REFERENCES

1. A. Borel, *Sur la cohomologie des espaces fibrés principaux et des espaces homogènes de groupes de Lie compacts*, Ann. of Math. (2) **57** (1953), 115–207.

2. G. Hochschild and J.-P. Serre, *Cohomology of group extensions*, Trans. Amer. Math. Soc. **74** (1953), 110–134.

3. J. Leray, *L'anneau spectral et l'anneau filtré d'homologie d'un espace localement compact et d'une application continue*, J. Math. Pures Appl. **29** (1950), 1–139.

4. R. C. Lyndon, *The cohomology theory of group extensions*, Duke Math. J. **15** (1948), 271–292.

5. J.-P. Serre, *Cohomologie des extensions des groupes*, C. R. Acad. Sci. Paris **226** (1950), 303–305.

6. ______, *Homologie singulière des espaces fibrés*, Ann. of Math. (2) **54** (1951), 425–505.

W. S. MASSEY

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 16, Number 1, January 1987

0273-0979/87 $1.00 + $.25 per page

Prescribing the curvature of a Riemannian manifold, by Jerry L. Kazdan, CBMS Regional Conference Series in Mathematics, vol. 57, American Mathematical Society, Providence, R. I., 1985, vii + 55 pp., $12.00. ISBN 0-8218-0707-2

It is generally believed that a most interesting area in nonlinear partial differential equations lies in the study of special equations, particularly those arising from geometry and physics. The monograph under review is an account of problems on the existence of a Riemannian metric with given curvature conditions. It contains some of the most important results in mathematics in recent years.

There are all kinds of curvatures: Gaussian curvature, scalar curvature, Ricci curvature, Riemannian sectional curvature, etc. When any one of them is prescribed, we get a system of partial differential equations on the fundamental tensor of the Riemannian metric. The problems have a meaning for a manifold without boundary, giving rise to some problems attractive because of their simplicity. But Chapter IV of these notes gives a treatment of some recent developments on boundary-value problems.

Even for the Gaussian curvature there are unanswered questions. To be the Gaussian curvature of a compact surface M^2, a function $K \in C^\infty(M^2)$ must satisfy a sign condition forced by the Gauss-Bonnet theorem. Kazdan and Warner proved that this is sufficient. It would be interesting to prove this by a conformal transformation of a given metric g_0 on M^2. There will be no difficulty if the Gaussian curvature K_0 of g_0 is negative. For $K_0 > 0$ there are further necessary conditions and it is not known whether they are sufficient. Even for the two-sphere $M^2 = S^2$ it is not known whether one could obtain a metric of constant curvature through the conformal transformation of a given one.

The simplest generalization of the Gaussian curvature to higher dimensions is the scalar curvature, which is a scalar invariant. By applying the Bochner technique to the Dirac operator, Lichnerowicz showed that if a compact spin manifold M^{4k} of dimension $4k$ has positive scalar curvature, its only harmonic spinor is zero, from which it follows by the Atiyah-Singer index theorem that its $\hat{A}$-genus is zero. Thus positive scalar curvature could have a topological implication. Following a question of Kazdan-Warner, R. Schoen and S. T. Yau, using minimal surfaces, proved that the three-dimensional torus T^3 (as well as many other three-manifolds) does not have a Riemannian metric of positive scalar curvature. M. Gromov and B. Lawson showed that this is also true for the n-dimensional torus T^n, by using the Dirac operator.

Generalizing the two-dimensional case, Yamabe had the idea of attaining a constant scalar curvature metric on a compact manifold M^n of dimension $n \geqslant 3$ by a pointwise conformal transformation. Important progress on this problem, the Yamabe problem, was achieved by T. Aubin and the work was recently completed by R. Schoen. The result is a positive answer to the Yamabe problem.

Concerning the Ricci curvature, the most important result is perhaps the Calabi conjecture, which was proved by S. T. Yau. Given a compact Kähler manifold M, its Ricci form is of type $(1,1)$ and belongs to the first Chern class $c_1(M)$ in the sense of de Rham cohomology. The Calabi-Yau theorem says that any closed form of $c_1(M)$ can be realized as the Ricci form of a Kähler metric of M. The theorem has many consequences. The proof of the theorem can now be simplified, using an interior estimate of L. C. Evans.

Another nice result on the Ricci curvature is the following theorem of R. Hamilton: Let (M^3, g_0) be a compact 3-manifold with positive Ricci curvature. Then there is a family of metrics g_t, $0 \leqslant t \leqslant \infty$, with positive Ricci curvature, where g_∞ is an Einstein metric and hence has constant positive sectional curvature.

The above are some highlights of the monograph. The monograph is informative, up-to-date, and contains a list of open problems. A natural question is: why Riemannian? A study of analogous problems on Finslerian manifolds, which are based on more general variational problems, will not only open to a new horizon, but also give a better understanding of the Riemannian case itself.

S. S. CHERN

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 16, Number 1, January 1987

0273-0979/87 $1.00 + $.25 per page

Infinite dimensional Lie algebras, by Victor G. Kac, second edition, Cambridge University Press, 1985, xvii + 280 pp., $24.95. ISBN 0-521-32133-6

The first edition of this book was published by Birkhäuser-Boston in 1983. It is testimony to the broad interest of the subject, barely twenty years old, and to the merits of the book that 1985 already saw the appearance of a second