impression that the authors have had little contact with the general mathematical community. This feeling is amplified by the authors' English, which is often far from the spoken language.

The book will probably be of most interest to people with a background in finite dimensional convexity, because they will at least see how that theory is related to C^*-algebras. I do not recommend a serious student of C^*-algebras to spend much time with the book.

ERLING STØRMER

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A general theory of optimal algorithms, by J. F. Traub and H. Wozniakowski, Academic Press, 1980, xiv + 341 pp., $36.00. ISBN 0-1269-7650-3

Let me begin by remarking that this book may not be well served by the particular conjunction of title and series (ACM monograph series) which suggest to me that the authors believe that their main audience will be found among computer scientists. I must disagree with this appreciation. In my opinion this is a book about certain aspects of applied mathematics, an ambitious, largely successful and therefore important book; and it is somewhat unfortunate that it is being noticed here several years after initial reviews appeared in computer science journals.

These remarks touch on the demarkation dispute between computer science and applied mathematics and perhaps deserve more explanation. One characteristic feature of the rapid evolution of computer science has been the way in which it has drawn on quite diverse subject areas as these become major sources of applications interest or of necessary development techniques, absorbed what has been needed, and passed on to other applications areas with new technological requirements and different problems. Interaction between computer science and the mathematical sciences has proved to be of continuing mutual benefit. Historically it has provided much of the impetus that has transformed such subjects as numerical analysis, formal systems, and complexity theory into important and flourishing branches of applied mathematics. However, many of the participants have understood neither the dynamics nor the strong pragmatic component in the computer science side of the interchange. It seems that as they become more deeply involved in the formal questions raised by their subject specialty they argue with increasing persistence that they are providing the theoretical basis for computer science. Unfortunately their chosen audience has not always taken note.

Numerical analysis and approximation theory seem the categories most appropriate to the present work. *A general theory of optimal algorithms* is certainly a tempting title, and the authors claim to subsume most, if not all of, computational complexity into their thesis; but the problem domain is analytic computational complexity or the complexity of problems which can only be solved approximately. Typical of their considerations is the problem of finding an optimal estimate of the value of a linear functional (of an unknown function) given certain data which is of two kinds: (a) the values of a finite number of other linear functionals which make up the problem information, and (b) a nonlinear restriction which limits the set of possible competing functions and which is available a priori. Here we think of the candidate functions being inside a ball defined by the restriction operator and intercepted by the hyperplane defined by the problem information. The possible values that the unknown functional can have are restricted by its intersection with the hypercircle defined in the above construction, and estimates, bounds, and optimality properties now follow easily. Perhaps the first use of this approach was due to Synge (1948), but the seminal paper was that of Golomb and Weinberger (1959) who described it in detail in a Hilbert space setting and showed how to find the solution explicitly in several cases. We now recognise they were constructing spline functions in the generalised sense introduced by Anselone and Laurent (1968) and others.

The first part of the book establishes a general framework into which the above example and much recent work (a suitable reference is Micchelli and Rivlin (1977)) fits easily. A very complete treatment is given, including some consideration of the problems introduced by nonlinearity in the information specified, and it seems likely that the extensions to include complexity considerations to permit discussion of optimal complexity algorithms for computing estimates to within a specified accuracy have been presented together for the first time. The importance of spline algorithms is underscored by providing a quantitative sense in which they are effective.

The second major topic treated is iterative information models for algorithms (a simple example is provided by Newton's method for finding a zero of a function). Here optimal order plays a role somewhat analogous to that of optimal approximation in the first section, and now exploration of the relationship between information and order is a major part of the development. The book concludes with a brief survey of the history of the subject and an annotated bibliography which provides extensive references to the Russian literature.

Inevitably there are restrictions to the general coverage promised in the title, and perhaps the most important of these are restrictions to exact information, to a restriction operator imposed a priori, and to exact arithmetic. The first is the most important because it excludes a whole spectrum of important problems in which the manner of specifying the set of allowable perturbations in recording the information may go a long way towards making the choice of restriction operator natural. Recent work in this direction is encouraging (for example, Wecker and Ansley (1983)). However, it is exciting that the authors have been able to treat such a wide range of problems with considerable authority. The book is a major contribution to research in numerical analysis and will prove of considerable interest to workers in approximation theory. If it has a defect it is that it makes few concessions to its readers, in particular in the choice of notation.

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M. R. OSBORNE

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Notes on real and complex C^*-algebras, by. K. R. Goodearl, Shiva Mathematics Series, Shiva Publishing, Ltd., Cambridge, Mass., 1982, 211 pp., $28.95. ISBN 0-9068-1216-X

The author of this book states his purpose clearly: "W[e] have tried to present to the non-specialist a view into the subject by means of its most striking theorems." He does not even hint at the vast range of the subject but merely covers a few things well. In a similar vein this review is directed not at the expert but at those who want to know why so many mathematicians study (and write books about) C^*-algebras. The review will follow the book in requiring C^*-algebras to be unital (i.e. to have multiplicative identity elements). It will also assume complex scalars except to discuss the real case.

The story begins with two classes of concrete examples: one commutative and the other not. Let S be a compact Hausdorff space. Let $C(S)$ be the space of all continuous complex valued functions on S. Under pointwise addition and multiplication $C(S)$ is a commutative algebra. That is, it is a linear space which is also a commutative ring under the same additive structure and with the scalar and ring multiplications agreeing as one would expect. In addition $C(S)$ has two other elements of structure which turn out to be crucial to its study. It has a norm (the supremum or uniform norm) defined by

$$\|f\| = \sup\{|f(s)| : s \in S\} \quad \forall f \in C(S)$$

under which it is a complete normed algebra or Banach algebra. It also has an involution $*$: $C(S) \to C(S)$ defined by

$$f^*(s) = f(s)^- \quad \forall f \in C(S), \forall s \in S,$$

where the bar denotes complex conjugation. We will say more about involutions below, but for now we remark that an algebra with a (fixed) involution is called a $*$-algebra (pronounced star-algebra) and a Banach algebra with an involution is called a Banach $*$-algebra.