an active and creative scientist. Such details are best left to the historians and to the book reviewers, who are usually delighted by the opportunity to fill them in.

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Commutative semigroup rings, by Robert Gilmer, Chicago Lectures in Mathematics, The University of Chicago Press, Chicago, IL, xii + 380 pp., 1984, $11.00. ISBN 0-2262-9392-0

Let R be a commutative ring and S a semigroup with respect to an operation $+$, not necessarily commutative. The semigroup ring $R[X; S]$ consists of formal sums $\sum_{i=1}^{n} r_i X^{s_i}$, $r_i \in R$, $s_i \in S$, with addition defined by adding coefficients, and multiplication defined distributively using the rule $X^s X^t = X^{s+t}$. For example, if N is the semigroup of nonnegative integers, then $R[X; N]$ is just the polynomial ring $R[X]$ in a single indeterminate X. Another important example is the semigroup ring $K[X; G]$, where K is a field and G is a finite group. The theory of semigroup rings divides much along the lines of these two examples. If $R[X]$ is taken as the starting point, then the tools and problems come from commutative algebra; if the starting point is $K[X; G]$, then the group G is the primary object of study, and the tools come from group representation theory and constitute a rich mixture of many other areas of mathematics. It should be emphasized that in the case of $K[X; G]$, the main interest is in a nonabelian group G; indeed, a large portion of noncommutative ring theory has been developed specifically in order to deal with this example. (A nice set of lectures on this aspect of the subject, with the ultimate goal of proving a couple of important theorems on finite groups, can be founnd in [**6**].)

To get an idea of the shift in emphasis imposed by restricting to a commutative semigroup S, as is done in this book, consider the question of semisimplicity of $K[X; G]$. A ring is called semisimple if its Jacobson radical is 0. For a commutative ring A, the Jacobson radical $J(A)$ is defined to be the intersection of the maximal ideals of A. A related notion is that of the nilradical $N(A)$, which is the intersection of the prime ideals of A. The ring A is called a Hilbert ring if every prime ideal is an intersection of maximal ideals, in which case, clearly, $N(A) = J(A)$. The definitions of $J(A)$ and $N(A)$ for a noncommutative ring are somewhat more complicated.

For a finite group G, $K[X; G]$ is semisimple, provided that G has no element of order p when char $K = p > 0$; this is Maschke's theorem and is fundamental for the classification of the representations of G (cf. [**6**, p. 244]). The statement remains true if, instead of being finite, G is taken to be an arbitrary abelian group (cf. [**4**, p. 73, Corollary 17.8]). To what extent does this latter result surface in the present book? The nearest theorem to it that I could find is Theorem 11.14, p. 140, which only yields the case that G is torsion-free. On the other hand, the case of a finitely generated abelian group, for instance, is fully covered by putting together a few results:

(1) Theorem 9.7, p. 103: *If* G *is a finitely generated group, then* $K[X; G]$ *is a Hilbert ring*,

(2) Corollary 9.3, p. 99: *If* D *is a domain of* $\operatorname{char} p > 0$, *then* S *is a* p*-torsion-free semigroup if and only if* $N(D[X; S]) = 0$;

(3) Corollary 9.14, p. 107: *If* D *is a domain of* $\operatorname{char} 0$ *and* S *is a cancellative semigroup, then* $N(D[X; S]) = 0$.

Thus, this book differs from the definitive works [**4**, **5**] of Passman on group rings in that its spirit is closer to that of a polynomial ring over an arbitrary commutative ring than to that of a group ring over a field. A sample result involving the Krull dimension of $R[X; S]$ is the following: If S is a cancellative monoid, then $\dim R[X; S] = \dim R[X; G] = \dim R[\{X_\lambda\}]$, where G is the quotient group of S and $\{X_\lambda\}$ is a set of indeterminates whose cardinality equals the torsion-free rank of G. This reduces the question of $\dim R[X; S]$ to that of the polynomial ring $R[\{X_\lambda\}]$.

The book begins with a chapter on commutative semigroups, followed by a chapter on zero-divisors, nilpotent elements, idempotents, and units of semigroup rings. There are then two chapters devoted to characterizing various kinds of semigroup rings, such as valuation rings, Prüfer rings, von Neumann regular rings, factorial domains, and Krull domains. The final chapter discusses Krull dimension and questions of isomorphism for semigroup rings. The author makes an effort to keep the semigroup S and the commutative ring R as general as possible, but, as the book progresses, S inevitably gravitates toward being a cancellative, torsion-free monoid, i.e., a semigroup with identity which can be embedded in a torsion-free group. As for R, it is not even assumed to have an identity to begin with, although this condition is readily imposed when it becomes expedient to do so. Note that if S is a semigroup without identity, then $R[X; S]$ is a ring without identity even when R has one.

There are a few brief remarks and references, intended to add perspective, interspersed throughout the text and appended to each section. These are sometimes so abbreviated that they leave the wrong impression. For example, the theorem on p. 76 that a finitely generated monoid S is finitely presented is correctly attributed to Redei, but the author fails to mention that the simple proof given here using the noetherian property of the semigroup ring $Z[X; S]$ is from the thesis of J. Herzog [**1**]. Another example is the passing assertion on p. 303 that the automorphisms of $k[x, y]$, k a field and x, y indeterminates, are not known and the follow-through reference on p. 314 to the Jacobian problem. As a matter of fact, the automorphisms of $k[x, y]$ have a long history, and there is a good classification of them; the monograph [3] of Nagata contains the details. Finally, any book of this length may be expected to have a few slips and misprints, and this one seems no exception. A couple that especially caught my eye are the reference on p. 21 to a paper of Ohm, which should be to another work, and the result of Heinzer-Ohm mentioned on p. 277, which is misquoted and is false as stated.

The book has been written with great patience, in a clean, crisp style, and includes almost all the definitions and details that one could hope for. It should make a valuable, easily accessible reference work. Much of the material covered is due to the author and his students, and, except for some overlap with the recent book [2] of Karpilovsky, most of it cannot be found in other books.

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Differential systems involving impulses, by S. G. Pandit and S. G. Deo, Lecture Notes in Mathematics, Vol. 954, Springer-Verlag, Berlin, 1982, vi + 102 pp., $8.00. ISBN 3-5401-1606-0

This book presents and studies a new class of generalized ordinary differential equations containing impulsive terms linearly. The review first introduces the discipline of generalized ODE, briefly describes the contents of the book, and offers comments on the treatment in the present literature. Readers familiar with the background, and those who do not believe that a book review is an excuse for an expository paper, may wish to begin around equation (11).

Consider ordinary differential equations in R^n,

$$\dot{x} = f(t, x) \quad (\text{i.e., } dx(t)/dt = f(t, x(t))). \tag{1}$$

If the right side, f: $R^{1+n} \to R^n$, is continuous, it is perhaps obvious what the solutions $x(\cdot)$ of (1) ought to be: an explicit definition is almost superfluous.

Applications soon dictated that continuity of f be relaxed. One studies 'block box systems'

$$\dot{x} = Ax + bu(t) \tag{2}$$

by examining the 'responses' $x(\cdot)$ to various 'inputs' $u(\cdot)$; and Laplace transform methods suggest that it is the discontinuous inputs that are crucial: e.g., a signum function, a unit stepfunction, or even a delta "function".

With discontinuities present in the term $u(\cdot)$ (the forcing term, or control), one can no longer successfully require that solutions $x(\cdot)$ satisfy the differential equation (2) for all t. Several plausible definitions of generalized solution come to mind:

(A) functions $x(\cdot)$ that are absolutely continuous (locally) and satisfy (2) for almost all t (absolute continuity cannot be relaxed to ordinary continuity