J. S. can be something of a knockout if his themes get hold of you. And his influence on what followed was (you may say) substantial.

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Computational analysis with the HP 25 pocket calculator, by Peter Henrici, John Wiley, New York, 1977, 280 pp.

Compact numerical methods for computers: linear algebra and function minimization, by J. C. Nash, John Wiley & Sons, New York, 1979, x + 227 pp., $27.50.

LINPACK: User's guide, by J. J. Dongarra, J. R. Bunch, C. B. Moler, and G. W. Stewart, Society for Industrial and Applied Mathematics, Philadelphia, 1979 368 pp., $14.00 list, $11.20 SIAM members.

For sometime beginning in the 1930's, Mathematics and Electrical Engineering had a fruitful liaison. From this liaison a new discipline, sometimes called Information Processing but more often Computer Science, was born. The infant discipline grew rapidly in strength and knowledge, and before long decided to set up its own establishment. Computer Science continued to prosper in this office-home which is today a model of industry and affluence. In it may be found many tools adapted from instruments invented by its parent disciplines, the ardor of whose liaison has meanwhile cooled.

The mathematical tools of Computer Science include a new concept of *real arithmetic*. The set of 'real numbers' in a computer is finite, and disjoint from that of 'integers'. 'Integer arithmetic' is exact, but 'real arithmetic' almost never is; thus in it, $(5/7) \times (7/5) \neq 1$. For this reason, one cannot test 'real' vectors (called 'arrays') for linear dependence on computers. Hence one cannot be sure in general whether a given matrix is singular, or just has a very large 'condition number'.

By way of compensation, many computers can be programmed to solve accurately in seconds 150 simultaneous equations in as many unknowns, at a cost of a dollar or so–and can find all the eigenvalues and eigenvectors of typical 80×80 matrices to eight significant decimal digits with equal facility. Truly enormous advances have been made in understanding computer arithmetic since von Neumann and his collaborators began to analyze probabilistically how many binary digits (or 'bits') had to be allocated to each 'real' number, in order to achieve eight digit accuracy in the final result. (In practice, the answer seems to be from 32 to 64.)

In improving its mathematical tools, Computer Science has received indispensable advice from its older half-sibling, Numerical Analysis, another child of Mathematics produced by earlier liaisons, but which has kept in much closer touch with its senior parent.

Due primarily to this advice, many ingenious tools for minimizing and maximizing functions are also available, as are programs for solving APPROXIMATELY a large variety of initial and boundary value problems intractable by the methods of Classical Analysis.

Thanks to the development of libraries of 'portable' Fortran software, usable on many makes of computer, the tools I have mentioned are becoming easier and easier for us mathematicians to use. Therefore it seems high time for us to become better informed about how at least some of our computational problems can be solved automatically.

Each of the three books under review should help to make this information available to mathematicians. But here their similarity ends: each is aimed at a different audience, and each has its own special philosophical, mathematical, and technical flavor.

Henrici's book is written for the "creative student" of any age, especially for gifted amateurs having a taste for constructive complex analysis as presented in his two-volume *Applied and computational complex analysis*. His new book shows by example how one can execute many algorithms of 'constructive analysis' by a programmable pocket computer with an 'assembler' language of about 6 words and having 60 words of storage. One can factor quartic polynomials, exponentiate and find reciprocals of power series, compute Bessel functions, and locate zeros of the Riemann zeta function on the critical line. The creative student of constructive analysis can learn much about good programming and good mathematics by trying to write and test programs for other algorithms in a similar style.

Nash, on the other hand, concentrates on algorithms for linear algebra and function minimization, scrupulously avoiding commitment to a particular programming language or size of computer. He presents his algorithms in English, leaving to the student the burden of translating them into debugged Fortran, Algol, Pascal, APL, or Basic. Although his exposition is very lucid and informative, it leaves the reader with a curious sense of being a spectator and not a participant.

The *LINPACK user's guide* is at the opposite extreme. At its core are 129 pages 'listing' 50 subroutines. These are written in standard Fortran and are also available on magnetic tape, hence easily executable at most computing centers. The *Guide's* main purpose is to document these programs. Written for computing professionals, it has the same sparkle as a manual explaining to expert repair mechanics the workings of an automobile engine.

Yet it is an important and meaty document, giving an authoritative picture of current mathematical software technology. Its programs have been exhaustively tested at a number of computing centers on a variety of machines, and can be certified as optimal (in the present state of the art) for solving many problems of linear algebra. Many millions of dollars of computing and programming time will be saved by using them! (The best direct and iterative methods for solving discretizations of linear elliptic boundary value problems are *not* included, however.)

Though the three books differ in most respects, they share a weakness: none of them points out the *limitations* of the methods that it explains. Thus the *LINPACK user's guide* nowhere mentions the fact that the rank of a general matrix cannot be computed in 'real arithmetic'; Nash does not comment on the possibility that his minimization algorithms fail for some functions (e.g. for $2x^2 - \pi x - 5\exp[-(10^3x)^2]$). Even Henrici, whose skillfully written programs would surely have delighted Euler or Gauss, fails to observe that the coefficients of his power series are not expressed on a computer as rational numbers, hence cannot easily be recognized as generating functions.

By making allowances for this minor weakness, the curious reader who browses through these books can acquire a realistic picture of the state of numerical mathematics today.

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Topologie und analysis, by Bernhelm Booss, Springer-Verlag, Berlin, Heidelberg, New York, 1977, xiv + 352 pp., $17.50.

The Atiyah-Singer index theorem; *an introduction*, by Patrick Shanahan, Lecture Notes in Math., vol. 638, Springer-Verlag, Berlin, Heidelberg, New York, 1978, v + 224 pp.

At the beginning of the twentieth century, mathematics had been greatly enlarged by the ideas of a number of giants, including Riemann, Cantor, Poincaré and Hilbert. Considerable effort was then expended on understanding and developing the whole new areas of mathematics which had been created. And this plus the general trend toward axiomatization meant that a parochial view largely dominated mathematics. In the last couple of decades, however, that has changed and some of the most exciting developments have come from the interaction, often in unexpected ways, of different parts of