of recursion theory. Broadly speaking, there are two possibilities. Firstly, we can try to generalise our recursion theory from $({}^{\omega}\omega)^m \times \omega^n$ to $({}^{{}^{\omega}\omega}\omega)^{\kappa} \times ({}^{\omega}\omega)^m \times \omega^n$, and so on. Once the "correct" definition of "recursive" has been made, many of the results considered in the book can be generalised, and Hinman provides several such generalisations. The second generalisation arises when we try to replace ω by a larger ordinal number. Not all ordinals α admit a reasonable "recursion theory". Those that do are called "admissible ordinals". Much is now known about such ordinals, and they play an important role in Set Theory as well as Recursion Theory–indeed some of the arguments employed in recursion theory on admissible sets have a distinctly set-theoretic flavor!

It should be said that, despite our brief reference to this section given above, Part C occupies fully one half of the book, and contains a vast amount of material. Indeed, as Hinman says in his Preface, this is the material of which his volume was originally intended to consist!

So how did I find the volume? Well, let me first of all admit to being a reluctant reader (of any serious text); as well as one with a marked tendency to miss all sorts of errors. Consequently, I read the book in a fairly "shallow" fashion, and gained a fairly good impression of an area in which I am not at all expert. Armed with a reasonable foreknowledge of basic recursion theory and set theory as I was, I found the going not too bad. But the book is plainly intended for the more dedicated reader, with most proofs given in some detail. My feeling (prejudice?) here is that the lone reader may well find the going heavy (I would have, had I tried to read through it in depth), so that it would be preferable to couple the reading with a series of seminars or discussions on the material. There is a large selection of exercises, distributed throughout the text, some easy, some not so easy, and some with hints. So as a "standard text" the book stands very well indeed.

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On uniformization of complex manifolds: The role of connections, by R. C. Gunning, Mathematical Notes no. 22, Princeton Univ. Press, Princeton, N. J., 1978, ii + 141 pp., $6.00.

In a celebrated inaugural address at Erlangen in 1872, Felix Klein defined geometry as the study of those properties of figures that remain invariant under a particular group of transformations. Thus, Euclidean geometry is the study of such properties as length, area, volume and angle which are all invariants of the group of Euclidean motions. In Klein's view, by considering a larger group one obtains a more general geometry. Thus Euclidean geometry is a special case of affine geometry. The latter in its turn is a special case of projective geometry. In any of these geometries, the group is relatively large. What Klein had in mind must be geometry of homogeneous spaces. For this reason, a homogeneous space G/H of a Lie group G is sometimes called a Klein space.

Elie Cartan and others extended Klein's Erlanger Program to include such geometric structures as Riemannian structures, affine connections and projective connections. On a Riemannian manifold, each tangent space has the structure of a Euclidean space while an affine connection defines how a tangent space, regarded as an affine space, should be developed onto the tangent space at an infinitesimally nearby point. In discussing projective connections, one replaces each tangent affine space by a tangent projective space. Thus, Klein's idea is still valid in differential geometry in the infinitesimal sense. Although a manifold itself may not be a Klein space, its tangent spaces are Klein spaces associated with a particular group of transformations.

Pseudo-groups of Lie-Cartan may be used also to unify certain geometric structures. A pseudo-group Γ of transformations of a space S is a locally defined transformation group so that if $f, g \in \Gamma$, their composition $g \circ f$ is defined to the extent that the range of f meets the domain of g. The pseudo-grop Γ_{A^n} of affine transformations on the affine n-space A^n consists of restrictions of affine transformations of A^n to open subsets of A^n. Every Lie group G has its infinitesimal version or linear approximation, called the Lie algebra of G. Similarly, every interesting pseudo-group of transformations has its infinitesimal version, which can be described as a sheaf of Lie algebras of vector field germs. In the sense that the classification of Lie algebras reduces to that of simple ones (which is, of course, far from being true), the classification of Lie algebras of vector field germs reduces to that of "primitive" ones, and this was essentially carried out by E. Cartan in 1902–1909. Thanks to joint efforts by many geometers in the 1960s, Cartan's results have been made precise with more elegant proofs.

Given a pseudo-group Γ one can speak of a Γ-structure. Thus, an affine structure on an n-dimensional manifold M is defined by a collection of coordinate charts $\{U_i, \varphi_i\}$ such that $\bigcup U_i = M$, $\varphi_i\colon U_i \to A^n$ and $\varphi_j \circ \varphi_i^{-1} \in \Gamma_{A^n}$. In other words, to give an affine structure to M is to cover M with coordinate charts in such a way that the coordinate changes are all affine transformations. Since an affine structure is locally indistinguishable from the global affine structure on A^n, it induces a natural flat affine connection, i.e., affine connection with vanishing torsion and curvature. Conversely, every flat affine connection induces an affine structure. The same can be said of other classical geometric structures. Schematically, we can summarize the situation as follows. (The second column generalizes the first and, in its turn, a special case, i.e., the flat case of the third.)

Classical Geometry	Γ- structure	Connection
Euclidean Geometry	(locally) Euclidean structure	Riemannian Connection
Affine Geometry	Affine Structure	Affine Connection
Projective Geometry	Projective Structure	Projective Connection
Möbius Geometry (Conformal Geometry)	Conformal Structure	Conformal Connection

So far, we have described "real" geometries on "real" manifolds. It is very natural to consider also their complex analogues with holomorphic geometric objects. Although Kähler or Hermitian geometry is an important complex analogue of Riemannian geometry, it is not a holomorphic analogue since it requires $d\bar{z}^1, \ldots, d\bar{z}^n$ as well as $dz^1, \ldots, dz^n$. (Although it is possible to build a holomorphic analogue of Riemannian geometry by considering holomorphic, nondegenerate, symmetric covariant tensor fields of degree 2, there are few (compact) complex manifolds admitting such tensor fields.) On the other hand, the concept of affine or projective connection extends more naturally to the holomorphic case. The subject of Gunning's lecture notes under review is precisely the study of holomorphic pseudo-group structures and holomorphic connections. Gunning has been exploring this subject for the past 15 years. Although several other people including Filmore, Scheuneman, Matsushima, Sakane and Vitter have made also important contributions to the theory of holomorphic affine connections and structures, the scope of the present notes is much wider.

We shall now go into some technical details of the notes. Gunning considers not only holomorphic affine connections but also holomorphic projective connections and holomorphic canonical connections. (By a canonical connection, he means a connection in the canonical line bundle.) These three types of connections and the corresponding pseudo-groups were not chosen at random. In Part I, he shows that, in dimension greater than 1, these and only these three pseudo-groups are (aside from the pseudo-group of all local holomorphic transformations) tangentially transitive in the sense that their linear isotropy groups are the largest possible group, i.e., the general linear group. Although the tangentially transitive pseudo-groups form a subclass of the class of primitive pseudo-groups, their classification is carried out here directly completely independent of Cartan's classification of primitive pseudo-groups.

In Part II, Gunning discusses holomorphic connections associated with each of the three tangentially transitive pseudo-groups. In contrast to the real differentiable case where the existence of a connection is a triviality, there is, in general, an obstruction to the existence of a holomorphic connection. Consider, for example, the case of a holomorphic affine connection. Given a complex manifold M covered by coordinate neighborhoods $U, V, \ldots$ with local coordinates $(u^1, \ldots, u^n), (v^1, \ldots, v^n), \ldots,$ we consider

$$c_{VU} = \sum \frac{\partial^2 v^i}{\partial u^j \partial u^k} du^j \otimes du^k \otimes \frac{\partial}{\partial v^i}.$$

Each c_{VU} is a holomorphic tensor field of covariant degree 2 and contravariant degree 1 defined on $U \cap V$. In other words, it is an element of $H^0(U \cap V;\ \Omega^1 \otimes \Omega^1 \otimes \Theta)$, where Ω^1 (resp. Θ) is the sheaf of germs of holomorphic 1-forms (resp. vector fields). Using the chain rule for several variables we can verify that $\{c_{VU}\}$ is a 1-cocycle and hence defines an element of $H^1(M;\ \Omega^1 \otimes \Omega^1 \otimes \Theta)$. This cocycle is cohomologous to zero if and only if there exists a 0-cochain $\{b_U\}$, $b_U \in H^0(U;\ \Omega^1 \otimes \Omega^1 \otimes \Theta)$, such that $c_{VU} = b_U - b_V$. If we write

$$b_U = \sum \Gamma^i_{Ujk}\, du^j \otimes du^k \otimes \frac{\partial}{\partial u^i},$$

then the equality $c_{VU} = b_U - b_V$ is nothing but the classical transformation law for the Christoffel symbols Γ^i_{Ujk}. This shows that a complex manifold M admits a holomorphic affine connection if and only if the element of $H^1(M; \Omega^1 \otimes \Omega^1 \otimes \Theta)$ defined by the 1-cocycle $\{c_{VU}\}$ is zero. This homological interpretation, admittedly tautological, is none the less useful in discussing holomorphic connections. For example, the vanishing of $H^1(M; \Omega^1 \otimes \Omega^1 \otimes \Theta)$ is a usable sufficient condition for the existence of a holomorphic affine connection.

For projective connections, the 1-cocycle $\{c_{VU}\}$ to be considered is given by

$$c_{VU} = \left[\frac{\partial^2 v^i}{\partial u^j \partial u^k} - \delta^i_j \frac{\partial \log \tau_{VU}}{\partial u^k} - \delta^i_k \frac{\partial \log \tau_{VU}}{\partial u^j} \right] du^j \otimes du^k \otimes \frac{\partial}{\partial v^i}$$

where $\tau_{VU} = \det(\partial v^i / \partial u^j)$. For canonical connections, one needs

$$c_{VU} = d \log \tau_{VU} \in H^0(U \cap V; \Omega^1).$$

An important necessary condition for the existence of a holomorphic affine connection is given by vanishing of Chern classes $c_i = 0$ for $i > n/2$, or more generally, $c_{i_1} \cdots c_{i_k} = 0$ for $i_1 + \cdots + i_k > n/2$, [2]. This condition can be strengthened to the vanishing of all c_i, $i > 0$, when M admits a Kähler metric. Similar necessary conditions are given for projective and canonical connections.

In Part III, Gunning discusses the existence problem for (flat and nonflat) holomorphic affine, projective and canonical connections on compact complex analytic surfaces. In the affine case, necessary conditions $c_1 = c_2 = 0$ eliminate very quickly many surfaces from the classification table of Kodaira. As one would expect, the surfaces of class VII_0, still not completely known, cause some difficulties. In fact, the argument on p. 113 is not complete. For the determination of compact analytic surfaces admitting holomorphic affine connections, we refer the reader to [2]. In the case of surfaces, it happens that the surfaces admitting holomorphic affine connections admit also holomorphic affine structures. In the projective case, a necessary condition $3c_2 = c_1^2$ disqualifies many surfaces while $c_1 = 0$ is a necessary and sufficient condition in the canonical case.

In his earlier and widely read notes, *Lectures on Riemann surfaces*, in the same Princeton Mathematical Notes series, Gunning has already discussed holomorphic affine and projective structures and connections on Riemann surfaces. He has shown there that a compact Riemann surface of any genus always admits projective structures while it admits affine structures only when the genus is one. In the present book, he shows that, in higher dimensions, even projective structures rarely exist. These structures are, none the less, of great interest. For example, the unknown surfaces of Class VII_0, if any, admit affine structures, [1], [2], and this fact may prove to be useful in determining all surfaces of Class VII_0.

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Principles of algebraic geometry, by Phillip Griffiths and Joseph Harris, Wiley, New York, 1978, xii + 813 pp., $42.00.

Algebraic geometry, as a mutually beneficial association between major branches of mathematics, was set up with the invention by Descartes and Fermat of Cartesian coordinates. Geometry was as old as mathematics; but it was not until the seventeenth century, more or less, that algebra had matured to the point where it could stand as an equal partner. Calculus too played a major role (tangents, curvature, etc.); in the early stages algebraic and differential geometry could be considered to be two aspects of "analytic" (as opposed to "synthetic ") geometry.

During the nineteenth century the horizons of the subject were expanded (to ∞!) by the development of projective geometry and the use of complex numbers as coordinates. Gradually, out of an intensive study of special curves and surfaces, the idea emerged that algebraic geometry should deal with an arbitrary algebraic subset of n-dimensional projective space over the complex numbers (i.e. a set of points where finitely many homogeneous polynomials with complex coefficients vanish simultaneously). This was the proper context for the working out of concepts like transformation groups and their invariants, correspondences, and "enumerative" geometry (how to count the number of solutions of a geometric problem).

In the middle of the nineteenth century, Riemann appeared on the scene like a supernova. His conceptions of intrinsic geometry on a manifold, topology, function theory on a Riemann surface, birational transformations, abelian integrals, and zeta functions, fueled almost all the subsequent developments. In the analytic vein, which is relevant to the book under review, some of the more prominent contributors have been Picard, Poincaré, Lefschetz, Hodge, Kodaira, and Hirzebruch. In particular Hodge and Kodaira used the theory of partial differential equations to establish basic results, some of which have not yet been proved otherwise.

It is not my purpose here to summarize the history of algebraic geometry (cf. **[D]**, **[Z]**), but rather to suggest that since it began algebraic geometry has been a prime exhibit of the unity of mathematics, an area where diverse methods from analysis, topology, geometry, algebra and even number theory have interacted in a marvellously fruitful way. Indeed, though the subject has sometimes grown in directions which seemed exclusively algebraic, geometric, or analytic, history teaches us that it will continue to flourish *only if* nourished by ideas from all the different fields.