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Bedienungsprozesse, by Gennadi P. Klimow, translated from the Russian by V. Schmidt, Birkhäuser Mathematische Reihe 68, Basel and Stuttgart, 1979, xi + 244 pp., $38.00.

Queues or waiting lines are among the richest sources of problems and examples in the theory of stochastic processes. The simplest queues serve well to illustrate the basic theory of Markov chains. The service systems, which are of research interest today, involve complex interactions of queues and are described in terms of typically multi-dimensional stochastic processes. Such descriptions hold, only if we are able to formalize the processes at all. Realistic queueing systems are formidable dynamic-stochastic systems indeed.

From isolated but distinguished contributions by Felix Pollaczek and A. Ya. Khinchin in the thirties and forties, the theory of queues emerged as a subdiscipline of probability theory during the years 1950–1960. Its growth since then has truly been astounding. I usually place the number of journal articles on queues at 7500 and know of some fifty books fully devoted to this subject. Both numbers probably underestimate the actual size of the literature. By those studying the job flows in telecommunication and manufacturing systems or inside computers or computer networks, the relevance of understanding the behavior of queues is taken for granted. In spite of this, several persons of learning have at various times declared the whole subject of queueing theory to be moribund or, what is sadder, irrelevant.

Why is there an apparent contradiction between the obvious vitality of the literature on queues and a fairly widespread, negative perception of its relevance? Why indeed is the theory of queues frequently singled out for criticism from both left and right? Persons with a need for practical results blame the formality and the lack of direct applicability of the published results. Others, with a predilection for abstract probability theory, feel that it suffices to formulate a minor variation of an existing queueing model, and one can "crank out" yet another paper in this already glutted field.

Both critics make valid points. The formalism and the repetitions of whiskered arguments are too obvious to be denied, particularly by those having a deep, almost passionate interest in this subject.

A historical comparison and perspective may be helpful. The development of queueing theory runs almost parallel to that of ordinary differential equations, which, as is well known, sprang from the study of mechanics during the eighteenth and nineteenth centuries. After the simple cases with elementary explicit solutions had all been treated, one saw a profusion of papers dealing with purely formal solutions in terms of ever more involved special functions. The relation to any concrete problems became more and more difficult to perceive and was, in most cases, nonexistent. The subject, dangerously near to collapse under its own weight, regained practical relevance and new mathematical vitality through the strongly interrelated developments of numerical methods and qualitative theory.

In the theory of queues, the formal period may be nearing its term. The need for informative, qualitative results on service systems is growing, and the ability of the classical formal, analytic methods to fill it is now recognized as being inadequate. New developments along the lines of qualitative and algorithmic analysis are now emerging, which offer renewed vitality to the field and new intellectual challenges to those working in it. Beautiful earlier contributions such as the Spitzer-Pollaczek formula, the Wiener-Hopf analysis of the $GI/G/1$ model and the combinatorial methods of Takács, among others will endure. These are results with a mathematical life of their own. The infinite minor variations upon classical models will appear as particular cases of broader structural results. They will hopefully vanish from the journals, unlamented by editors and readers alike.

The preceding discussion is necessary to place the recent book by Professor G. P. Klimov in a proper perspective. It is a well written and mathematically sound treatment of queueing theory, but it belongs squarely to the formal period. Among books on queues it is rather thin. Of the 236 pages of actual text, 54 review the basic theory of renewal processes and other point processes, and 37 pages are devoted to a review of general mathematical background material, such as Lebesgue-Stieltjes transforms, simple differential equations, Poisson-Charlier polynomials and Markov processes.

Two long chapters, 3 and 4, deal with the so-called classical queueing models with one or more servers. There are brief treatments of priority queues, server breakdowns and control of queues, and a very detailed discussion of Lindley's equation by the Wiener-Hopf technique. The formalism of the author's presentation may be illustrated by two examples. In many derivations, he uses D. van Dantzig's *method of collective marks*, without due credit to its originator. This is a simple, but clever device, which allows one to think of probability generating functions or Laplace-Stieltjes transforms of nonnegative random variables as *probabilities* of events in an associated *Gedankenexperiment*. Doing so, permits one to prove by interpretation a number of transform formulas, which otherwise require some tedious computations. From extensive classroom experience, I have learned not to use it, except as a tool for verification. Only the most astute students can keep track of the various assumptions of independence used along the way, and all are unfortunately encouraged to look only for solutions in terms of transforms.

The second example is related to the latter comment. The author presents the classical functional equation for the transform of the busy period for the $M/G/1$ queue. He also obtains transform expressions, similar to the Pollaczek-Khinchin formula, for the stationary waiting time distributions of several queueing models. In both cases, the transform formula hides a very useful integral equation of Volterra type. The former is nonlinear and somewhat challenging; the latter is linear, elementary and beautifully suited for numerical solution. More importantly, the transform-free equations reveal structural properties which can be widely generalized and which lead to unified algorithmic approaches. Transform solutions do not, in general, offer these advantages.

Chapters 5 and 6 deal with queues in which the server's time is allocated to several arrival streams. This choice of topic reflects the author's research interests. It is the most novel area treated in this book and makes for interesting, if somewhat specialized, reading.

It was with great anticipation that I turned to Chapter 7, whose title announces the discussion of a statistical estimation method for queues. The statistical analysis of queues and other probability models is regularly described as one of the most significant areas for research, but it just does not seem to get beyond the embryonic stage. The author's treatment is disappointing. Chapter 7 contains a general description of statistical risk theory and a few classical procedures from the theory of life testing. Its relevance to the difficult inferential problems of queueing theory is tenuous.

Both the index and the bibliography of this book are so sketchy and incomplete as to be useless. The author occasionally uses unfortunate terminology. A Poisson process with group arrivals is called a nonregular Poisson process. The editor has supplied a number of helpful footnotes, which clarify such matters. It is nice to tell the student that a recurrent stream is the same as a renewal process and that a stationary ordinary stream without aftereffects is more commonly known as a Poisson process.

In summary, this book is safe and useful in the hands of experienced queueing theorists. Because of the formality of its treatment and the total lack of discussion of the insights into *the behavior of queues*, which the mathematical analysis should provide, I would never consider its use as a text book.

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