Conclusion. It is fitting that the first major book on percolation theory be authored by Harry Kesten, the major contributor to the field in recent years. The expert in the field will find this book indispensable, while it supplies a good introduction for the nonexpert. It is not intended to be a reference volume covering the entirety of percolation theory, but is limited to rigorous results for the classical models. In fact, so much has happened in recent years that it would be difficult to produce a single volume which is both rigorous and comprehensive. The monograph will help consolidate and unify the theory of the classical percolation models, although a tidy systematic understanding still lies in the distant future, considering the major unsolved problems and many loose ends that remain. Certainly the volume deserves the title of the series: "Progress in Probability and Statistics".

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 11, Number 2, October 1984

0273-0979/84 $1.00 + $.25 per page

Geometric aspects of convex sets with the Radon-Nikodým property, by Richard D. Bourgin, Lecture Notes in Math., vol. 993, Springer-Verlag, Berlin, 1983, xii + 474 pp., $22.00. ISBN 3-5401-2296-6

In the Spring of 1973, Jerry Uhl and I were putting the finishing touches on a manuscript entitled, *Vector measures*. With his usual wisdom, Jerry suggested we put the manuscript aside for six months or so. His reasoning went more or less like this: we're happy with what we've done *now*, so, if it still looks good to us in six months—all the better; besides, maybe something beautiful will happen in the meantime that really ought to be included. Jerry and I are unabashed optimists and so Jerry's suggestions offered extremely appealing alternatives.

Lefty Gomez once said, "I'd rather be lucky than good" and, with little choice available, this reviewer agrees with such sentiments. During the Summer of 1973, the main themes of the subject of vector measures attained a clarity far exceeding that which even Jerry and I could have hoped for. As a result, there followed a period of three years of continued analysis, synthesis and expansion before the revised manuscript was finally submitted. Still, the fact that the book was not already obsolete depended largely on our decision (again on Jerry's sage advice) to include only those results that seemed to be in final, definitive form.

No doubt the advances experienced over the Summer of 1973 that most affected our manuscript concerned Banach spaces having the "Radon-Nikodym property". Recall, if you will, that a Banach space X has the Radon-Nikodym property if, given a probability space (Ω, Σ, μ) and an additive measure F: $\Sigma \to X$ for which $\|F(E)\| \leqslant \mu(E)$ holds for each event E, then there exists a Bochner integrable F: $\Omega \to X$ such that $F(E) = \int_E f\,d\mu$ holds for each and every event E.

Separable duals have the Radon-Nikodym property, a fact whose discovery can be blamed equally on I. M. Gelfand, N. Dunford and B. J. Pettis. Also, reflexive spaces have the Radon-Nikodym property; this was shown to be so by Dunford, Pettis and R. S. Phillips.

One (among many) equivalent formulation: X has the Radon-Nikodym property if and only if for every finite measure space (Ω, Σ, μ) every operator T: $L_1(\mu) \to X$ is representable, i.e., of the form $Tg = \int \vec{f} g\, d\mu$ for some essentially bounded measurable function $\vec{f}$: $\Omega \to X$. Now, the representability of such operators plays a central role in the development of the topological theory of tensor products; this was understood and exploited by A. Grothendieck. Indeed, one of the highlights of the earliest version of *Vector measures* was an exposition of one of Grothendieck's many surprises: If X^* has the Radon-Nikodym property and the approximation property, then X^* has the metric approximation property. At the heart of the proof of this result is the fact that a dual X^* has the Radon-Nikodym property if and only if integral operators into X^* are nuclear.

Aside from similar external characterizations of the Radon-Nikodym property (and their applications), little of substance was known about this rather formal notion. Well, there was one fascinating exception, a theorem of H. Maynard, which complemented earlier work of M. A. Rieffel but, as of Spring 1973, the Maynard theorem stood alone as an *internal* characterization of the Radon-Nikodym property; Maynard's theorem: a Banach space X has the Radon-Nikodym property if and only if each closed bounded convex subset K of X is s-dentable, i.e., each such K admits, for each $\varepsilon > 0$, a point x_ε which cannot be represented in the form $\Sigma_n \lambda_n x_n$ of an infinite convex combination of points x_n of K separated from x_ε by at least ε.

To be sure, it was the hope for, and anxious anticipation of, essential refinements of Maynard's theorem that made the temporary shelving of our manuscript palatable to Jerry and me. Our patience was to be richly rewarded.

In the Summer of 1973, Maynard's s-dentability theorem was improved by W. J. Davis, R. E. Huff and R. R. Phelps, resulting in what is now known as the dentability theorem: a Banach space X has the Radon-Nikodym property if and only if every closed bounded convex set K in X admits, for each $\varepsilon > 0$, a point x_ε which is not in the closed convex hull of points in K at least ε away from x_ε. Within weeks of this discovery, J. Lindenstrauss established that in spaces with the Radon-Nikodym property every nonempty closed bounded convex set has an extreme point (and is, indeed, the closed convex hull of its extreme points); the validity of the converse remains unknown. Soon after, R. R. Phelps showed that precisely in spaces with the Radon-Nikodym property do nonempty closed bounded convex sets find themselves the closed convex hull of their strongly exposed points. C. Stegall told why the dual of a separable Banach space *has* to be separable to enjoy the Radon-Nikodym property, and R. E. Huff and P. D. Morris took Stegall's lead to show that a dual space has the Radon-Nikodym property whenever each of its nonempty closed bounded convex sets has extreme points. Huff and Morris went on to find extreme points in nonconvex sets provided they live in spaces with the Radon-Nikodym property, and G. A. Edgar gave a stunning version of Choquet's representation theorem for noncompact convex sets as long as they sit in separable spaces with the Radon-Nikodym property.

The rush was on. A geometric lode had been struck and its mathematical miners were quick to react. Convexity was seen in duality with smoothness, and the strong differentiability spaces of E. Asplund were established as precisely the preduals of (dual) spaces with the Radon-Nikodym property (I. Namioka, R. R. Phelps, C. Stegall). The noncompact Choquet theorem of Edgar was extended, generalized and scrutinized (G. A. Edgar, R. D. Bourgin). Rosenthal's dichotomy and all its attendant wonders were exploited (H. P. Rosenthal, T. Odell, R. Haydon); a remarkably fine line between the Radon-Nikodym theory for the Bochner and Pettis integrals soon emerged. The special character of the Radon-Nikodym property in Banach lattices was exposed (M. Talagrand, N. Ghoussoub, J. Bourgain). New examples of spaces with the Radon-Nikodym property were discovered, spaces equipped with sufficient pathology to redirect worthwhile research efforts (J. Bourgain and F. Delbaen, P. McCartney and R. O'Brien, J. Bourgain and H. P. Rosenthal). These advances were either too new or too poorly understood by Jerry and me to be included in the final manuscript of *Vector measures*.

Now, a clear exposition of these topics (and much, much more) is to be found in Bourgin's monograph. Bourgin presents the theory from the beginning, concentrating on detailing the geometric aspects of *convex sets* with the Radon-Nikodym property, avoiding natural pitfalls encountered in a somewhat technically complicated situation, and successfully conveying the essential methods a student of the subject needs to master. I hasten to add that the material covered is quite up to date with fresh informative treatments afforded results that actually appeared *after* the publication of Bourgin's monograph.

There are, of course, omissions and exclusions. After careful exposition of the Bourgain-Stegall cycle of ideas concerning the Bishop-Phelps theorem, it would have been enlightening for some if the author had mentioned the beguiling mystery of the validity of the Bishop-Phelps theorem for *complex* Banach spaces, especially since a consequence of the theorem of Bourgain is that the Bishop-Phelps theorem holds in *complex* Banach spaces with the (real) Radon-Nikodym property. Similarly, a striking application of the Huff-Morris theorem (concerning the existence of extreme points in any nonempty closed bounded subset of a Banach space with the Radon-Nikodym property) is P. Mankiewicz's proof that complex Banach spaces with the Radon-Nikodym property have unique complex structure; the omission of this result is unfortunate, again because the question of uniqueness of complex structures on complex Banach spaces (the complex Mazur-Ulam problem) is open in general. Almost nothing is said about the role of the Radon-Nikodym property in the study of operator ideals, a subject arguably central to the study of the geometry of Banach spaces. Again, nothing is said about the part played by the Radon-Nikodym property in abstract harmonic analysis, both commutative and noncommutative. All this is nitpicking though since the objective of the monograph is *not* to tell everything there is about the Radon-Nikodym property, but rather to tell about a substantial amount of certain geometric aspects of the Radon-Nikodym property. In this regard, Dick Bourgin has done an admirable job.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 11, Number 2, October 1984

0273-0979/84 $1.00 + $.25 per page

Differential geometry of foliations, by Bruce L. Reinhart, Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 99, Springer-Verlag, Berlin, 1983, ix + 194 pp., $42.00. ISBN 3-5401-2269-9

A structure on a differentiable manifold of dimension m can be defined by requiring that there exist an atlas whose coordinate transformations satisfy a special condition. In the case of a foliated structure of dimension p, the transformations must map the points in their domains lying in a p-plane parallel to some fixed subspace $R^p \subseteq R^m$ into a p-plane of the same type. Thinking of the layers of an onion or the pages of a magazine suggests the right mental picture when $p = 2$ and $m = 3$. A structure defined by an atlas determines a reduction of the structure group of the tangent bundle to the group consisting of the tangent maps to coordinate transformations. (A similar game can also be played with higher order jet bundles.) Integrability problems in differential geometry are concerned with reversing this process, that is, with determining if a given reduction can be realized in the way described, at least up to some kind of equivalence. For instance, Reeb's "Problème Fondamental" in the first monograph on foliations [2] was to determine whether a manifold that admits a continuous field of p-planes can also be given a foliated structure of dimension p.

Reinhart, with the pardonable exaggeration of an enthusiast, subtitles his book *The fundamental integrability problem*. I would have been more comfortable with the indefinite article, but he probably wanted to show both his intention of including much more within foliations than the minimal structure described above and that the field is of fundamental importance as a rich