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*Information and exponential families in statistical theory*, by O. Barndorff-Nielsen, Wiley, New York, 1978, ix + 238 pp., \$33.00.

If we take  $n$  samples of a random variable which is normally distributed with unknown mean  $\mu$  and known variance  $\sigma^2$ , then the sample mean  $\bar{X}$  is normally distributed with mean  $\mu$  and variance  $\sigma^2/n$ . Thus the probability that  $\sqrt{n}(\bar{X} - \mu)/\sigma$  lies between  $-1.96$  and  $1.96$  is, from the standard normal table, equal to  $.95$  and hence it seems that the probability that  $\mu$  lies between  $\bar{X} - 1.96\sigma/\sqrt{n}$  and  $\bar{X} + 1.96\sigma/\sqrt{n}$  is also  $.95$ . The latter statement, which seems to the naive reader to be equivalent to its immediate predecessor, makes no sense since the fixed number  $\mu$  either lies in the given interval (a 95% confidence interval) or it doesn't and most elementary books on statistics take some pains to explain this fact. What the statement really means is that if we use this method over and over again to infer that  $\mu$  is in the given interval we will be right about 95% of the time. Unfortunately this limit of frequency concept coincides with most people's intuitive idea of probability and very likely was used that way earlier in the same book. The same situation obtains in the more realistic situation of an unknown  $\sigma$  except that the  $t$ -distribution is used instead of the normal and many other statistical problems lead to this sort of impasse.

The apparent duality here; each choice of  $\mu$  determining a distribution of  $\bar{X}$  and each sample mean  $\bar{X}$  seeming to determine a distribution of  $\mu$ ; has led to a great deal of heated dispute among statisticians, who tend to be rather feisty anyhow. The first great storm center seems to have been the fertile brain of R. A. Fisher. Fisher's idea of fiducial inference is not much mentioned anymore but the debate goes on as the book under review illustrates.

The book is divided into three parts the first of which is concerned, among other things, with the above mentioned duality. In fact,  $x$  serves as the probability variable while  $\omega$  is reserved for the parameter. This emphasizes the duality perhaps at the expense of occasionally confusing the over-conditioned probabilist. Barnard's notion of l.o.d.s. functions is introduced but not extensively developed. The likelihood function and Barndorff-Nielsen's somewhat controversial plausibility function seem to be the only well-known examples. Much of this part is devoted to the, more or less, dual notions of sufficiency and ancillarity. In fact four different definitions are given of each,

$B$ -sufficiency and  $B$ -ancillarity corresponding to the usual concepts. Several reprinted talks with accompanying discussions on these subjects have appeared lately. One, by the author, is in *J. Roy. Statist. Soc. Ser. B* **38**(1976) no. 2, 103–131 and the others in which he participated can be found by consulting the *Math. Rev. Indices* which list discussants as well as authors.

A short middle section gives some special mathematical results and leads to the final part on exponential families. There the regular theory of exponential families is developed and a number of examples are given. The existence and uniqueness of maximum likelihood and maximum plausibility estimates is discussed as is prediction by both methods for these families. The final chapter then deals with the existence and character of sufficient and ancillary statistics for exponential families.

The mathematical and statistical prerequisites for the book are modest and most of the proofs are pretty straightforward. A little measure theory and a standard senior statistics course should suffice. This does not mean, unfortunately, that the book is easy to read. The ratio of definitions to theorems is very high as is common in this subject. This makes for a lot of cases in which it is easier to follow the proof than it is to figure out what the theorem really says. This is mainly a problem of attention span (the reviewer's is somewhat below average) but is a problem.

On the positive side, the author provides many good examples, i.e. examples which are relevant to current statistical practice while also illustrating the points involved. Each chapter contains a complements section and a notes section which between them tell a great deal about who did what, how it could be generalized, and who did it differently. All in all, an interesting book for the specialist but a little heavy going for the general mathematical public.

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*The collected works of Harold Davenport*, edited by B. J. Birch, H. Halberstam, and C. A. Rogers, Volumes I, II, III, and IV, Academic Press, London, New York, San Francisco, 1977, xxxiii + 1910 pp., \$30.35, \$29.35, \$34.25, \$20.35.

The *Collected Works of Harold Davenport* fill four handsome volumes. Three photographs and the facsimile of a handwritten page form the frontispieces.

Almost a decade has passed since Davenport died, but few people realize that, in part, perhaps, because some of his work continued to appear as late as 1975.

As one starts browsing through the almost 2000 pages, one of the first impressions is that of an unusual fluency of the exposition not only in English, but also in German and French. When one then looks at the frontispieces one remembers the man (most of us will remember him like the photos of volumes 3 and 4) and recognizes his handwriting (at least this