20. A. R. Forsyth, *Theory of differential equations*, Vols. 5 and 6, Dover, New York, 1959.
21. E. Hopf, *The partial differential equation* $u_t + uu_x = \mu u_{xx}$, Comm. Pure Appl. Math. **3** (1950), 201–230.
22. ______, *Generalized solutions of non-linear equations of first order*, J. Math. Mech. **14** (1965), 951–974.
23. S. N. Kruzkov, *Generalized solutions of nonlinear first order equations with several independent variables*. II, Mat. Sb. **72**; English transl. in Math. USSR Sb. **1** (1967), 93–116.
24. P. D. Lax, *Nonlinear hyperbolic equations*, Comm. Pure Appl. Math. **6** (1953), 231–258.
25. ______, *The initial value problem for nonlinear hyperbolic equations in two independent variables*, Ann. Math. Studies **33** (1954), 211–229.
26. ______, *Weak solutions of nonlinear hyperbolic equations and their numerical computation*, Comm. Pure Appl. Math. **7** (1954), 159–193.
27. ______, *Hyperbolic systems of conservation laws*. II, Comm. Pure Appl. Math. **10** (1957), 537–566.
28. O. A. Oleinik, *Discontinuous solutions of non-linear differential equations*, Uspehi Mat. Nauk. **12**; English transl. in Amer. Math. Soc. Transl. (2) **26** (1957), 95–172.

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Surfaces and planar discontinuous groups, by Heiner Zieschang, Elmar Vogt, and Hans-Dieter Coldewey, Lecture Notes in Math., vol. 835, Springer-Verlag, Berlin, 1980, x + 334 pp., $21.00. ISBN 0-3871-0024-5

There is a basic connection between the study of combinatorial group theory and Riemann surfaces which arises as follows: If S is a compact Riemann surface of genus $g \geqslant 2$, then its universal cover is U, the unit disc in the complex plane, and S can be represented as U/Γ where Γ is the group of covering transformations. Γ is generated by $2g$ transformations $a_1,\ldots, a_g$, $b_1,\ldots, b_g$ which satisfy the relation $\prod_{i=1}^{g}[a_i,b_i] = 1$. Here $[c,d] = cdc^{-1}d^{-1}$ and a_i and b_i are Möbius transformations which leave U invariant. The group Γ is an example of a planar discontinuous group and Γ is said to represent S. The single relation above gives a finite presentation of Γ with $2g$ generators. The situation for noncompact surfaces is more complex, but similar.

The book under review begins with combinatorial group theory and develops that part of combinatorial group theory which is relevant to Teichmüller theory and the theory of Riemann surfaces. Teichmüller theory and Riemann surfaces is an interesting field to work in because it lies at the intersection of so many fields: complex analysis, topology, algebraic geometry, differential geometry, combinatorial group theory, and geometric topology. Most of the basic theorems in the subject can be proved using the methods of any one of these fields. When one translates from the language of one field to that of another, different aspects of the theory become either more or less clear and elegant.

An example of where the combinatorial approach is especially nice is the application of the Reidemeister-Schreier methods to the theory of automorphisms of Riemann surfaces: A homeomorphism of a Riemann surface induces an outer automorphism of the fundamental group of the surface and an automorphism of the first homology group (which is the abelianized fundamental group). An automorphism h of S (i.e. a conformal self-map) corresponds to a self-homeomorphism of finite order. Using analytic methods, Hurwitz proved that an automorphism which induces the identity on homology must be the identity [**6**]. This result has many generalizations. Accola [**1**] used differentials to obtain a strong generalization. Marden, Richards, and Rodin [**12**] also obtained strong generalizations using analytic methods. Grothendieck and Serre [**4**] used algebraic methods to prove that if h induces the identity on the first homology group with coefficients in Z_n for any $n > 2$, then h is the identity.

All of these results tell what an automorphism of a Riemann surface cannot do. They are of the form: "If an automorphism h has such and such an action on homology, then h must be the identity." On the other hand, applying Reidemeister-Schreier methods to the appropriate group-theoretic situation, one can actually write down the matrix of the induced action on homology for an automorphism of prime order [**3**]. This explicit matrix representation implies many of the classical results.

If h is such an automorphism, and $L(H)$ the group of all lifts to U of h and its powers, then $L(H)$ is a finite extension of Γ and Γ is isomorphic to the fundamental group of S. $L(H)$ is a finitely presented group whose presentation is related to Γ and to the fixed point structrue of h. The action of h on the fundamental group of S corresponds to the action of $L(H)$ by conjugation on Γ. Reidemeister-Schreier theory allows one to obtain a finite presentation for a subgroup Γ of finite index in a group $L(H)$ with a known presentation, and thus to calculate the conjugation action. This is a straightforward algebraic calculation, and the corresponding action on homology comes from merely abelianizing.

Zieschang, Vogt, and Coldeway treat this from an even broader perspective, applying Reidemeister-Schreier to any pair of groups to obtain the most general possible results along these lines including Zimmerman's generalized Riemann-Hurwitz formula [**15**].

This book is not only for people interested in Riemann surfaces. It will also be read by two other audiences. The first consists of people concerned with combinatorial group theory per se. There are two well-known books in this area, one by Magnus, Karrass and Solitar [**11**] and the other by Lyndon and Schupp [**7**]. The original German version of this text [**14**] had a large influence upon the latter book as acknowledged by its authors in several places. For an excellent summary and historical perspective on the problems of combinatorial group theory which will also place this book in that perspective see Gilbert Baumslag's review of the first book [**2**].

The second audience consists of those people who wish to see an account of the classification of all planar discontinuous groups. These groups are also known as noneuclidean crystallographic (NEC) groups. These are discrete groups generated by real Möbius transformations with determinant 1 together with orientation-reversing reflections and glide reflections. This book provides a unified treatment of the classification which originally depended upon the work of many authors writing from different points of view. These authors include Macbeath, Wilke, Hoare-Karrass-Solitar, and Zieschang (see [**8**, **9**, **13** and **5**]). The generality needed for arguments involving *all* NEC groups (and not just Fuchsian groups) at times obscures the simplicity of the combinatorial approach to Riemann surface theory.

From the perspective of the book a planar discontinuous group arises as follows: A *surface* is a connected two-complex where each directed edge appears at least once and at most twice in boundary paths such that for any two edges and any initial point P there is a star around P beginning with one edge and ending with the other. A *planar net* is an open surface, E, where each vertex has a finite star and each simple closed path bounds a disc. There is a certain notion of automorphism of a planar net (a mapping which preserves faces, edges, and vertices in such a way that boundary relations are preserved and such that each part of the net has a preimage). A *planar discontinuous group* is an equivalence class (E, G) where E is a planar net and G is a group of automorphisms of E. (Here equivalence means that one pair can be obtained from another by certain chains of "extensions" and "reductions".) Intuitively, a planar net is a tiling of the plane and a planar discontinuous group is a group for which a tile is a fundamental domain.

The book begins with the elementary theory of free groups and graphs and proceeds through 2-dimensional complexes and combinatorial presentations of groups. It then turns to surfaces and gives a nice exposition of covering surfaces, intersection numbers of curves (one topic which cannot be treated algebraically to my knowledge) and mappings. The classification of planar discontinuous groups and the theory of automorphisms of such groups follows. The final two chapters are devoted to the complex analytic theory of Riemann surfaces and planar discontinuous groups and the topological theory.

Short historical summaries explaining both where some of the work originally appeared and recent developments are to be found at the end of many sections of the book. These summaries place the roughly four hundred items in the bibliography in perspective and make interesting reading. However, the authors have not attempted to list a complete bibliography or to accurately assess every article.

The book assumes only very basic knowledge of group theory and complex analysis and can be read by any graduate student. Each chapter contains a nice series of exercises. The book is an English translation and an expansion of an earlier text by the same authors and emerged from notes taken by E. Vogt at lectures delivered by H. Zieschang. It also contains contributions by D. Coldeway and J. Stillwell. Stillwell has done a good job of translating the original German text.

REFERENCES

1. R. D. M. Accola, *Automorphisms of Riemann surfaces*, J. d'Analyse Math. **18** (1967), 1–5.

2. Gilbert Baumslag, *Review of combinatorial group theory by Magnus, Karrass and Solitar*, Bull. Amer. Math. Soc. (N.S.) **1** (1979), 950–953.

3. Jane Gilman, *On a matrix representation for automorphisms of Riemann surfaces*, Linear Algebra and its Applications **17** (1977), 139–147.

4. A. Grothendieck and J.-P. Serre, *Techniques de contruction en geometrie analytique*, Seminaire H. Cartan, Paris, 1960/61. Exp. 7–12 and the Appendix by Serre.

5. A. H. M. Hoare, A. Karrass and D. Solitar, *Subgroups of NEC groups*, Comm. Pure Appl. Math. **26** (1973), 731–744.

6. A. Hurwitz, *Algebraische Gebilde mit eindeutigen Transformationen in sick*, Math. Ann. **41** (1893), 403–442.

7. R. C. Lyndon and P. E. Schupp, *Combinatorial group theory*. Ergeb. Math. Grenzgeb. vol. 89, Springer-Verlag, Berlin, 1977.

8. A. M. Macbeath, *Groups of hyperbolic crystallography*, Math. Proc. Cambridge Philos. Soc. **31** (1975), 699–712.

9. ______, *The classification of noneuclidean plane crystallographic groups*, Canad. Math. J. **6** (1976), 1192–1205.

10. W. Magnus, *Noneuclidean tessalations and their groups*, Academic Press, New York, (1974).

11. W. Magnus, A. Karrass and D. Solitar, *Combinatorial group theory*, Wiley, New York, 1966.

12. A. Marden, I. Richards and B. Rodin, *Analytic self-mappings of Riemann surfaces*, J. D'Analyse **18** (1967), 197–225.

13. M. C. Wilke, *On noneuclidean crystallographic groups*, Math. Z. **91** (1966), 87–102.

14. H. Zieschang, E. Vogt and H.-D. Coldeway, *Flächen und ebene diskontinuierliche Gruppen*, Lecture Notes in Math., vol. 122, Springer-Verlag, Berlin and New York, 1970.

15. B. Zimmerman, *Eine Verallgemeinerung der Formel von Riemann-Hurwitz*, Math. Ann. **229** (1977), 279–288.

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Mathematical theory of entropy, by Nathaniel F. G. Martin and James W. England, Encyclopedia of Mathematics and its Applications, vol. 12, Addison-Wesley Publishing Company, Reading, Mass., 1981, xxi + 257 pp., $29.50. ISBN 0-2011-3511-6

Topics in ergodic theory, by William Parry, Cambridge Tracts in Mathematics, vol. 75, Cambridge Univ. Press, Cambridge, England, 1981, x + 110 pp., $23.95. ISBN 0-5213-3986-3

An introduction to ergodic theory, by Peter Walters, Graduate Texts in Math., vol. 79, Springer-Verlag, Berlin and New York, 1982, ix + 250 pp., $28.00. ISBN 0-3879-0599-5

Ergodic theory is concerned with the action of a transformation T or group of transformations G on a space X. The space X usually has some measure-theoretic, topological, or smooth structure which T or G preserves. Typical of the kinds of questions asked is the early recurrence theorem of Poincaré, which states that if T preserves a finite measure then almost every point in any set of positive measure must return to the set under the action of T. The much deeper convergence theorems of von Neumann and Birkhoff triggered considerable