the sources of pure mathematics often lie in works that today's mathematicians would consider to be "applied" mathematics or "physics". This defect damages most severely the researches of the eighteenth century, in which "applied" mathematics had not been invented, and mathematics was divided into "pure" and "mixed"; in Samuel Johnson's words, "pure considers abstracted quantity...; mixt is interwoven with physical considerations." Another is the writers' tendency to assume that rigor was sought for rigor's sake, which while true of some works of some mathematicians was not at all characteristic of the search for and achievement of rigorous