mann? Weierstrass? Cantor? Borel? Lebesgue? Hilbert? L. Schwartz?") and some are simply impossible ("There are two major types of nuclear explosive devices. Describe the mathematical formulation of the action in each case.")

At the close of his provocative essay on the gap between disciplines, Freeman Dyson writes of ways to bring mathematics and physics back together. It is at present not realistic, he says, to expect members of one group to make original contributions to work of current interest in the other. The fields have drifted too far apart and their union is too large for a single intelligence to span. What can be done, at least for the time being, is to establish contact through papers of a special kind: when a new result in one field shows promise of attracting interest in the other, a review article gathers up points of contact and proposes areas of collaboration. It will, of course, take a change of attitude, the invention of new kinds of reward, and a few reforms in graduate education to make this happen. It is perhaps just barely possible. And what about a more intimate reconciliation of the sciences in the long future? Here our imaginations must range more freely. The solution–a dangerous one, says Dyson–lies in the hands of the biologists who will ultimately discover ways of extending human memory and intelligence to the point where the whole of science is once again comprehensible to one human being. Meanwhile we must do what we can with the natural mind as it is given to us.

An excellent book on the present status and possible future of useful mathematics would be a step in the direction that Dyson envisions. Professor Murray's first attempt falls short of the requirement. Perhaps he, or another mathematician of equal distinction and equal dedication to the task, will give it another try.

EVERETT HAFNER

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 3, Number 1, July 1980

0002-9904/80/0000-0310/$01.50

Relativistic theories of materials, by Aldo Bressan, Springer Tracts in Natural Philosophy, vol. 29, Springer-Verlag, Berlin-Heidelberg-New York, 1978, xiv + 290 pp.

Einstein's general relativity is primarily a unification of gravity with spacetime geometry: the curvature of a four-dimensional Lorentzian manifold signals the presence of gravity. But the theory can be regarded as a complete description of at least macrophysics; it necessarily deals with electromagnetism and matter in addition to gravity. In fact its most important specific postulate, the Einstein field equation $G = T$, describes, roughly speaking, how matter and electromagnetism generate gravity. The equation relates a purely geometric object with a physical, almost anti-geometric one: G, the Einstein curvature, is determined at a spacetime point by certain averages of the sectional curvatures there; T, the stress-energy tensor field, is determined by electromagnetism and matter. Einstein's own attitude toward this contrast is given, for example, by his comments on the equation in his autobiographical contribution to *Albert Einstein, Philosopher-Scientist* (Paul A. Schlipp editor, Library of Living Philosophers Inc., 1949). At that time he felt current theories of electromagnetism and matter required replacement by a more geometric theory like his "unified field theory" so that the equation would ultimately be replaced by a wholly geometric one. Today, Einstein's attempts along these lines are regarded as obsolete. But the problem of how to combine matter models with his theory of gravity remains pressing.

Our most nearly fundamental and most nearly satisfactory models of matter and electromagnetism are furnished by special relativistic quantum theory–say quantum field theory to be specific. How to combine these models systematically with general relativity is simply not yet known. One would clearly have to deal with a quantum modification of general relativity, for example, the usual scalar curvature might somehow be replaced by an operator field. Many quantum theorists have argued that in a fully quantized theory gravity might even lose its special role as a carrier of spacetime geometry, becoming in effect little more than another form of "matter". This seems very unlikely to the reviewer, but time will tell.

More modest, and less fundamental, are current attempts to combine quantum matter theories with nonquantum general relativity, that is, to construct an approximation appropriate whenever quantum gravitational effects are negligible. One can take over the partial differential equations of special relativistic quantum field theory with comparative ease and comparatively little ambiguity. However, as is typical, the less local parts of the pre-general-relativistic theory are much harder to handle. For example, to decide whether gravity can create lots of quantum particles in a certain situation one must (presumably) be able to decide whether or not a given quantum matter field on a given spacetime corresponds to zero incoming particles. At the moment, the answers to this and a number of other similarly "global" questions are at best known only in a few special cases.

In any case, to describe stars and the other macroscopic objects of primary interest in the present, almost exclusively astophysical applications of general relativity, one normally uses drastically simplified matter models much less fundamental than those mentioned above. An example is one-particle kinetic theory. Roughly speaking, one there describes a gas as a collection of nonquantum, point particles, undergoing instantaneous collisions every once in a while; the influence of the gas on other matter, electromagnetism and gravity is approximated by smoothed out, average effects. This model is easy to take over into general relativity and is very useful there.

Many intentionally oversimplified matter models useful in nonrelativistic physics are even less microscopic. It is primarily with these that Bressan's book deals. He shows how one takes over into relativity various standard idealizations: the concept of a rigid body; the thermodynamic laws for fluids in which heat conduction can occur; the basic concepts of elasticity theory; and many others, including some rather detailed and specialized models (for example §78 of the book has the title *Magneto-elastic acceleration waves in Piezo-elastic ideal conductors*).

The treatment is very systematic and thorough. Some knowledge of classical tensor analysis and of nonrelativisitic material models is assumed. The author works with local coordinates throughout, usually making no attempt to consider situations where spacetime has a topology different from that of R^4. This classical approach is in fact rather appropriate for the subject matter.

The book's biggest drawback is its excessively formal character. Whether and how to take over a particular nonrelativistic, macroscopic idealization into relativity is only partially a question of whether the appropriate differential geometric formalism can be set up. To get a real sense of the uses and limitations of some model one also needs to analyze some specific physical situations to which the model is relevant and needs to investigate the model's relation to less phenomenological, more microscopic models. Such discussions are regrettably rare in the book. But within its own framework the book is highly competent. It will remain a useful reference for quite some time.

R. K. SACHS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 3, Number 1, July 1980

0002-9904/80/0000-0311/$02.25

Rational quadratic forms, by J. W. S. Cassels, London Mathematical Society Monographs No. 13, Academic Press, London-New York-San Francisco, 1978, xvi + 413 pp., $36.50.

The focal point of the book under review, the classification of quadratic forms over **Z**, can be formulated very simply. If

$$f = \sum f_{ij} x_i x_j \quad \text{and} \quad g = \sum g_{ij} y_i y_j$$

are nondegenerate quadratic forms in n variables $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ respectively, with coefficients $f_{ij} = f_{ji}$ and $g_{ij} = g_{ji}$ in **Z**, is it possible to determine whether or not f and g are equivalent over **Z**, i.e. whether or not there is a linear change of variables

$$y_j = \sum t_{ij} x_i$$

with (t_{ij}) an invertible matrix over **Z** which will transform g into f? This is closely related to the question of describing those integers that are represented by g, and to the more general question of which quadratic forms are represented by g over **Z**. All these problems can, of course, be formulated over any integral domain and not just over **Z**. In particular, they can be formulated over an arbitrary field where it can be shown, rather simply, that every quadratic form is equivalent to a diagonal form provided the characteristic of the field is not 2. If the field in question is **R**, then g is equivalent to a diagonal form

$$\sum_{1}^{r} x_i^2 - \sum_{r+1}^{n} x_j^2,$$

and r and n provide a complete set of invariants for equivalence over **R**. This is Sylvester's Theorem. It is the classification theorem over **R**. Forms over **R** with $r > 0$ and $n > r$ are called indefinite, with $r = n$ positive definite, and so on. Forms over **Z** are called indefinite if they are indefinite when viewed over **R**, and so on. It is important to make the distinction between definite and