connections with classical transforms are discussed. The final Chapter 12 is a study of convolution integral equations involving the H-function of two variables.

The Appendix lists useful formulas for special functions of one and several variables, along with a brief account of an H-function of several variables. The bibliography contains more than one thousand references.

The wealth of material is, in the reviewer's opinion, sensibly selected and arranged. Proofs are not necessarily given; and in some cases the authors deal summarily with large families of formulas by merely deriving the *numbers* of such formulas. Moreover, it should be noted that H-symbols are satisfactorily printed, in spite of their typographical complexity.

The book, providing a comprehensive account of a subject widely scattered in the literature, will be of value to all researchers and students in the field of special functions.

REFERENCES

1. A. M. Mathai and R. K. Saxena, *Generalized hypergeometric functions with applications in statistics and physical sciences*, Lecture Notes in Math., vol. 348, Springer-Verlag, Berlin, Heidelberg and New York, 1973.

2. ______, *The H-function with applications in statistics and other disciplines*, Wiley Eastern Ltd., New Delhi, Bangalore and Bombay, 1978.

PER W. KARLSSON

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 14, Number 1, January 1986

0273-0979/86 $1.00 + $.25 per page

The theory of topological semigroups, by J. H. Carruth, J. A. Hildebrant, and R. J. Koch, Pure and Applied Mathematics, Marcel Dekker, Inc., 1983, vi + 244 pp., $34.75. ISBN 0-8-8247-1795-3

BACKGROUND. Very few subject areas in mathematics can assign their creation to a single individual. Topological semigroups is one of the exceptions. Alexander Doniphan Wallace is universally acknowledged to have fostered the idea of studying continuous, associative multiplications on Hausdorff spaces, and no practitioner could wish for a more colorful father of the subject.

An excellent teacher of graduate students and an able wordsmith, Wallace was the natural choice of his descendants to chronicle the growth of the subject. Regrettably, the continuing press of administrative duties prevented this project from every being seriously begun. An early hint of what might have been is contained in the Bulletin article [4] that summarized his 1955 address to the Society—one of the most cited sources ever in American mathematics publishing.

By the early 1960s, the only book in print on the subject (besides Wallace's jealously guarded course notes) was a Centrum tract by Paalman-De Miranda [3], an effort clearly not intended to serve as a text. The appearance of the first volume of Clifford and Preston's work in algebraic semigroups [1] made the lack of an equivalent book on the topological side even more noticeable. At length, two of Wallace's erstwhile colleagues at Tulane undertook to remedy this shortage. In 1966 the first full-fledged book on topological semigroups, by Karl Hofmann and Paul Mostert, appeared [**2**]. During the period of manuscript preparation, the authors were jointly involved in the creation of an important new theorem (the "second fundamental theorem" of the subject), and chose to include this new material in their book. Inevitably, their work came to be regarded as more of a research monograph than a text. By the early 1970s, twenty years after the birth of the subject, an adequate introductory text still did not exist.

Good graduate-level teaching in mathematics seems often to produce more of the same. Wallace had several students who are adept graduate teachers, R. J. Koch principal among them. Under the urging of J. H. Carruth, the two of them began work to supply the evident need. Shortly thereafter, J. A. Hildebrant enlisted to complete the cast of authors. As in any joint effort, progress was in fits and starts, and a decade passed before intention became fact.

THE BOOK. While it would be inaccurate to characterize this volume as the book Wallace never wrote, Chapter One, on fundamental concepts, reproduces the bulk of the material present in Wallace's course notes, a fact duly acknowledged in the introduction. Gone is the florid language—no more "mobs" or "clans"—but the early results that make the subject appealing to graduate students are all presented herein. The remarkable Swelling Lemma, which states that, in a compact semigroup, closed sets cannot grow under translation action by an element, has a new proof. Nets are ubiquitous, reflecting the year J. L. Kelley spent at Tulane during Koch's graduate student days. The importance of idempotents and their associated maximal semigroups is noted, along with the general reliance on compactness to produce these objects. The standard material on closed congruences has been updated by the inclusion of the work of Lawson and Madison on locally compact, σ-compact semigroups.

The first half of Chapter Two forms an extremely valuable portion of the book, illustrating techniques for constructing both examples and new classes of semigroups. Split extensions, Rees products, inverse limits, and adjunction techniques are all discussed. Several simple but effective line drawings are provided to illustrate low-dimensional examples. The latter half of the chapter discusses duality, compactifications, free topological semigroups, and coproducts. Matters suddenly become intensely homological, in contrast with the set-theoretic flavor of the book up to this point. While this material certainly belongs to the general topic of semigroup constructions, it tempts the neophyte to flee from this sudden hail of arrows launched in his direction. In the reviewer's mind, it could well have been banished to volume two.

Internal structure is the topic of Chapter Three. Monothetic semigroups are done in detail, as are both the algebraic and topological structure theorems for completely simple semigroups. All the relations of Green are discussed at length, and the important bicyclic semigroup is introduced in this chapter. The Schutzenberger group of an H-class is done in its algebraic version. The details of the compact topological version are sketched, but the reader is referred to [2] for further information. In particular, the interesting fact that an H-class of a compact semigroup must be (topologically) homogeneous is not mentioned, although it is implicit in Theorem 3.61.

Chapter Four treats I-semigroups; that is, semigroups with underlying topological space an arc, in which one noncut point acts as identity and the other as a zero element. This material has been completely understood for a long time, and the text mirrors this fact with an extremely telegraphic presentation. The fact that I-semigroups ("threads" in the old days) are so well behaved played a large role in encouraging further research in compact semigroups. Theorem 4.23, due to R. C. Phillips, states that, in a compact semigroup, translation of an I-subsemigroup by an element is either a point or an arc. This is one of the most useful results in the theory.

One of the most difficult theorems in the study of compact semigroups concerns the existence of a local 1-parameter semigroup at the identity of a continuum monoid possessing an (otherwise) idempotent-free neighborhood. The original result, due to Mostert and Shields, required first handling the case in which $H(1)$, the maximal semigroup of the identity, is a Lie group. The mysteries of the generalized limit were invoked, along with Gleason's cross-section theorem. This theorem is the principal point of Chapter 5; however, the argument given here is the much improved version due to Carruth and Lawson in 1970. Character theory is used to whittle $H(1)$ down to circle size. Gone is the generalized limit, replaced by certain uniformity properties that are easy to understand. Putting this proof between hard covers is reason enough for the existence of this volume.

The sixth and final chapter entails an introduction to the theory of compact divisible semigroups, an area of particular interest to one of the authors and one in which a good bit of work has been done in recent years. Cone semigroups and the exponential function are discussed at length, and Hildebrant's work on sub-unithetic semigroups is presented in detail. An allusion to the "backwards gamma" theorem of Anne Lester Hudson is made in the closing remarks of the chapter. This theorem states that a sequence of successive square roots in a compact semigroup has the property that its cluster points all lie in a compact Abelian subgroup whose identity acts as an identity for each member of the sequence. In the opinion of the reviewer, this result has had a great influence on later divisibility research, and deserves both a precise statement and a proof within this chapter.

ORCHIDS AND ONIONS. This book is a very valuable addition to the literature on topological semigroups. Not only does it gather the scattered contributions of thirty years into one volume—it presents them in an organized manner, with clear and sometimes new proofs, and with consistent notation and symbols. Not the least of its virtues is an enormous bibliography, to which the authors devoted much time, attention, and twenty pages. The amount of mathematical material present in this work is considerable. The reviewer had the opportunity to class test it in a one-semester course, skipped the back half of Chapter Two, and still ran out of time before he got to Chapter Six.

On the other side of the coin, this book is still, strictly speaking, not a text. Exercises for the student to worry about are virtually nonexistent, although many proofs leave sufficient gaps to provide challenge. The removal of some peripheral material to exercises would give the book more of a Clifford-Preston flavor, while allowing some contact with every chapter in a one-semester course. Complaints about choice of content should be forestalled until the appearance of a second volume, due out in the near future and promising cohomology, semilattices, Lie semigroups, and other topics of current interest. It strikes the reviewer that cohomology, which has provided the subject with some of its most elegant theorems, would have been well invested in the first volume. Nevertheless, the reviewer believes Wallace would be happy with this book, and in this subject there can be no better compliment.

References

1. A. H. Clifford and G. B. Preston, *The algebraic theory of semigroups*. I, Math. Surveys, 7, Amer. Math. Soc., Providence, R.I., 1961.

2. K. H. Hofmann and P. S. Mostert, *Elements of compact semigroups*, Merrill Books, Inc., Columbus, Ohio, 1966.

3. A. B. Paalman-De Miranda, *Topological semigroups*, Mathematical Centre Tracts, 2nd ed., Mathematiche Centrum Amsterdam, 1970.

4. A. D. Wallace, *On the structure of topological semigroups*, Bull. Amer. Math. Soc. **61** (1955), 95–112.

D. R. Brown

BULLETIN (New Series) OF THE AMERICAN MATHEMATICAL SOCIETY
Volume 14, Number 1, January 1986

0273-0979/86 $1.00 + $.25 per page

Analytic functional calculus and spectral decompositions, by Florian-Horia Vasilescu, Mathematics and its Applications, Volume 1, D. Reidel Publishing Company, Dordrecht, Holland, 1982, xiv + 378 pp., Dfl. 180,–, U.S. $78.50. ISBN 90-277-1376-6

A linear transformation T acting on a finite-dimensional complex vector space $\mathscr{X}$ can always be decomposed as $T = D + N$, where (i) D is diagonalizable and N is nilpotent; and (ii) $DN = ND$; moreover, such a decomposition is unique with respect to the conditions (i) and (ii), and both D and N are indeed polynomials in T. When $\mathscr{X}$ is an infinite-dimensional Banach space, such a representation for a bounded operator T is no longer true, but an important class of transformations introduced and studied by N. Dunford [3] in the 1950s possesses a similar property.

By definition, a *spectral operator* T acting on $\mathscr{X}$ is one for which there exists a spectral measure E (i.e., a homomorphism from the Boolean algebra of Borel subsets of the complex plane $\mathbf{C}$ into the Boolean algebra of projection operators on $\mathscr{X}$ such that E is bounded and $E(\mathbf{C}) = I$) satisfying the following two properties: (1) $TE(B) = E(B)T$; and (2) $\sigma(T|_{E(B)\mathscr{X}}) \subset \overline{B}$, for all B(Borel) $\subset \mathbf{C}$. Such an E is called a *resolution of the identity* for T, and is