

this book than what the mathematician is. I believe that this would be an excellent choice for a more advanced course in dynamical systems which emphasized applications. Such a course might have as a prerequisite the material of one of the other books and should be supplemented with some original sources.

## REFERENCES

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*Special functions in queuing theory and related stochastic processes*, by H. M. Srivastava and B. R. K. Kashyap, Academic Press, Inc., New York, 1982, xii + 308 pp., \$37.50. ISBN 0-1266-0650-1.

When the early mathematical models of queues were formulated in the beginning of this century, some of the main concepts in the more general field of stochastic processes were not fully developed. Hence these models were at best crude approximations of reality. As more and more tools from the theory of stochastic processes were made available, it was possible to formulate models which were more realistic descriptions of the actual phenomena under study. As a simple illustration, one may refer to the underlying parameters of a simple queue, which by and large were assumed to be fixed, but more realistically vary in accordance with some underlying random fluctuations such as the queue size. Evidently the analyses of such models were more involved with the end results not being in terms of simple functions, but often involving in some form one or more of the family of functions—known as special functions—i.e., gamma, incomplete gamma, beta, incomplete beta, Bessel, modified Bessel, hypergeometric, etc.

What then can be learned from the intersection of the vast amount of literature available in these two areas of mathematics, namely theory of special functions and queuing theory? This well-researched book provides a source for those who attempt to seek answers to this question. Its extensive bibliography lists recent contributions of many authors who have analyzed queuing models which differ from well-known early formulations in that they are often more realistic descriptions of reality. Precisely, the authors attempt to demonstrate how and where special functions appear in the analysis of special queuing models.

Initially a few themes in queuing and special functions are reviewed, though the short treatment of queues is dwarfed by the more exhaustive presentation of the latter. Gamma and beta functions for instance are developed in detail; hypergeometric functions and their transformations, as well as properties, are described. So are the generalized hypergeometric functions  $E$ ,  $G$ , and  $H$ . Orthogonal polynomials are reviewed.

The book is then divided into six essentially self-contained chapters. Chapter 1 begins with a detailed analysis of the simplest and most widely studied models— $M/M/1$  (Poisson arrivals and service completions). It is demonstrated how modified Bessel functions appear in the transition probabilities related to this model. Such results are then extended to multiple server systems known for many years. The chapter ends, however, with the analysis of this model with added restrictions which have been studied more recently, such as queues with bounded waiting times.

Queues in which customers receive service in accordance with different priorities have been widely studied in recent years. Chapter 2 is devoted to this topic and the special role special functions play in their analysis. Various priority schemes may, of course, be considered, each resulting in different forms of solutions for the underlying basic, and often common, equations. In this chapter queues with bulk arrivals and batch services are considered as well.

Chapter 3 is devoted to a comprehensive review of the more recently studied topic of queues with variable parameters. Though a lot is learned about solutions of models with such alternative formulations, much more is left to be done. The book's emphasis is again on the role special functions play; indeed it is instructive to see how the solutions reduce to their corresponding constant rate forms. In the context of variable rates, balking and renegeing behavior of customers is also reviewed. Some results are available for such models and are presented in this chapter as well. Multiple server systems with variable rates are also reviewed and analyzed. This chapter contains many recent results published, not only in journals with wide circulations, but also less available journals and monographs. Chapters 4 and 5 treat queues where only one or neither of the underlying processes is Poisson. The authors go through the same steps as they did in Chapter 1 with a wide review of available literature. Time dependent rates are considered as well.

Chapter 6 essentially stands separate from the rest of the book. It is not on the subject of queues, but rather on other subjects where stochastic modeling is employed. Models are picked from such related fields as population genetics, reliability and linear programming and analyzed with the end results involving special functions in some fashion.

Students of applied probability and mathematically-oriented individuals in operations research would learn a lot from reading this book. The book is as well intended for those students of special function theory who would like to see areas of application. The authors deserve special credit for bringing together a vast amount of information from various sources.

Like any branch of mathematical sciences, queuing theory is clogged with repetitious articles often providing only a slight variation of the same basic equations. The fact remains, however, that when the two (or more) underlying

processes in a queuing system push against one another, many problems arise that are not only challenging but indeed of important practical consequence. Many such problems pertinent to simpler models are already resolved. Much more, however, remains to be done with complex systems without the inclusion of the traditional assumptions.

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*Approche élémentaire de l'étude des fonctions arithmétiques*, by J. M. de Koninck and A. Mercier, Les Presses de L'Université Laval, Quebec, 1982, xvii + 309 pp., \$24.00. ISBN 2-7637-6947-0

The authors of this book assert that their object is to provide a book whose material can serve as a second course in number theory. It is their view that from a pedagogical viewpoint, the best way to stimulate interest in mathematics among beginners is to expose them to subjects which have an appeal to the intuition—one such subject being number theory. Number theorists can hardly quarrel with this enlightened viewpoint!

The preparation required to read this book is the content of a standard course in elementary number theory. Topics included in the book are: elementary results on the distribution of primes, the elementary proof of the prime number theorem, Dirichlet's theorem on primes in an arithmetic progression, arithmetic functions and the associated Dirichlet series, order of magnitude of arithmetic functions, asymptotics of additive and multiplicative number theoretic functions, asymptotics of sums of arithmetic functions over subsets of  $\mathbb{N}$ , sieve methods and applications.

The authors have written a nice book with an abundance of exercises. While much of the material is standard, the specialist will find material on additive and multiplicative functions presented here for the first time in an easily accessible form.

The style is clear and the explanations easy to follow. It should be said that some of the material is of a very specialized nature and not entirely appropriate to a second semester course in number theory. Of course, this is a matter of individual taste, but the reviewer would favor topics more central to the discipline.

However, there is, fortunately, ample material in the book and a choice of excellent topics can be made for a second semester course based on these topics. The specialized material remains for those interested in delving more deeply into properties of arithmetic functions.

On the whole, the book is to be well recommended.

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