Two chapters are devoted to group extensions. The cohomology and homology groups of the group G with coefficients in the $\mathbf{Z}G$-module A are defined by

$$H^n(G, A) = \operatorname{Ext}^n_{\mathbf{Z}G}(\mathbf{Z}, A), \qquad H_n(G, A) = \operatorname{Tor}_n^{\mathbf{Z}G}(\mathbf{Z}, A),$$

and the lower-dimensional ones are interpreted. Of particular interest are $H^2(G, A)$, the group of extensions of A by G, and $H_2(G, A)$, the Schur multiplier of G according to a formula by Hopf.

There is a final chapter on spectral sequences, which occupies about one sixth of the book and which emphasizes their use as a technique for computing homology. The reviewer admits regretfully that he did not read this chapter.

It is unusual for textbook writers to bother much about who originated what. The present author should therefore be applauded for going out of his way to attribute credit for theorems and proofs.

I found this book pleasant and stimulating reading. Its enthusiastic style is infectious and managed to rekindle my interest in the subject.

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Schottky groups and Mumford curves, by L. Gerritzen and M. van der Put, Lecture Notes in Math., vol. 817, Springer-Verlag, Berlin and New York, 1980, viii + 316 pp., $19.54.

I suspect that even in this modern age of categories and functors, most mathematicians still view p-adic analysis with a certain amount of disdain. If you're one such person, before you flip back to the research announcements, please allow me to show you a very important result, due to John Tate, that will change your mind, I hope.

We begin by reviewing some well-known concepts. Let H be the upper half-plane $\{x + iy \in \mathbf{C} \mid y > 0\}$. Let $\tau \in H$ and let $L = \{\mathbf{Z} + \mathbf{Z}\tau\}$. Thus, L is a two-dimensional lattice and it is a standard beautiful fact that $\mathbf{C}/L = E$ "is" an elliptic curve. This may be seen in either of the following ways:

(a) The field of L-invariant meromorphic functions on $\mathbf{C}$ forms an elliptic function field; or,

(b) the Riemann Surface E may be embedded into $\mathbf{P}^2(\mathbf{C})$ as a nonsingular cubic

$$y^2 = 4x^3 - g_2 x - g_3,$$

via the use of the classical Weierstrass $\wp$-function: In this last case we set, as functions of E,

$$\Delta = g_2^3 - 27g_3^2 \quad \text{and} \quad j = (12g_2)^3/\Delta.$$

It is well known that $\Delta \neq 0$ and that j characterizes E up to isomorphism (algebraic or complex, they are equivalent). We can, in fact, study such cubics over any algebraically closed field in a similar manner.

Now the act of forming the quotient of $\mathbf{C}$ by L can be viewed as having two steps: First we form the quotient of $\mathbf{C}$ by $\mathbf{Z}$ under the exponential map $e^{2\pi iz}$. If we do so, we find $\mathbf{C}/\mathbf{Z} \simeq \mathbf{C}^*$, and the image of L is the *multiplicative* lattice generated by $q = e^{2\pi i\tau}$. Then, we divide $\mathbf{C}^*$ by $\{q^{\mathbf{Z}}\}$ to obtain E. As functions of q we find

$$\frac{\Delta}{(2\pi)^2}(q) = q \prod_{n=1}^{\infty} (1 - q^n)^{24},$$

and

$$j(q) = 1/q + 744 + 196884q + \cdots.$$

Okay, so far so good, everything is nice and complex analytic, and we are now ready for Tate's result. Let k be a finite extension of $\mathbf{Q}_p$ (the p-adic numbers) or a formal Laurent series field over a finite field. Let $|\ |$ be the norm on k, $R = \{x \in k \,|\, |x| \leq 1\}$ and $M = \{x \in R \,|\, |x| < 1\}$. Finally, let K be an algebraic closure of k equipped with the canonical extension of $|\ |$, and let $q \in k^*$ with $1 > |q|$. It is easy to see that the expansions for $j(q)$ and $\Delta/(2\pi)^{12}(q)$ converge to elements of k. Tate's remarkable result is that $K^*/\{q^{\mathbf{Z}}\}$ "is" an elliptic curve over k with invariants $j(q)$ and $\Delta/(2\pi)^{12}(q)$ in the sense that the field of $\{q^{\mathbf{Z}}\}$ invariant meromorphic functions on K^* (i.e.; functions that are quotients of everywhere convergent Laurent-series over k) is an elliptic function field with the above invariants. Not all elliptic curves arise in this fashion, but Tate shows that those curves which arise in this fashion are precisely the ones with "split multiplicative" reduction at M. *The main point is that it is precisely Tate's construction that allows one to make a complete study of the q-expansions of classical modular forms which are computed through complex analysis.* These ideas have been exploited by Katz, Mazur, Serre, Deligne, Ribet, Rapoport, etc., in a variety of directions.

I hope that you are now convinced of the necessity of p-adic analysis and its subtlety and are intrigued enough to finish the review.

Two questions come immediately to mind: What about the analogue of the second interpretation of $\mathbf{C}/L$, and, what about generalization to curves of higher-dimensional genus?

To answer the first question, Tate developed his theory of "rigid analytic spaces". This is a theory of p-adic continuation that allows one to give the space $K^*/\{q^{\mathbf{Z}}\}$ a structure similar to the complex structure on $\mathbf{C}^*/\{q^{\mathbf{Z}}\}$. In particular, one has the all important G.A.G.A. Theorem for this theory. So, as $K^*/\{q^{\mathbf{Z}}\}$ is a complete, connected, nonsingular analytic space of dimension one equipped with a group structure it, again, "is" an elliptic curve. The embedding can be given by using q-expansions of classical modular forms. The topics of rigid spaces is treated briefly in Chapter III of the book, but for a complete study one can now refer to J. Fresnel and M. van der Put, *Géométrie analytique rigide et applications*, Birkhauser 1981. (It should be pointed out that there was an earlier theory of p-adic analytic continuation developed in the 50's by Krasner and used by Dwork, etc.)

The answer to the second question is due to David Mumford and forms the subject of the book being reviewed. Before stating Mumford's ideas we recall briefly the classical theory of Schottky groups: Let $\Gamma \subseteq \mathrm{PGL}(2, \mathbf{C})$ be a Kleinian group; so Γ acts discontinuously at at least one point of $\mathbf{P}^1(\mathbf{C})$ under the standard action. Assume also that Γ is free and contains no unipotent elements. Then a result due to B. Maskit tells us that Γ is a *Schottky group*, i.e., if we set Ω = set of points of $\mathbf{P}^1(\mathbf{C})$ where Γ acts discontinuously, then Ω is connected and Ω/Γ is a compact Riemann surface. It is precisely an analogue of this Schottky uniformization that Mumford uses to give a vast generalization of Tate's results.

So, a subgroup $\Gamma \subseteq \mathrm{PGL}(2, k)$ is called a *Schottky group* if and only if

(a) Γ is finitely generated,

(b) Γ has no nontrivial elements of finite order, and

(c) Γ is discontinuous.

By a result of Ihara, Γ is a free group and we let $\{\gamma_1, \ldots, \gamma_g\}$ be a set of generators. We let $\Omega \subseteq \mathbf{P}^1(K)$ be the set of ordinary points of Γ, i.e., those points where Γ acts discontinuously. Then Ω has a rigid analytic structure as a connected subject of $\mathbf{P}^1(K)$. The action of Γ on Ω has a fundamental domain given by removing $2g$ disjoint open discs in $\mathbf{P}^1(K)$. The main result of Mumford is that Ω/Γ is analytically isomorphic to a complete nonsingular connected curve of genus g. As with elliptic curves, not all curves arise in this fashion, but those that do may be characterized by their particularly simple reductions at M. Such curves are called by the authors "Mumford curves". They treat this construction from both a geometric and function-theoretic viewpoint.

Once one has a Mumford curve, one can ask about its Jacobian, automorphism group, etc. Manin-Drinfeld and Myers, have shown that the Jacobian of a Mumford curve can be constructed as an "analytic torus", which is a generalization of Tate's construction of elliptic curves to higher-dimensional abelian varieties. Furthermore, F. Herrlich has shown that the order of the automorphism group of a Mumford curve is much smaller than the estimate given classically by Hurwitz. Both subjects are given a detailed treatment by the author.

Mumford's work was one motivating factor in Drinfeld's analytic theory of $\mathbf{P}^1(K) - \mathbf{P}^1(k)$, k a function field over a finite field, and this subject is treated in the last chapter.

In all, the book is a clear and thorough introduction into the beautiful theory of Mumford curves and their geometry.

One related subject, which unfortunately wasn't treated, is the work of I. V. Cherednik (*Algebraic curves uniformized by discrete arithmetic subgroups*, Uspehi Mat. Nauk **30** (1975), no. 3, 181–182; see also V. G. Drinfeld, *Coverings of p-adic symmetric regions*, Functional Anal. Appl. **10** (1976), 107–115). This work establishes that certain Shimura curves parametrizing abelian varieties can be uniformized as quotients of $\mathbf{P}^1(K) - \mathbf{P}^1(k)$, $k = \mathbf{Q}_p$. This again illustrates the link between complex and p-adic analysis and perhaps will be treated in later books on the subject.

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