set of irreducible unitary representations for each real semisimple G that are attached to the nilpotent G orbits in Lie(G). Experimental evidence indicates that these problems are inextricably connected. Miraculously, their resolution should have implications for automorphic forms (cf. [**1**]). Jantzen has found a subject perfectly suited for an advanced text: one which has reached not the top of the mountain, but a solid ledge with a beautiful view.

REFERENCES

1. J. Arthur, *On some problems suggested by the trace formula*, Lie Group Representations in Harmonic Analysis, II (R. Herb, R. Lipsman and J. Rosenberg, eds.), Lecture Notes in Math., vol. 1041, Springer-Verlag, Berlin and New York, 1984.
2. L. Auslander and B. Kostant, *Polarization and unitary representations of solvable Lie groups*, Invent. Math. **14** (1971), 255–354.
3. D. Barbasch and D. Vogan, *Problems in primitive ideal theory*, Proc. Durham Conf. on Enveloping Algebras (to appear).
4. I. N. Bernstein, I. M. Gelfand and S. I. Gelfand, *Structure of representations generated by vectors of highest weight*, Functional Anal. Appl. **5** (1972), 1–8.
5. W. Borho, *A survey on enveloping algebras of semisimple Lie algebras*, Proc. Windsor Conf. on Lie Algebras and Related Topics (to appear).
6. W. Borho, P. Gabriel and R. Rentschler, *Primideale in Einhüllenden auflösbarer Lie-Algebren*, Lecture Notes in Math., vol. 357, Springer-Verlag, Berlin and New York, 1973.
7. W. Borho and J. C. Jantzen, *Über primitive Ideale in der Einhüllenden einer halbeinfachen Lie-Algebra*, Inven. Math. **39** (1977), 1–53.
8. J. Dixmier, *Algèbres enveloppantes*, Gauthier-Villars, Paris, 1974.
9. M. Duflo, *Sur la classification des idéaux primitifs dans l'algèbre enveloppante d'une algèbre de Lie semi-simple*, Ann. of Math. (2) **105** (1977), 107–120.
10. Harish-Chandra, *Representations of semi-simple Lie groups*. I, Trans. Amer. Math. Soc. **75** (1953), 185–243.
11. J. E. Humphreys, *Introduction to Lie algebras and representation theory*, Springer-Verlag, Berlin and New York, 1972.
12. J. C. Jantzen, *Moduln mit einem Höchsten Gewicht*, Lecture Notes in Math., vol. 750, Springer-Verlag, Berlin and New York, 1979.
13. A. Joseph, *Goldie rank in the enveloping algebra of a semisimple Lie algebra*. I, II, J. Algebra **65** (1980), 269–283, 284–306.
14. ______, *Goldie rank in the enveloping algebra of a semisimple Lie algebra*. III, J. Algebra **73** (1981), 295–326.
15. D. N. Verma, *Structure of certain induced representations of semi-simple Lie algebras*, Dissertation, Yale Univ., 1966.

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Complex analysis, a functional analysis approach, by D. H. Luecking and L. A. Rubel, Universitext, Springer-Verlag, New York, 1984, vii + 176 pp., $16.00. ISBN 0-3879-0993-1

1. Function theory in functional analysis. Many branches of mathematics owe a debt to classical function theory. This is especially true of functional analysis. Here the archetypal application, due to M. H. Stone (in his famous 1932 book

—see A. E. Taylor [**1971**] for history), is the following: Let $\mathscr{B}(\mathscr{H})$ denote the algebra of continuous linear operators on the (complex) Hilbert space $\mathscr{H}$. The equation

$$\|T\| := \sup\{\|Tx\|: x \in \mathscr{H}, \|x\| \leqslant 1\}$$

defines a submultiplicative norm which is complete in $\mathscr{B}(\mathscr{H})$; i.e., it makes $\mathscr{B}(\mathscr{H})$ a unital Banach algebra. The *resolvent set*

$$\rho(T) := \{\lambda \in \mathbf{C}: T - \lambda I \text{ is invertible in } \mathscr{B}(\mathscr{H})\}$$

is open for each T, contains all λ with $|\lambda| > \|T\|$, and in this set $f(\lambda) := (T - \lambda I)^{-1}$ is holomorphic and vanishes at infinity. These facts are very elementary and easy to confirm. If $\rho(T)$ were all of $\mathbf{C}$, f would be an *entire* function, and so by Liouville's theorem it would be identically 0—a manifest absurdity. It follows that the *spectrum* $\sigma(T) := \mathbf{C} \setminus \rho(T)$ is never empty. This (now canonical) proof was later used by I. M. Gelfand [**1941**] to draw the same conclusion with an arbitrary (complex) unital Banach algebra in the role of $\mathscr{B}(\mathscr{H})$. This is the cornerstone of his famous representation theory of commutative Banach algebras.

2. The space of holomorphic functions. On the other hand, the set $H(\Omega)$ of holomorphic functions on a region Ω in $\mathbf{C}$ is a vector space, even an algebra with unit. The topology τ of uniform convergence on compact subsets of Ω renders all the algebraic operations in $H(\Omega)$ (and differentiation as well) continuous, and a classical theorem of Weierstrass assures us that $H(\Omega)$ is complete with respect to this very natural convergence concept. Thus $H(\Omega)$ is a *topological vector space*, even a Fréchet space, since this convergence concept can evidently be realized with a complete translation-invariant metric. In other words, $H(\Omega)$ is an example of the kind of objects studied in linear functional analysis. Since so much knowledge about $H(\Omega)$ was accumulated already in the 19th and early 20th centuries, this is a very useful example both for testing general conjectures and for enriching textbook presentations of linear functional analysis; and, indeed, several aspects of the abstract theory (e.g., Montel spaces) evolved from this example. On the other hand, some of the powerful tools of functional analysis might shed new light on this old friend. It is true that some of these tools, like the Baire category theorem, were prefigured in the classical investigations of $H(\Omega)$, but many, like the Hahn-Banach and Krein-Milman theorems, were not; and these could be expected to lead to new knowledge about $H(\Omega)$, as has indeed been the case. Especially when we look at certain subspaces of $H(\Omega)$ which can be made into Banach spaces or Banach algebras (with norm topologies stronger than τ), like the Hardy spaces H^p when Ω is a disc or a half-plane, do we see a rich and fully developed wedding of function theory with functional analysis. (For this panorama see K. M. Hoffman [**1962**], P. L. Duren [**1970**], J. Garnett [**1981**], and S. D. Fisher [**1983**].) As for $H(\Omega)$ itself, systematic investigation of it as a topological vector space began in the 1950s, with duality questions being foremost in the early work of G. Köthe, A. Grothendieck, J. Sebastião e Silva and C. L. da Silva Dias.

3. The functional analysis viewpoint: duality and approximation. Let us look at some of the viewpoints, techniques, and questions about holomorphic functions which functional analysis engenders. The most prevalent classical context where the matter can be perceived is approximation—e.g., Mergelyan's theorem. We wish to approximate a holomorphic function f by polynomials (globally), so we want f to lie in the closed linear span of the functions z^n. Because of the Hahn-Banach theorem, this means that every continuous linear functional which annihilates all the z^n must annihilate f. It is apparent that knowledge about the dual space $H(\Omega)^*$ is therefore important. Or we could extend the functional further (by Hahn-Banach) to the space $C(\Omega)$ of continuous functions on Ω and then use the fact (Riesz representation theorem) that the linear functionals on this space are integration with respect to compactly supported Borel measures. It is this latter technique which is featured in the elegant functional analysis proofs of L. Carleson [**1965**] and J. Garnett [**1968**] and is pervasive in the monographs cited earlier. This argument can also be applied to the more elementary Runge theorem. The measures involved here are really quite simple, and it is worthwhile to take a direct approach to them via Cauchy's integral theorem. If Γ is a smooth curve in Ω and φ is holomorphic in $\mathbf{C}\setminus K$ for some compact $K\subset\Omega\setminus\Gamma$, then $\Phi(f):=\int_\Gamma f\varphi$ defines a continuous linear functional on $H(\Omega)$. That every continuous linear functional arises in this way was discovered by L. Fantappiè in his complicated studies of "analytic functionals" in the late 1920s. R. Caccioppoli [**1931**] simplified the matter considerably (but the time was not ripe for considering this integral formula as a representation of the dual space $H(\Omega)^*$). The idea here is to write

$$f(z)=\frac{1}{2\pi i}\int_\Gamma f(w)(w-z)^{-1}\,dw$$

for appropriate Γ (after Cauchy's theorem), remember that the integral is a limit of Riemann sums locally uniformly with respect to z, pass Φ through this limit (Fubini in disguise), and get $\Phi(f)=\int_\Gamma f\varphi$ for the function $\varphi(w):=(2\pi i)^{-1}\Phi(f_w)$, where $f_w(z)=(w-z)^{-1}$. Thus the measure mentioned above is just φ times arclength of Γ. The function φ is called the *Fantappiè indicatrix* of Φ. (In the measure-theoretic approach we have the function $\hat{\mu}(w)=\int(w-z)^{-1}\,d\mu(z)$, now called the *Cauchy transform* of μ.) This function is holomorphic for large w, as one sees by commuting Φ with the series $\sum_{n=0}^{\infty}z^nw^{-n-1}$ for f_w, and vanishes at infinity. In fact, φ is holomorphic in a whole neighbourhood of $\mathbf{C}\setminus\Omega$, but is not quite uniquely determined by Φ. To get unique determination and a satisfactorily complete duality theory, one introduces the space $H_0(\mathbf{C}\setminus\Omega)$ of *germs* of holomorphic functions defined in neighbourhoods of $\mathbf{C}\setminus\Omega$ and vanishing at ∞. Then $H(\Omega)^*=H_0(\mathbf{C}\setminus\Omega)$.

Particularly illuminating is the case when Ω is a disc (of radius $R\leqslant\infty$, say). Then it is possible to choose for Γ a circle; the function φ consequently has the form

$$\varphi(w)=\sum_{n=0}^{\infty}\varphi_n w^{-n-1},$$

and for $f(z) = \Sigma_{n=0}^{\infty} f_n z^n$ we have

$$\Phi(f) = \sum_{n=0}^{\infty} f_n \varphi_n.$$

Here the numbers $\varphi_n = \Phi(z^n)$ satisfy $\overline{\lim}\, |\varphi_n|^{1/n} < R$. Thus we see how the duality problem can be viewed from the perspective of sequence spaces (as was done by O. Toeplitz already in 1937). Questions about the denseness in $H(\mathbf{C})$ of linear combinations of derivatives $\{f^{(n)}: n \in \mathbf{Z}^+\}$ or of translates $\{T_w f: w \in W\}$, where $T_w f(z) := f(z + w)$ and f is entire, which are naturally suggested by the functional analytic viewpoint, are very easily resolved with the help of this sequential representation of Φ. For example (Laurent Schwartz, V. Ganapathy Iyer), if W is infinite and possesses at least one cluster point in $\mathbf{C}$, then $\operatorname{span}\{f^{(n)}: n \in \mathbf{Z}^+\}$ and $\operatorname{span}\{T_w f: w \in W\}$ have the same closure; if, moreover, f is of minimal exponential type ($f(z)e^{-\varepsilon|z|}$ is bounded for each $\varepsilon > 0$), then these spans are dense in $H(\mathbf{C})$.

4. Duality and interpolation. It is an elementary general fact that the dual X^* of a topological vector space X is itself a topological vector space when endowed with the pointwise topology, and that the only linear functionals on X^* which are continuous in this topology are the evaluations at points of X. In view of the above-described representations of $H(\Omega)^*$, this means that each continuous linear functional F on $H_0(\mathbf{C} \setminus \Omega)$ is implemented by a function $f \in H(\Omega)$ in the sense that, for each $\varphi \in H_0(\mathbf{C} \setminus \Omega)$, $F(\varphi)$ equals the value which the functional Φ, determined by φ, has at f. If we examine more closely the mechanism by which germs determine functionals, we see that this means the following: For each compact $K \subset \Omega$ there is a contour $\Gamma = \Gamma_K$ in $\Omega \setminus K$ which surrounds K (once with positive orientation) such that

$$(*) \qquad F(\varphi) = \int_\Gamma f\varphi, \qquad \varphi \in H(\mathbf{C} \setminus K).$$

This aspect of duality is especially useful in dealing with interpolation problems. Consider the classic one: Mittag-Leffler's "Anschmiegungssatz". The sequence $\{z_n: n \in \mathbf{N}\}$ of distinct points of Ω has no cluster point in Ω and for each n there are given complex numbers $\{a_{n,k}: k = 0, 1, \ldots, m_n\}$. It is desired to show that an $f \in H(\Omega)$ exists such that $f^{(k)}(z_n) = a_{n,k}$ for all n and k. To this end, look at the vector subspace S of $H_0(\mathbf{C} \setminus \Omega)$ spanned by the functions $(w - z_n)^{-k-1}$ ($n \in \mathbf{N}$, $k = 0, 1, \ldots, m_n$) and define a linear functional F on S by $F((w - z_n)^{-k-1}) = 2\pi i a_{n,k}/k!$. Show that F is continuous, extend it to an element of $H_0(\mathbf{C} \setminus \Omega)^*$, and implement this extension with an $f \in H(\Omega)$. With $K = \{z_n\}$, $(*)$ gives

$$2\pi i a_{n,k}/k! = F\left((w - z_n)^{-k-1}\right) = \int_\Gamma f(w)(w - z_n)^{-k-1}\, dw,$$

and by Cauchy's theorem this integral equals $2\pi i f^{(k)}(z_n)$. Hence, f has the desired properties.

5. Algebraic questions. Once $H(\Omega)$ is thought of as a ring, certain interesting algebraic questions (not all of which require functional analysis techniques for their resolutions) suggest themselves: What are the closed ideals of $H(\Omega)$?

What are the ring automorphisms of $H(\Omega)$? What are its ring derivations? If $Z \subset \Omega$ and $N: Z \to \mathbf{N}$, then the set $I(N)$, consisting of all $g \in H(\Omega)$ that have at each $z \in Z$ a zero of order at least $N(z)$, is an ideal and is closed. The reason is that differentiation is a continuous linear operator on $H(\Omega)$, and the order of a zero is specified by the vanishing of a derivative. If Z has a cluster point in Ω, then $I(N) = \{0\}$. If Z has no cluster point in Ω, then a slightly stronger form of the interpolation theorem in §4 furnishes an $f \in H(\Omega)$ that has, at each $z \in Z$, a zero of order $N(z)$ and no other zeros in Ω. Clearly, then, g/f is a well-defined holomorphic function in Ω for each $g \in I(N)$, whence $I(N) = fH(\Omega)$. If $h \in H(\Omega)$, and we take $Z = h^{-1}(0)$ and $N(z)$ to be the multiplicity of z as a zero of h, then $I(N) = hH(\Omega)$, so every principal ideal is closed. Conversely, if I is a closed ideal, and we take $Z = \bigcap\{ g^{-1}(0): g \in I\}$ and $N(z)$ to be the smallest order z experiences as a zero of a function in I, then $I(N) = fH(\Omega)$ for an appropriate f as above. We have $I \subset I(N)$, and a Hahn-Banach argument, using the integral representation of functionals, will show that $I = I(N)$, so I is principal (theorem of O. F. G. Schilling). It is also true that every finitely generated ideal is already a principal ideal (theorem of J. H. M. Wedderburn and O. Helmer). But there are nonclosed maximal ideals and (theorem of I. Kaplansky) nonmaximal prime ideals.

Among the obvious ring automorphisms of $H(\Omega)$ are the maps $f \to f \circ \varphi$, where φ is a conformal automorphism of Ω. These are even algebra isomorphisms. If ψ is a conformal map of Ω onto $\Omega^* := \{\bar{z}: z \in \Omega\}$, then $f \to \bar{f} \circ \bar{\psi}$ is also a ring (but not an algebra) automorphism, since $z \to \bar{f}(\bar{z})$ is holomorphic on Ω^* for each $f \in H(\Omega)$ (look at difference quotients!). Conversely, if Φ is a ring automomorphism of $H(\Omega)$, then $-1 = \Phi(-1) = \Phi(i^2) = [\Phi(i)]^2$, so the continuous function $\Phi(i)$ on the connected set Ω assumes only the values $\pm i$. It follows that either $\Phi(i) = i$ or $\Phi(i) = -i$. In the first case Φ has the form $f \to f \circ \varphi$; in the second case, the form $f \to \bar{f} \circ \bar{\psi}$ (theorem of L. Bers, S. Kakutani, and C. Chevalley). To get hold of such a φ or ψ, we note the obvious fact that a principal ideal is maximal if and only if it has the form $(z - \omega 1)H(\Omega)$ for some $\omega \in \Omega$, and then we exploit the fact that Φ must permute these ideals among themselves. We let $\varphi(\omega)$ denote the unique zero of the principal maximal ideal which Φ maps onto $(z - \omega 1)H(\Omega)$. Thus, for $g \in H(\Omega)$,

$$\Phi(g)(\omega) = 0 \Leftrightarrow g(\varphi(\omega)) = 0,$$

so for all $f \in H(\Omega)$, $\Phi(f - f(\varphi(\omega))1)(\omega) = 0$; that is,

$$\Phi(f)(\omega) = \Phi(f(\varphi(\omega))1)(\omega).$$

Now if we know that Φ is an *algebra* automorphism, that is, Φ is complex linear (resp., conjugate linear), then $\Phi(f)(\omega) = f(\varphi(\omega))$ (resp., $\bar{f}(\varphi(\omega))$) follows from the above, and, in particular, $\Phi(z) = \varphi$ or $\bar{\varphi}$. Thus the case of algebra maps is easy. The nontrivial part of the theorem is showing that the ring map must be either linear or conjugate linear. This characterization of ring isomorphisms of $H(\Omega)$ leads to the following curiosity: since the radius ratio R/r is a conformal invariant of the annulus $\Omega = \{z \in \mathbf{C}: r < |z| < R\}$, this ratio is locked up in the ring structure of $H(\Omega)$; that is, in principle, it can be expressed in purely ring-theoretic terms. But how? An explicit construction was given by A. Beck [**1964**] and I. Richards [**1968**].

A ring derivation is an additive map Δ: $H(\Omega) \to H(\Omega)$ which satisfies $\Delta(fg) = f\Delta g + g\Delta f$. Evidently, for any $h \in H(\Omega), f \to hf'$ defines such a map. And this is the whole story: there are no others (theorem of J. Becker).

6. Luecking and Rubel's book. I turn now to the book of Luecking and Rubel. It is very attractive and quite inexpensive. All the above themes (and more) are developed *ab ovo*. The authors contend that even at the elementary level the functional analysis viewpoint is the "right one", and they strive to make their account elementary and self-contained. The reader is referred elsewhere for proofs of one or two standard facts, like the Stone-Weierstrass and Tietze extension theorems, and a one-semester, undergraduate-level complex variables course is a formal prerequisite according to the Preface, but actually Cauchy's theorem for a rectangle and the power series development that flows from it are all proved in the text. Duality arguments based on the integral representation provide a unifying theme. This is presented first for Ω a disc (where the sequence-space point of view can also be brought in, as indicated in §3 above) and then for general Ω. Not all the applications of functional analysis are particularly economical or striking; in some cases considerable stage preparation, involving the traditional arguments and methods, is necessary. The verification that this subspace is closed or that linear functional is continuous in order to infer theorem S from functional analysis result T may involve many of the steps in the classical proof of S. The authors acknowledge this and turn it to advantage: sometimes they give both constructive and abstract treatments (e.g., of the Mittag-Leffler theorems), and sometimes in the exercises they indicate refinements of the arguments which will convert an existence proof in the text (e.g., that of Runge's theorem) into a recipe for constructing the function. They then invite the reader to "note the typical contrast between constructive and abstract methods: constructive methods are stronger in that explicit formulas and estimates are obtained while abstract methods are stronger in that more general theorems usually result and interesting connections with apparently unrelated problems are often exposed".

Besides the topics outlined above, there are some Baire category results (the holomorphic functions on the disc Ω which have no analytic continuation are a residual set in the complete metric space $H(\Omega)$), the Riemann mapping theorem, the strong form of Rouché's theorem and the D. Challener and L. A. Rubel converse of it, several interesting interpolation results including interpolation by gap series (where diophantine approximation plays a role), and a short final chapter which discusses first-order (in the sense of predicate logic) conformal invariants. In connection with the "span of translates" issue raised in §3 above, the authors construct an entire function f such that the set $\{T_w f$: $w \in \mathbf{C}\}$ of translates itself is dense in $H(\mathbf{C})$ (G. D. Birkhoff, W. Seidel, and J. L. Walsh). An elementary construction due to C. Blair and L. A. Rubel is even more striking. They manufacture a "triply universal function". This is an entire function f such that, for an appropriate choice of constants in the n-fold iterated integral $f^{(-n)}$ of f, *each* of the sequences $\{f^{(-n)}\}_{n=0}^{\infty}$, $\{f^{(n)}\}_{n=0}^{\infty}$, and $\{T_n f\}_{n=0}^{\infty}$ is dense in $H(\mathbf{C})$. The results about ring isomorphisms and derivations in $H(\Omega)$ (§5 above) were extended and simplified quite a bit by the reviewer and S. Saeki [**1983**]. Both of these latter papers were apparently of too recent origin to be included in this book. Actually, this book had apparently been in gestation for some time and was adumbrated by the paper of L. A. Rubel and B. A. Taylor [**1969**], which the reader can profitably consult for a short excursion into some of the ideas discussed here, as well as some variations on the proofs in the book.

In conclusion, I feel the authors completely achieved their goal and have presented their case in a very lively, concise manner. The density of errors is very low (but regrettably there is no index). The chapters are short, and each is followed by a number of relevant, accessible exercises. The book is rewarding reading for cognoscenti and students alike.

BIBLIOGRAPHY

A. Beck, *On rings on rings*, Proc. Amer. Math. Soc. **15** (1964), 350–353. MR **29** #1347; Zbl **135**, p. 289.

C. Blair and L. A. Rubel, *A triply universal entire function*, l'Enseign. Math. (to appear).

R. B. Burckel and S. Saeki, *Additive mappings on rings of holomorphic functions*, Proc. Amer. Math. Soc. **89** (1983), 79–85, MR **84i**:30063; Zbl **532**. 30040.

R. Caccioppoli, *Sui funzionali lineari nel campo delle funzioni analitiche*, Rend. Accad. Lincei Roma (6) **13** (1931), 263–266; ibid. **15** (1932), 713–714. Jbuch **57**, p. 470; **58**, p. 427; Zbl **2**, p. 31; **5**, p. 14.

L. Carleson, *Mergelyan's theorem on uniform approximation*, Math. Scand. **15** (1965), 167–175. MR **33** #6368; Zbl **163**, p. 86.

P. L. Duren, *Theory of H^p-spaces*, Academic Press, New York & London, 1970. MR **42** #3552; Zbl **215**, p. 202.

S. D. Fisher, *Function theory on planar domains: a second course in complex analysis*, Wiley, New York, 1983. Zbl **511** #30022.

J. Garnett, *On a theorem of Mergelyan*, Pacific J. Math. **26** (1968), 461–468. MR **38** #1532; Zbl **165**, p. 169.

______, *Bounded analytic functions*, Academic Press, New York & London, 1981. Zbl **469** #30024; MR **83g**:30037.

I. M. Gelfand, *Normierte Ringe*, Mat. Sb. (N.S.) **9(51)** (1941), 3–24. Jbuch **67**, p. 406; Zbl **24**, p. 406; MR **3**, p. 51.

K. M. Hoffman, *Banach spaces of analytic functions*, Prentice-Hall, Englewood Cliffs, N.J. 1962. MR **24** #A2844; Zbl **117**, p. 340.

I. Richards, *More on rings on rings*, Bull. Amer. Math. Soc. **74** (1968), 677. MR **37** #1980; Zbl **169**, p. 409.

L. A. Rubel and B. A. Taylor, *Functional analysis proofs of some theorems in function theory*, Amer. Math. Monthly **76** (1969), 483–489; ibid. **77** (1970), 58. MR **40** #384; Zbl. **175**, p. 443.

A. E. Taylor, *Notes on the history of the uses of analyticity in operator theory*, Amer. Math. Monthly **78** (1971), 331–342, 1104. MR **43** #5531; Zbl **211**, p. 304. (See also MR **58** #21239 and Zbl **211**, p. 423.)

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Spectral theory of Banach space operators, by Shmuel Kantorovitz, Lecture Notes in Mathematics, Vol. 1012, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 179 pp., $9.50, 1984. ISBN 3-5401-2673-2

The earliest results of the spectral reduction theory for bounded and unbounded selfadjoint operators can be found in works by D. Hilbert, F. Riesz