REFERENCES

1. F. A. Bogomolov, *Classification of surfaces of class* VII_0 *with* $b_2 = 0$, Math. USSR-Izv. **10** (1976), 255–269. See also the review by M. Reid in Math. Reviews, vol. 55, MR 359.

2. M. Inoue, S. Kobayashi and T. Ochiai, *Holomorphic affine connections on compact complex surfaces*, J. Fac. Sci. Univ. Tokyo (to appear).

SHOSHICHI KOBAYASHI

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 2, Number 1, January 1980

0002-9904/80/0000-0012/$01.75

Principles of algebraic geometry, by Phillip Griffiths and Joseph Harris, Wiley, New York, 1978, xii + 813 pp., $42.00.

Algebraic geometry, as a mutually beneficial association between major branches of mathematics, was set up with the invention by Descartes and Fermat of Cartesian coordinates. Geometry was as old as mathematics; but it was not until the seventeenth century, more or less, that algebra had matured to the point where it could stand as an equal partner. Calculus too played a major role (tangents, curvature, etc.); in the early stages algebraic and differential geometry could be considered to be two aspects of "analytic" (as opposed to "synthetic ") geometry.

During the nineteenth century the horizons of the subject were expanded (to ∞!) by the development of projective geometry and the use of complex numbers as coordinates. Gradually, out of an intensive study of special curves and surfaces, the idea emerged that algebraic geometry should deal with an arbitrary algebraic subset of n-dimensional projective space over the complex numbers (i.e. a set of points where finitely many homogeneous polynomials with complex coefficients vanish simultaneously). This was the proper context for the working out of concepts like transformation groups and their invariants, correspondences, and "enumerative" geometry (how to count the number of solutions of a geometric problem).

In the middle of the nineteenth century, Riemann appeared on the scene like a supernova. His conceptions of intrinsic geometry on a manifold, topology, function theory on a Riemann surface, birational transformations, abelian integrals, and zeta functions, fueled almost all the subsequent developments. In the analytic vein, which is relevant to the book under review, some of the more prominent contributors have been Picard, Poincaré, Lefschetz, Hodge, Kodaira, and Hirzebruch. In particular Hodge and Kodaira used the theory of partial differential equations to establish basic results, some of which have not yet been proved otherwise.

It is not my purpose here to summarize the history of algebraic geometry (cf. **[D]**, **[Z]**), but rather to suggest that since it began algebraic geometry has been a prime exhibit of the unity of mathematics, an area where diverse methods from analysis, topology, geometry, algebra and even number theory have interacted in a marvellously fruitful way. Indeed, though the subject has sometimes grown in directions which seemed exclusively algebraic, geometric, or analytic, history teaches us that it will continue to flourish *only if* nourished by ideas from all the different fields.

It seems inevitable in mathematics that powerful methods will eventually be pushed to the point of excess, either of generality or complexity. In a period of less than fifteen years, Grothendieck, building on Serre's foundational work, introduced revolutionary concepts and techniques (enough, according to Dieudonné, to keep several generations of algebraic geometers occupied). In this comparatively brief time, the enormous energy of Grothendieck and his school resulted in a shelf-full of seminars and IHES publications whose sheer mass threatened to unbalance algebraic geometry. The inhuman task of working through thousands of pages made the essential contributions virtually inaccessible to anyone who was not in contact with Grothendieck, either first hand in Paris, or second hand in Cambridge, Princeton, or Moscow. This led to some resentment, grumblings about "empty generality", "fashions in mathematics", etc. The situation is now much improved. In the past five years a number of first-rate down to earth introductory texts have appeared (cf. [H], [S]), with the foreseeable consequence that algebraic geometry should once again occupy its proper place in the general mathematical consciousness.

It is perhaps not out of place here to mention recent indications that algebraic geometry may be applied in a nontrivial way to the real world (Yang-Mills theory, systems theory, . . .)

Among the above-mentioned introductory texts the emphasis has been largely algebraic. In the books of Mumford and Shafarevich various topics are treated with transcendental methods, so as to establish the importance of such methods in algebraic geometry. But before Griffiths' and Harris' *Principles of algebraic geometry*, there was no systematic gathering together of the modern transcendental methods–basically those of differential geometry on complex manifolds, like Hodge theory on Kähler manifolds, currents, Chern classes, residues etc.–together with extensive illustration of how they can be applied to particular situations. So this book could fill an essential gap in the presentation of algebraic geometry to the mathematical public. Overall, the book should be most successful. This is first of all because of the choice of topics–the authors have a firm grasp of what is fundamental. Secondly there is the underlying philosophy that, in the best tradition of the subject, emphasis must be placed on the interaction between general theory and particularly interesting examples. So it is that about half the book deals with the general theory of complex manifolds and algebraic varieties (interspersed with examples), while the other half has a wealth of applications to curves and their Jacobian varieties, surfaces, and finally (seventy pages!) to one example of a three-dimensional variety, the quadric line complex. There appears to be no mention of the general algebraic approaches of Weil, Zariski, or Grothendieck. The basic objects of study throughout are algebraic varieties of the most concrete kind–subvarieties of complex projective space.

The difficulties, didactic and otherwise, associated with writing a text on algebraic geometry are discussed in [H] and [S]. The potential student will not find that this book provides easy or smooth access to algebraic geometry as such. The authors say in their preface that it is a "presentation of some of the main results . . . not meant to be a survey of algebraic geometry, but rather designed to develop a working facility with specific geometric questions";

they do not use the word "introductory". The formal definition of algebraic variety first appears on p. 166: "An algebraic variety $V \subset \mathbf{P}^n$ is the locus in $\mathbf{P}^n$ of a collection of homogeneous polynomials $\{F_\alpha(X_0, \ldots, X_n)\}$" ("locus" is not defined). Turning to Chapter two to find out what an algebraic curve is, one is confronted in the first paragraph with terms like "Riemann surface", "Kähler manifold", "$\bar{\partial}$-Laplacian", "harmonic form", etc. A Riemann surface is defined on p. 15 to be a one-dimensional complex manifold; according to the preface, the book strives to be self-contained, so "complex manifold" is defined (p. 14); but "dimension" is not. The other terms have been defined earlier in the book in a summary of harmonic theory on compact complex manifolds. Before saying "As the reader is no doubt aware, laying the proper algebraic foundations for the subject of algebraic geometry is an all-consuming task" (p. 678), the authors might have reflected on the fact that in the books of Hartshorne, Mumford, and Shafarevich, the discussion of algebraic varieties begins on page one.

The claim of "self-containedness" is hedged somewhat in the introduction to Chapter zero: "it is tacitly assumed that the reader has some familiarity with the basic objects discussed". The reader had better be familiar, for example, with the fundamental tool of cohomology of sheaves; he or she is not likely to learn it from the brief discussion in Chapter zero (where it is carelessly stated that any short exact sequence of sheaves on a topological space gives rise to a long exact sequence in Čech cohomology). For the de Rham and Dolbeault theorems, he/she should also know that on a differentiable manifold, singular, simplicial and Čech cohomology agree. And lots more too. In fact the entrance requirement for this book is about two years of solid graduate level topology, differential geometry and complex variables.

Furthermore, the book has numerous local deficiencies which may adversely affect its value. It has obviously been written rather hurriedly. "Local" refers to misprints, inadequate references (internal and external), obscurities, and outright errors, which do not however vitiate the important results. Of course such slips must occur in a book this size, and an obsessive search for them would be fatuous. But it seems to me that they occur with disturbing frequency, to the point where a conscientious reader could become frustrated and even discouraged.

But this is not the right note to close on. Globally, *Principles of algebraic geometry* is an impressive scholarly work, not only as a compendium of basic analytic methods, but also as a guidebook to the vital geometric core of the subject. My advice to a student of algebraic geometry would be: "Start with one of the other introductory books, but have it in mind to get thoroughly familiar with this one. You will probably need a knowledgeable teacher to help you over the rough spots. If it makes you feel better, think of this book as a set of lecture notes, or even as a fantastic collection of exercises, with copious hints. This is very high quality mathematics; put forth the effort and learn as much of it as you can."

References

[D] J. Dieudonné, *Cours de géométrie algébrique*, vol. 1. Presses Universitaires de France, Collection Sup, Paris, 1974.

[H] R. Hartshorne, Review of D. Mumford's *Algebraic geometry* I: *Complex projective varieties*, Bull. Amer. Math. Soc. (N. S.) 1 (1979), 513–515.

[S] S. Shatz, Review of R. Hartshorne's *Algebraic Geometry*, Bull. Amer. Math. Soc. (N. S.) 1 (1979), 553–560.

[Z] O. Zariski, *Algebraic surfaces* (2nd supplemented edition), Springer-Verlag, Berlin and New York, 1971.

JOSEPH LIPMAN

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 2, Number 1, January 1980

0002-9904/80/0000-0013/$01.75

Differenzenapproximationen partieller Anfangswertaufgaben, by R. Ansorge, Leitfäden der angewandten Mathematik und Mechanik (LAMM), Volume 45, B. G. Teubner, Stuttgart, 1978, 298 pp., DM 29.80.

Exactly thirty years ago, when I was about to develop a serious interest in some numerical aspects of partial differential equations, two well-known mathematicians gave me the benefit of their deeper insights in the form of two predictions. "Digital computing machines will never successfully compete with analog computers. Their vaunted speed is no use, since they break down all the time" was one prediction. "The role of functional analysis in the theory of partial differential equations will always remain mostly decorative. The important ideas can equally well be expressed in the language of traditional analysis" was the second statement. The quaintness of those utterances in retrospect from 1979 came vividly to my mind when I was reading this book by R. Ansorge. Such a thorough and detailed investigation into the nature of finite difference methods would not now be considered a worthwhile effort if the first prediction had been right; and the book would not begin–as it does–with two sections entitled *Function-analytic formulation of initial value problems* and *The concept of a generalized solution*, if the language and methods of Functional Analysis had not, by now, deeply penetrated all work on partial differential equations.

There have been other widespread, more specific, predictions concerning trends in the numerical analysis of partial differential equations which would have pushed finite difference methods into the background, if they were true. One was that, as the available error estimates for these methods were, by necessity, always statements on the orders of magnitude only, rather than explicit realistic inequalities, they would be increasingly regarded as unreliable and worthless. Another one was the expectation that techniques of the Galerkin type, i.e., approximations in suitably constructed finite dimensional subspaces, such as those furnished by the finite element method would completely supersede the less flexible "old-fashioned" procedure of replacing derivatives by difference quotients in a grid.

For initial value problems, at least, as distinguished from boundary value problems, it is, however, still true that difference approximations are of paramount computational interest.

In the early history of this subject the name of Lewis F. Richardson stands out [5]. His grandiose scheme of an enormous staff of pencil pushing human computers numerous enough to solve with adequate accuracy the hyperbolic