FC-group (one in which the conjugacy classes are finite), and on normal subgroups of $U(\mathbf{Z}/n\mathbf{Z}[G])$.

The third chapter of the book concerns the isomorphism problem discussed earlier, i.e. the problem of determining when $RG \cong RH$ implies $G \cong H$. Many basic results are discussed, but Dade's important example is unfortunately omitted. Chapter IV deals with the related problem of uniqueness of the coefficient ring: does $RG \cong SG$ imply $R \cong S$? As for the isomorphism problem, it is easy to see that the answer is negative in general, but one may still search for special conditions under which it becomes affirmative. Since little is known about this problem, the author confines his attention to the case where $G = \langle x \rangle$ is infinite cyclic. Hence, RG in this case is the ring $R\langle x \rangle = R[x, x^{-1}]$ of Laurent polynomials over R. For this special context, some results of Sehgal and Parmenter are presented, which show the answer to the uniqueness problem to be affirmative for some special classes of rings (perfect, commutative von Neumann regular, commutative local and a few others).

Further, there is a chapter on Lie properties of KG. Here, KG is viewed as a Lie algebra by the usual device of defining $[a, b] = ab - ba$. One may then ask for conditions that KG be solvable, nilpotent or whatever, when viewed as a Lie algebra in this fashion. As a sample, we cite the theorem of Passi, Passman and Sehgal: let K have characteristic $p \geqslant 0$. Then KG is Lie solvable if and only if G is p-abelian, if $p \neq 2$ or $p = 2$ and G has a 2-abelian subgroup of index at most two. (Here, G is called p-abelian if its derived group is a finite p-group; 0-abelian if it is abelian in the usual sense.)

The book concludes with a compilation of research problems which were stated at various points in the text. Forty-two such problems (of clearly variable difficulty) are nicely organized, with comments showing the connections between them. This chapter should be very useful for researchers in the field (especially beginners), and the author is to be congratulated for providing it.

The entire book is well written and carefully organized. It is inherent in the material that some of the proofs are computational and somewhat boring, but the dilettante can easily skip over the tedious parts, and follow the flow of ideas. The format and typography are uninspired, but straightforward enough to be undistracting. In all, this book is quite pleasing, as specialized works go, and I think that anyone with any interest in group rings will find some valuable nuggets in it.

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K-theory, an introduction, by Max Karoubi, Springer-Verlag, Berlin, Heidelberg, New York, 1978, xviii + 308 pp., $39.00.

What is a real vector space of dimension -2? What is an abelian group of order $1/3$? Assuming that the reviewer retains some measure of sanity, which has not been proved and is beginning to seem doubtful, are these concepts meaningful and useful?

One is fully aware that in antiquity the concepts of negative number, zero, and fractions were not fully understood. We now understand these notions, presumably. Why is it any less reasonable to interpret having a vector space of dimension -2 as giving an IOU for a 2 dimensional vector space than it is to consider having -2 dollars as being in debt?

Having now given the entire program away, how do you do these things? You begin with a collection in which you have some notion of addition or multiplication and formally introduce inverses to get negatives or fractions. The collection is forced to form a group, just as you create the integers from the natural numbers. You need to be a bit careful or you may lose everything. If you considered all vector spaces over the reals then $R^n \oplus R[x]$ is isomorphic to $R[x]$, and R^n would be equivalent to $R[x] - R[x] = 0$.

What is amazing is that the modern use of these ideas waited for the study of coherent algebraic sheaves by A. Grothendieck where the group obtained from sheaves was used to prove a version of the Riemann-Roch theorem (see **[6]**). A topological version of these ideas was presented by M. F. Atiyah and F. Hirzebruch **[3]** and the subject quickly exploded into a major tool of modern topology.

In differential geometry, one of the important global ideas is the notion of the tangent vector bundle of a smooth manifold. If one is given a compact manifold M^n, then one can find an imbedding of M in a Euclidean space R^N, and the collection of normal vectors also forms a vector bundle of fiber dimension $N - n$. This bundle is called the normal bundle (of the imbedding) of M, and in any attempt to eliminate dependence of the normal bundle on the imbedding, one is almost forced to consider "the" normal bundle of M as a formal difference–the tangent bundle of a Euclidean space minus the tangent bundle of M–which is the inverse except for a dimension choice.

For Atiyah and Hirzebruch, the starting notion was that of a complex vector bundle. For complex vector bundles over a compact Hausdorff space X, the resulting group of classes was denoted $K(X)$. The impressive power of their work comes from the fact that the functor assigning to X the group $K(X)$ is actually part of a cohomology theory $X \to K^*(X)$ based on the Bott periodicity theorem. Now the tendency is to proceed in the opposite direction and derive Bott periodicity as a consequence of the calculation of $K(S^2)$.

For a compact space, the real (complex) vector bundles over X can be interpreted as being the finitely generated projective modules over the algebra $C(X)$ of continuous real (complex) valued functions on X. (The conditions just say one has a direct summand of a finite number of copies of $C(X)$ with finiteness keeping you out of the $R[x]$ problem and the direct summand making your inverse exist as the complement except for a dimension choice.) This was noted by R. G. Swan **[10]**. Since that time, the idea of forming a group of modules over a ring has come to be a major tool in algebra, and has led to an entire area of research called Algebraic K-theory (see **[5]**). Much work has been directed at finding the good analogue of $K^*(X)$ (see **[9]**). As an aside, one should note that the original Grothendieck work on sheaves is really the analogue in algebraic geometry of the vector bundles of topology, so algebra started it all.

In analysis, these ideas have also begun to appear. The work of M. F. Atiyah and I. M. Singer [**4**, and much more] on the index of elliptic operators is all phrased in terms of the K-theory of vector bundles over X. The idea is that $f\colon V \to W$ with V, W infinite dimensional can easily have $W - V = \operatorname{coker} f - \ker f$ finite dimensional. For the functional analyst, the work of L. G. Brown, R. G. Douglas, and P. A. Fillmore [**7**] has shown that many questions of operator theory are conveniently studied in terms of the homology theory $K_*(X)$ dual to $K^*(X)$, with a general operator algebra being treated as an analogue of $C(X)$.

These examples indicate places where the concept of formal inverse is truly useful in many areas of mathematics. Phrasing problems in terms of a group formed from a class of objects having no genuine inverses permits the methods of algebra to come to bear on the problems while frequently retaining the essentials of the questions. Obviously, if any group valued invariant will solve your problem, this one has to work, because it contains the most information.

This book is basically an exposition of the standard material of K-theory as it appears in topology. One first must describe precisely what vector bundles are and then define the K-theory of a category (i.e., the way you formalize the group you get by creating inverses). Being interested in topology, one then discusses Bott periodicity, and computes the K-theory of standard spaces. To justify it all, you conclude with applications–particularly the elegant K-theoretic solution to the problem of finding the maximum number of linearly independent vector fields on spheres (Adams [**1**]).

The most obvious new features of the presentation are the author's notion of taking the K-theory of a Banach category (for vector spaces, $\operatorname{Hom}(V, W)$ is a vector space, and if you generally insist that $\operatorname{Hom}(X, Y)$ for your category have a lot of structure, then the K-groups of the category are richer) and the notion of K-theories $K^{p,q}(\mathcal{C})$ based on Clifford algebras à la M. F. Atiyah, R. Bott, and A. Shapiro [**2**] to do real Bott periodicity (instead of just using Clifford algebras, actually construct K-theories for these algebras and analyze their relationships). These techniques have been exploited in the author's papers and are certainly part of the author's contribution to the subject.

Generally, my impression of the book is quite favorable. It seems, for example, to be quite well written and polished. I'm not at all sure that there was a burning need for this book, however. A number of books are available to provide considerable overlap as text and the original papers are an excellent place to learn this material. The obvious competitor is Husemoller's book [**8**] which has the advantage of covering more of the related topological ideas, although most topologists tend to disparage it.

I do think the author has perpetrated a number of minor travesties. "Quasi-vector-bundles" and "pseudo-abelianizations of additive categories" are unfortunate. The historical notes are mostly "who proved what first", and don't add a lot. Finally some of the problems are a bit overwhelming, as for Example 9.27 in Chapter I for which it would be desirable to have read

Chapter IV. I must say, however, that no book is ever without some problems and this book seems far better than average. A graduate student in topology would gain a lot from reading this book and wouldn't suffer too much. He would probably need to consult some other sources, which wouldn't be hazardous to his education.

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Modern methods in partial differential equations, an introduction, by Martin Schechter, McGraw-Hill, New York, 1977, xv + 245 pp.

In the theory of linear partial differential equations, one is given an equation of the form

$$Pu = \sum_{|\alpha| \leq m} p_{(\alpha)}(x) D^{\alpha} u = f, \qquad x \in \Omega, \tag{1}$$

generally supplemented by boundary conditions or one or more hypersurfaces in Ω, and one asks questions about the solutions of (1), typically in one of the following three categories:

(2) Existence.

(3) Uniqueness.

(4) Qualitative behavior.

The last category is quite broad; one is asking what the solutions look like. One wants to know "everything" about them, ideally; such properties as regularity, propagation of singularities, and estimates in various norms are special cases, but of course endlessly more questions arise, such as behavior of nodal sets, decay of solutions, location of maxima, limiting behavior under (possibly quite singular) perturbations of the equation or the boundary, spectral behavior of P, and many more.