3-dimensional case of invariant measures on planesets is not the best approach. Surely this topic is more naturally included with those of Chapter 5. How much better to begin with some interesting examples based on the work of Sylvester, Pólya-Szegö, and Pleijel, thus giving the reader a feel for the nice ideas to be treated in depth further on. Incidentally, of the various minor misprints, the misspelling of the name *Schläfli* seems the most eye-catching.

For the casual library reader who wishes to know more about the nature of Ambartzumian's work, we recommend first reading the author's introduction, and then moving to A. Baddeley's well-written Appendix A, which gives an overview of much of the material in the first six chapters of the text and, in addition, several related topics such as Hilbert's Problem IV. As one reads the text, the papers of Baddeley and K. Piefke referenced in Appendix A will also be of interest.

Ambartzumian has established a base camp in a little-explored area of geometry. From here a number of interesting problems can be seen from a new perspective. With luck a boom town could arise. At the very least this work is a significant contribution to the foundations of integral geometry.

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BULLETIN (New Series) OF THE
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Stochasticity and partial order, doubly stochastic maps and unitary mixing, by Peter M. Alberti and Armin Uhlmann, Mathematics and its Applications, Vol. 9, D. Reidel Publishing Company, Dordrecht, Holland, 1982, 123 pp., $28.50. ISBN 0-9277-1350-2

Many interesting theorems in functional analysis have their origin in non-trivial finite dimensional results. The book under review provides an example of such a development. It starts with some classical results in convexity theory which go back to Birkhoff and Rado, and it ends up with theorems on C^*-algebras.

Let me be more specific. The starting point is the real vector space l_n^1 in n dimensions with an ordering defined by $a = \{a_1, \ldots, a_n\}$ is said to be positive if each $a_i \geqslant 0$. An $n \times n$ matrix T is called doubly stochastic if it is a convex combination of isometries S on l_n^1 such that $a \geqslant 0$ implies $Sa \geqslant 0$. Such matrices define an ordering on l_n^1 by $a \succ b$ if there is a doubly stochastic matrix T with $a = Tb$. In the second chapter, l_n^1 is replaced by the complex $n \times n$ matrices M_n equipped with the trace norm, and the doubly stochastic matrices are replaced by positive linear trace preserving unital maps T: $M_n \to M_n$. Since spectra of selfadjoint matrices belong to l_n^1, the results for l_n^1 are applicable; for example, if we again define $a \succ b$ if $a = Tb$ with T as above, then for a and b selfadjoint, $a \succ b$ if and only if $\mathrm{Spec}(a) \succ \mathrm{Spec}(b)$ in l_n^1. Furthermore, this is equivalent to a belonging to the convex hull

$$\mathrm{conv}\{ ubu^* : u \text{ unitary in } M_n\}.$$

If we look at the above from the point of view of C^*-algebras, we quickly see connections. If Tr denotes the usual trace on M_n, a linear functional on M_n is of the form $\omega(a) = \mathrm{Tr}(ab)$ for some $b \in M_n$, and the norm of ω is the trace norm of b. Furthermore, ω is positive if and only if b is positive, and ω is a state if, in addition, b satisfies $\mathrm{Tr}(b) = 1$. The main idea of the book under review is that the state space of a C^*-algebra can be studied from the point of view of stochasticity as discussed in the first two chapters. The natural extension of $\succ$ to the dual of a C^*-algebra is, by the last result in the preceding paragraph, that $\rho \succ \omega$ if ρ belongs to the w^*-closure of the convex hull of all functionals ω^u, defined by $\omega^u(a) = \omega(uau^*)$, where $u \in \mathscr{U}(A)$—the unitary group in A. The key technical tools are the so-called Ky Fan functionals, which, for states and positive operators, are shown to be defined by $K(\omega,a) = \sup\{\omega^u(a) \colon u \in \mathscr{U}(A)\}$. A key result is that $\rho \succ \omega$ if and only if $K(\rho,a) \leqslant K(\omega,a)$ for all positive $a \in A$. If M is a von Neumann algebra we have duality in $K(\cdot,\cdot)$ in the sense that if p and q are projections, then $p \succ q$ in the usual sense of von Neumann algebras if and only if $K(\omega,p) \geqslant K(\omega,q)$ for all states ω. Much of the book is devoted to this duality and to studying states which are maximal with respect to $\succ$. The maximal states will, for finite von Neumann algebras, be traces, and, for those of type III with separable preduals, be all states. The theory is closely related to center valued traces and their generalizations to convex maps.

Thus the authors more or less accomplish what they intended, namely, "to explain and to prove certain relations between stochasticity and partial order." I cannot say, however, that I am satisfied. Perhaps it is that the book lacks scope. A look at its reference list may explain why. Except for references to the authors themselves and to papers in mathematical physics, the list contains practically no references to papers in C^*-algebras published after 1970. This is reflected in the text, where there are some comments on the relationship to physics but hardly any to recent mathematics. It would, for example, be interesting to have at least some comments on the relationship to the non-abelian ergodic theory developed in the 1970s or to the different results that exist on the faces of the state space of a C^*-algebra. I am really left with the impression that the authors have had little contact with the general mathematical community. This feeling is amplified by the authors' English, which is often far from the spoken language.

The book will probably be of most interest to people with a background in finite dimensional convexity, because they will at least see how that theory is related to C^*-algebras. I do not recommend a serious student of C^*-algebras to spend much time with the book.

ERLING STØRMER

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A general theory of optimal algorithms, by J. F. Traub and H. Wozniakowski, Academic Press, 1980, xiv + 341 pp., $36.00. ISBN 0-1269-7650-3

Let me begin by remarking that this book may not be well served by the particular conjunction of title and series (ACM monograph series) which suggest to me that the authors believe that their main audience will be found among computer scientists. I must disagree with this appreciation. In my opinion this is a book about certain aspects of applied mathematics, an ambitious, largely successful and therefore important book; and it is somewhat unfortunate that it is being noticed here several years after initial reviews appeared in computer science journals.

These remarks touch on the demarkation dispute between computer science and applied mathematics and perhaps deserve more explanation. One characteristic feature of the rapid evolution of computer science has been the way in which it has drawn on quite diverse subject areas as these become major sources of applications interest or of necessary development techniques, absorbed what has been needed, and passed on to other applications areas with new technological requirements and different problems. Interaction between computer science and the mathematical sciences has proved to be of continuing mutual benefit. Historically it has provided much of the impetus that has transformed such subjects as numerical analysis, formal systems, and complexity theory into important and flourishing branches of applied mathematics. However, many of the participants have understood neither the dynamics nor the strong pragmatic component in the computer science side of the interchange. It seems that as they become more deeply involved in the formal questions raised by their subject specialty they argue with increasing persistence that they are providing the theoretical basis for computer science. Unfortunately their chosen audience has not always taken note.

Numerical analysis and approximation theory seem the categories most appropriate to the present work. *A general theory of optimal algorithms* is certainly a tempting title, and the authors claim to subsume most, if not all of, computational complexity into their thesis; but the problem domain is analytic computational complexity or the complexity of problems which can only be