worthwhile work, but a decade later the subject has lost much of its interest. Not every answer deserves a new question. The past ten years have produced a spate of counterexamples in the type of harmonic analysis considered here. No doubt more will be forthcoming, and one can expect that the resolution of some of the unsolved problems listed by Graham and McGehee will show a lot of cleverness and ingenuity. But it is time to do something else. The general questions which can be posed in terms of locally compact abelian groups really come down to **T**, **Z** and **R**. There are still some things to do in $\mathbf{R}^n$. I would rather see some positive results related to nice subsets of $\mathbf{R}^n$ than more counterexamples for perfect nowhere dense subsets of $\mathbf{R}^1$. Also, commutative methods applied to noncommutative Lie groups have yielded some interesting results, but the authors say little about this.

In summary, Graham and McGehee have written an interesting monograph, and written it well, but it is an epitaph for an epoch in Harmonic Analysis, 1940–1980. Rest in peace.

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Operator algebras and quantum statistical mechanics, Volumes I and II, by Ola Bratelli and Derek W. Robinson, Springer-Verlag, New York-Heidelberg-Berlin; Volume I, *C^* and W^*-algebras, symmetry groups, decomposition of states*, 1979, xii + 500 pp., $36.00; Volume II, *Equilibrium states, models in quantum statistical mechanics*, 1981, xi + 505 pp., $46.00.

The theory of operator algebras was initiated by von Neumann in 1927, and Murray and von Neumann in 1936. One of the principal motivations of Murray and von Neumann for the theory was an application to a quantum mechanical formalism. Nevertheless, until the late fifties, there was no significant interplay between operator algebras and quantum physics except for Segal's pioneering works on a C^*-quantum mechanical formalism. Up to the time, the theory had more or less developed independently of physics. In fact, most researchers of the theory were mathematicians, and their interests were mainly concerned with structure theory and representation theory, which are the traditional research subjects in ring theory and group representation theory. In 1957, Haag introduced the notion of the quasi local structure of von Neumann algebras in field-theoretic models and emphasized its importance to quantum physics. Ever since, many mathematical physicists have joined in the study of operator algebras, and the fruitful collaboration between mathematicians and physicists has made remarkable progress in the theory and significant applications to quantum physics, so that nowadays the so-called C^*-quantum physics becomes one of the central branches in the theory. Physical theories consist essentially of two elements, a kinematical structure describing the instantaneous states and observables of the system, and a dynamical rule describing the change of these states and observables with time. In the C^*-quantum physics, the infinite system is an appropriate C^*-algebra or von Neumann algebra and its elements represent kinematic observables, and the states on the C^*-algebra or the normal states on the von Neumann algebra describe the instantaneous states of the system. For a complete physical description it is necessary to specify the dynamical law governing the change with time of observables or the states. This dynamics is given by a continuous one-parameter group of *-automorphisms of the algebra or more generally by a *-derivation in the algebra as the infinitesimal change of the system. Consequently, the physical theory in operator algebras becomes richer than the general theory, because one has to deal with operator algebras, one-parameter groups of *-automorphisms, *-derivations, various symmetry groups of *-automorphisms, invariant states under those automorphism groups and so on. Naturally, researchers in the C^*-quantum physics were forced to redevelop the theory in relevant directions. New concepts were introduced, new techniques applied, new important results obtained, and new advanced theory is being formed. The C^*-quantum physics is most successful in abstraction of quantum lattice systems, so that there is a good possibility that the theory of quantum lattice systems will be well developed within the C^*-framework, though it has as yet made no significant contribution to the phase transition theory which is one of the most important subjects in statistical mechanics. The phase transition theory would be the future subject with which the C^*-quantum physics has to be seriously concerned.

In the book under review, the authors describe the elementary theory of operator algebras and parts of the up-to-date advanced theory which are of relevance to mathematical physics. Subsequently they describe various applications to statistical mechanics. The selection of topics from the advanced theory in this book is reasonable, though it somewhat leans towards the research directions and tastes of the authors. The reference list is not complete, so that references on the historical background in some sections are not enough. However these minor criticisms are negligible. By and large the authors have excellently done the difficult job of exposing the subject matter which is a mixture of standard theory and new research work which has not previously appeared in book form. It is a good textbook for mathematicians and physicists who want to learn the C^*-quantum physics. In the following, I will review the book chapter by chapter. It consists of two volumes. The first volume is devoted to mathematical theory of operator algebras and their dynamics, and the second to its applications to quantum statistical mechanics, and to the models of quantum statistical mechanics. The first volume contains four chapters and the second contains two chapters. The chapters of the second volume are numbered consecutively with those of the first. Chapter 1 is a brief historical introduction. The historical introduction is not concerned with the theory of operator algebras, but it is concerned with the interplay between operator algebras and quantum physics. Chapter 2 is C^*-algebras and von Neumann algebras. It discusses the elementary theory of operator algebras, Tomita-Takesaki theory and the standard form of von Neumann algebras, quasilocal algebras, and miscellaneous results and structure. The authors select material from the general theory of operator algebras which is needed for quantum physics. Chapter 3 is groups, semigroups, generators. In this chapter, the authors discuss mainly derivations, automorphism groups and generation problems. Chapter 4 is decomposition theory. Here various decompositions of states are treated. The authors use a modern method developed recently by many researchers, which combines the reduction theory of von Neumann with the Choquet theory. In the ergodic decomposition which is of importance in mathematical physics, the notion of G-abelianness introduced by Lanford and Ruelle is used.

The contents of the first volume is rich enough to use it as a textbook for advanced graduate students in the field of functional analysis. For physics students, there might be too much abstraction. Chapter 5 is states in quantum statistical mechanics. Here the authors describe continuous quantum systems, KMS states, and stability and equilibrium. The material prepared in Volume 1 is seriously used in the two sections of KMS states, and stability and equilibrium. Chapter 6 is models of quantum statistical mechanics. Here, the authors describe quantum spin systems and continuous quantum systems. This chapter is most instructive for C^*-algebraists, though there is little involvement of the theory of operator algebras. The reason is that it would be an interesting problem to extend various results in this chapter to more general C^*-dynamics.

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Global Lorentzian geometry, by John K. Beem and Paul E. Ehrlich, Pure and Applied Mathematics, vol. 67, Dekker, New York, 1981, vi + 460 pp.,

The past two decades have witnessed an enormous growth in the development of global methods in Lorentzian geometry. The time seems ripe for a systematic treatment of global Lorentzian geometry written in the language of